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A decision aid for me, Neolithic man and other impaired decision makers

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A Decision Aid for Me, Neolithic Man and other Impaired Decision Makers.

An exploration of a data envelopment approach to the analysis of decisions and its usefulness under assumptions of cognitive impairment.

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Summary

This thesis concerns the development of a family of normative multiple attribute decision analysis models, which owe their inspiration to the ideas of Data Envelopment Analysis, and which are styled by the acronym DORA-D. Their use has been put in the particular context of the cognitive facility of decision makers and, in particular, the argued impairment of decision makers across a range of decision related skills. These include the ability to articulate objectives, the ability to discriminate and the lability of preference and value, the ability to compute value, the ability to express preference between choices with variation in several attributes and to trade-off differences, and facility with cardinal probability. Although, the approach developed can accommodate a range of methods of value elicitation, the author has been concerned that it can accommodate minimalist assumptions of facility. The use of the ideas of Evolutionary Psychology as a "touchstone" for judging and balancing these assumptions, is examined.

The style of the work is one of a personal exploration, based on a personal problem, the author's own investment decision making. However, his preoccupation has been with the development of devices, and the adoption of different perspectives to decision analysis problems, which are useful to other analysts on a broad range of problems, and which can be taken further by other researchers. Various extensions to the technique and supplementary devices, some of which may be useful in association with other decision aids, are suggested. These include the concept of Fundamentally Decomposed Preference, and a simplified approach to the analysis of configural problems.

Various simulations, testing the efficacy of alternative elicitation mechanics in association with the techniques, are reported.

Key Words

decision analysis; multiple attributes; data envelopment analysis; principles of modelling; objectives; preference; value; value function; value lability; evolutionary psychology; cognition; rationality; linear programming; non-linear programming; portfolio analysis; modern portfolio theory; decomposition; efficiency; problem structuring; configurality; prudential algebra.

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Glossary

$\succsim$	Is not less preferred than or Is weakly preferred to
$\succ$	Is strictly preferred to
$\sim$	Is indifferent between or Is of equal value to
[m,n] choice	A binary pair of choices where m+n attributes differ between the two choices and one choice is superior with respect to m attributes and the other choice superior with respect to the other n attributes. The special case of [1,1] choice is also referred to as Fundamentally Decomposed Choice.
adaptation	Principally used with its biological/EP meaning: that is the process by which generations of an organism become better suited to their environment by natural selection, or a function, feature or trait generated by such process.
ancestral environment	The environment in which the mental facilities of the human mind evolved beyond those of other primates, to its present competence. More formally, the Environment of Evolutionary Adaptation. Corresponds to the Pleistocene (2mya) and before, up to the Neolithic period (10,000 ya) by which it is sometimes stylised here.
Attribute	A quantitatively measurable or classifiable property of a decision option or choice whose magnitude is related to the quality or value of the option or choice.
Best Dominated Choice	BDC. An artificial or virtual choice defined by magnitudes of attributes of a set of efficient options. The BDC is defined by a set of attribute magnitudes (monotonically increasing with decision "goodness") such that each attribute magnitude is at the maximum value consistent with it being dominated by *all* specified efficient options. The vector of nadir attribute values.
Beta	A measure of non-diversifiable risk, reflecting the degree to which the risk associated with an individual investment is correlated with the risk of a "portfolio" consisting of all investments in the market.
BPL	Best Possible Light
CAF	Comparative Advantage Function. The value function that gives rise to the MCA. It is the Value Function that shows an option in its Best Possible Light.
CAPM	Capital Asset Pricing Model
CCR	The Charnes, Cooper, Rhodes DEA model
Choice	1. A set of Attributes not necessarily corresponding to an Option but which can be valued, or over which a decision maker can express a preference relative to another Choice. A Choice in this use has the same characteristics as an Option but may be an artificial and not directly implementable package of attributes. 2. A binary pair or set of Choices over which an expression of preference is sought.
Comparison Set	The set of options or choices that are *explicitly* included in analyses to define Frontier Constraints, which constrain the value function or to limit the CAF of an option under evaluation. Frequently the option under evaluation is excluded from the Comparison Set. Options that are already established as not potentially optimal may be excluded from the Comparison Set to speed LP execution, or to minimise distortions if mildly mis-specified non-linear valuation is treated as linear.
Complexity Indicator	A binary pair of integers [m,n] describing the number of attributes differing for a pair of choices. See [m,n] choice.
Configurality	1.The dependence of preference and decision value on the inter-relationship or configuration of attribute values not just independent magnitudes; embracing Conjunctive and Disjunctive decision making and "cross-product" interactions. 2. The value of the parameter r in the General Configural Model.

Criterion Space	The domain defined by the parameters of a value function. In a linear model relating attributes to value, it is the hyperspace defining all possible weights of the attributes. There is a mapping between Criterion Space and the "hypersurface" of Decision Space containing efficient options, insofar as a point in Criterion Space will define an efficient option. However, a specific efficient option will correspond to a region within Criterion Space, not generally a point.
DEA	Data Envelopment Analysis.
Decision Maker	A person or group of people with the authority to select an option and whose values, or interpretation of other stakeholders' values, determine the selection that is made. The term embraces all groups of people involved in collective consideration of decisions and includes committees, Boards, and situations in which one person or group of people make proposals to another. In all instances such a group is considered as a single entity ultimately having or behaving as if it had a single mind. This thesis does not concern itself dynamics of such groups or with preference conflict resolution within them, treating them as a black box. Singular pronouns are invariably used, whilst recognising that only exceptionally is a decision exclusively made by one person.
Decision Space	That hyperspace defined by feasible decision attribute values. In the case of discrete alternatives it is the convex hull containing all feasible options.
Decomposition	The process by which a single Option or group of Options may be translated into derivative Choices over which a Decision maker can more reliably express preferences in a manner which allows the selection of an Option or Reduction in the potentially optimal Options. Usually one or more [m,n] options are translated into choices of Reduced Order.
DORA-D	Contracted acronym for Decision Option Reduction Analysis using concepts from Data Envelopment Analysis.
Efficient Peer	... of an inefficient option. The efficient option/s from amongst all efficient options that have the highest valuation when evaluated using the CAF of the inefficient option.
EP	Evolutionary Psychology
Facet	A group of efficient options or choices that can be simultaneously co-optimal. All facets are efficient, though this adjective is sometimes used as a reminder.
Franklin Decomposition	The derivation, from a pair of efficient choices, of a set of [s,1] choices (where s is a low integer) over which preference may be expressed. Its suggested use in Dora-D is, thus far, confined to [1,1] decompositions. Distinguished from Larichev Decomposition.
Frontier Constraint	A representation within a mathematical programming formulation that limits the coefficients or other parameters of a value function, arising from a requirement of Dora-D that no choice within the Comparison Set, or portfolio constructed therefrom, may within the method be assigned a value of greater than an arbitrary number (in this thesis,1)
Frontier Probing	A methodology for establishing value functions which would result in valuations of feasible portfolios which are not permitted though not yet prevented in the model, and specifying explicit constraints to avoid continuing violation. A metaphor for the process is that the existing constraint envelope or "frontier" is "probed" for violating situations and, when found, the hole in the frontier is plugged by a new constraint.

Fundamentally Decomposed Choice/ Preference, FDC, FDP	FDC. A binary pair of choices (either existing or generated by decomposition) such that the value of all but two attributes are equal for both choices and where one choice is superior with respect to one attribute and the second choice is superior with respect to the other. Also classified as a [1,1] choice. Such choices are considered here to be potentially the most meaningful given the mental competencies of human decision makers. Fundamentally Decomposed Preference, FDP, embraces the process.
General Configural Model	A valuation model in which attribute magnitudes are related to value by a model of the form $\sum a_i x_i^r$ a strategically equivalent valuation to that given by the Minkowski metric $V(X) = (\sum a_i x_i^r)^{1/r}$.
he, she, him, her, his, her	Generally I have sought to reflect that Decision Makers and Analysts are men and women. Occasionally, where I have felt that this would lead to clumsy communication, or inadvertently, I have used terms of either gender but in all instances a gender non-specific pronoun is implied. In illustrating some aspects of Evolutionary Psychology a sexual distinction, apparent from the context, is intended.
Holistic Integration	The selection of a desired solution from decision options, after consideration of attributes of those options with intuitive or only limited conscious processing of the information available.
impaired, impairment	Related to any general inability of the unaided human mind to comprehend or process information relevant to a decision. Does not imply abnormality.
Information Gain	A measure of the reduction of option variety secured by analysis and preference elicitation, based on the number of remaining potentially optimum options compared with the original number of options. See Chapter 9 for definition.
Initial Option Reduction	The process, within the methodology developed here, by which all options are decreased to a sub-set of potentially optimal options, depending only or largely on information relating to the magnitudes of the attributes of the options, excluding, or largely excluding, information relating to a decision maker's preferences.
Larichev Decomposition	The derivation from a set of efficient options of a set of [1,1] choices over which preference may be expressed to enable the reduction in the efficient options.
Latitude	That variation in the specification of a value function that can be sustained, without inconsistency with the explicit preferences of the decision maker. Within a linear programming model it can be operationalised as that portion of Criterion Space that is feasible with respect to preference constraints.
LP, NLP, MOLP, MIP, MP	Linear Programming, Non-Linear Programming, Multiple Objective Linear Programming, Mixed Integer Programming, Mathematical Programming
Maximal Efficient Choice	A set of attribute magnitudes (monotonically increasing with decision "goodness"), where all attribute magnitudes correspond to those of the Best Dominated Choice for a specified set of efficient options with the exception of only one attribute. The magnitude of the excepted attribute is equal to the highest magnitude occurring for that attribute amongst the specified set of efficient options. Any binary pair of a set of Maximal Efficient Choices will be a [1,1] choice.

MCA MCA(AP) MCA(OI)	Maximal Comparative Advantage. A valuation of an Option or Choice using that value function, of all those permitted within the valuation Latitude, which gives that Choice the highest value relative to the value of the best alternative Choice/s in the Comparison Set, measured using the same function. In this thesis the value of the best of the Comparison Set (usually excluding the option under consideration), is arbitrarily assigned a value of 1. Also referred to as BPL valuation. Analogous to Efficiency or Super-efficiency in DEA. The suffix (AP) refers to "under Andersen-Petersen conditions" where the option under evaluation is explicitly excluded from the Comparison Set. (OI) indicates that the option under evaluation is within the Comparison Set absolutely constraining the MCA to1.
Mechanic	A particular process, device or procedure governing the form in which preference or value information is elicited from a decision maker and represented as constraints in an analytic model. It is ascribed specialist meaning in distinction from the more general term "mechanism".
Modified Minkowski Metric	The function $\sum a_i x_i^r$ used in the General Configural Model.
MPT	Modern Portfolio Theory
Option	An object, action or policy, defined by Attributes, that may be selected by a decision maker for implementation. Distinguished from Choice.
Performance Gain	A measure of the expected value of remaining potentially optimum options, related to the value of the optimum and the value of all options. See Chapter 9 for definition.
Portfolio	A decision defined by selections from inter-dependent sub-options where more than one such sub-option is required to be simultaneously selected.
Preference Bracketing	The specification of an interval with respect to a variation in the magnitude, x, of one attribute A, given a specified alternative favourable movement, y, in a second attribute B, *ceteris paribus*, such that a decision maker is confident that he or she will prefer x to y for any x above the upper bound and will prefer y to x for any x below the lower bound.
Preference Constraint, Value Constraint	A representation within a mathematical programming formulation that limits the coefficients or other parameters of a value function, constructed following the expression by a decision maker of a preference between Choices or Options, by an expression of relative value of Choices, or by arbitrary limitations on parameters reflective of a decision maker's intentions. Contrasted with Frontier Constraint.
Preferential Independence	Here implies Mutual Preferential Independence unless otherwise stated. The property by which the preference for one choice over another, differing only in the magnitudes of a sub-set of attributes, is not affected by the magnitudes of the non-differing attributes.
Reason	Process by which people assess information by connected thought, ie draw inferences by conscious deliberation.
Reduced Order, Higher Order	Reduced Order refers to the characteristic where a binary [m',n'] choice has m'<m and n'≤n when compared with a choice classified [m,n]. Higher order is the converse.
Reduction	The process by which decision options are progressively shortlisted by excluding those which cannot be optima within constraints specified by a decision maker and incorporated in the analysis model.
Reference Set	Those other options constraining the upper valuation of a particular option. Also referred to as the Peer Group.

Representative Efficient Set	A sub-set of all efficient options which, if it does not embrace the optimum option, embraces an option which a decision maker could not realistically be expected to distinguish from an optimum decision on value grounds, given his/her mental competencies and potential preference lability. It is a set such that at least one member has an MCA above a threshold level of meaningful distinction under all feasible CAFs.
Review group	The set of shares considered by the writer for inclusion in his investment portfolio.
SEU	Subjective(ly) Expected Utility.
spandrel	A genetic concomitant of an adaptation not itself having adaptive value.
Strategic Equivalence	... of value functions. The property possessed by functions which give rise to the same ranking by value of all possible options and where all instances of indifference under one function are also evaluated as indifference under the other.
Subsequent Option Reduction	Stages after Initial Option Reduction by which options are further reduced on the basis of information relating to decision makers' preferences.
Test Complement	Used in project portfolios only. A portfolio excluding all projects within the Test Portfolio and including all projects outside it.
Test Portfolio	A starting portfolio used in the Frontier Probing Method of extended Dora-D and in forming Project Portfolios. The method seeks the efficient peer of such a portfolio, within a valuation latitude which is consistent with a decision maker's declared preferences and other pre-emptive constraints.
Vague	of objectives. Being well understood but incompletely articulated; inexactly or only partially expressed in quantitative terms.
Value Constraint	Any constraint limiting the Latitude of the Value Function. Contrasts with Frontier Constraints.
Value Function	A function defining the value, or possible value, to a decision maker of an option or choice, in terms of the magnitudes of the attributes of the choice. Used in the thesis in preference to Utility.
Virtual Frontier Constraint	A Frontier Constraint that exists within the real problem but is not explicitly included in the mathematical representation of the problem. In Frontier Probing it is a member of the set of unspecified constraints.

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Chapter 1 Introduction

1.1 Objective, approach and origins

This thesis concerns the development of a normative decision analysis technique, or rather an analytic technique within the context of an approach to decision making. The aim of the research that underlies it, was to find a novel integrated decision analysis methodology to enable a better decision making for certain types of decision problem. Its emphasis was on problem structuring and model formulation, rather than the development of mathematical theory or computational technique, at one end of the spectrum, or embracing the wider, softer issues of the process or systems of decision making at the other. It depends on one and serves the other. It has the characteristic of "hard methodology" but is based on the assumption that decision making is a soft process undertaken by real people with aims, values and insights but with limited facility to process the information that is relevant.

The object was to advance methodology, though the work described is "on a problem". It is problem centric and not technique oriented in the sense that it seeks to address methods of solving real problems, but it is the methodology, not the specific problem, that is the focus of the research. It is concerned with quantitative analysis methodology of complex many-factor situations. It is not essentially concerned with exploring the mathematics of decision taking, but the expression of models and the exposition of technique in the area involves mathematical syntax, and some arguments may be dependent on declared formal axioms and conjectures concerning the existence of exploitable relationships. However, the formal mathematical approach of theorem and proof is not part of the core background of the writer, nor in the spirit of what is presented here. It is design, problem representation, or formulation, which is the principal concern. In this it rests most comfortably within the discipline of Operational Research.

It is the impression of this writer, and no stronger claim is made, that the validity of quantitative analytic tools developed by mathematical modellers for decision aid, have been particularly related to their mathematical coherence and conformity with axioms of good (logically rational) decision making. There has, perhaps, been less attention paid to ensuring, on the one hand, that models do not make assumptions beyond inherent human capability (ie assuming as simple to comprehend that which is difficult) and, on the other, that they do not assume gratuitous complexity and sophistication in the ability of the mind to process decision information (eg, developing models of unconscious process structure, which cannot be discerned in the conscious expressions by a decision maker of his or her objectives and values). As Buchanan, Henig and Henig (1998) observe, " ...at the centre of the decision process lies the mind of the decision maker". It is the decision maker's objectives, values, and, importantly, in my view, his or her frailties in the cognition and communication of these, that in like manner should be at the centre of the attention of the modeller and aid designer. These observations will be discussed in detail later in the thesis.

In developing the approach described here, I have been particularly concerned with the validity of model formulation and analytic technique in the context of the competencies, in particular the cognitive facility, that people have to make decisions and to analyse them. I set the techniques developed within conservative, even minimalist assumptions of human facility, or rather, argued premises (though these remain capable of accommodating higher levels of competence should a decision maker or analyst feel that they can fruitfully adopt them or simply disagree with the assumptions). In seeking these premises I address some psychological issues and, in an attempt to articulate a unifying touchstone I make use of some of the ideas of Evolutionary Psychology. As far as I am aware this has not been applied to the elucidation of decision making skills before. I believe that this constitutes a rich vein of research opportunity in its own right, perhaps a unifying glue, for illuminating a large volume of disparate empirical research which addresses people's decision making. My purpose remains far more modest; merely to provide a context for technique design.

Foremost amongst the ideas I question, is the concept of the articulated hard quantitative objective or criteria (including hard multiple or alternative objectives). In this research, and in the techniques developed, I have taken the viewpoint that a decision maker will have a good qualitative understanding of what he or she wishes to achieve and a tight grasp of the (sometimes well quantified) decision factors or *attributes* contributing to the goodness or badness of a decision. These are relevant to, but not wholly descriptive of the objectives, and they relate to objectives through one-many and/or many-one connections. (This is in distinction from the viewpoint of Keeney and Raiffa (1976, p34) who see an attribute as a measurement used to represent an objective, that is, in one-one relationships). In the view here, objectives are perceived as initially being *vague* (a word to which I will ascribe specialist meaning) and the decision process as being one in which an initially wide *latitude* of hard valuation possibilities, consistent with the decision maker's declared preferences, are refined into a single quantified objective or value function, through the elucidation of further preferences informed by the analysis. This process can be conceptualised as an inductive-deductive loop; deducing optimal decision candidates consistent with objective latitude, articulating preference, inducing reduced objective latitude, deducing a lesser list of decision candidates etc.

This thesis is the product of a personal exploration. It derives from, and was directed by, a particular and personal decision problem. I sought to reveal, develop and invent by seeking solutions to the problem of generating a portfolio of shares to meet my personal needs. Although the case is specific to me, I will suggest that it has a generic structure which could be exploited in other decision making situations inside and outside the financial portfolio area, and many features that can be exploited in multiple attribute selection problems generally.

In this exploration I adopted a dual persona. I was analyst or researcher but was also the decision maker. The nature of the issues and techniques explored were motivated in part (and in common with most research) by a personal interest in exploring ideas, which might have usefulness. However, the quest for usefulness started from considerations of personal meaningfulness to this researcher. Approaches, paradigms, and techniques, which provide insight to me as a decision maker, are the unashamed source. Equally, whilst the task was to find a "better way", this, in the first instance, was simply better for me as decision maker on my own problem.

Of course a subjective or private "better" would not of itself advance "knowledge". It would if it were "better" for some objective reason; if arguments of general external validity were to be advanced (for example, if it were quicker, more accurate, overcame criticisms made of other methods, more philosophically complete etc.). But it might still do so even if the reasons were to be internal to the researcher ("I find it easier to relate to this than method X in this instance, though it does the same thing"). The personal or private perspective in the latter situation still constitutes a valid contribution to public knowledge and an appropriate epistemology for technique development, *unless* it is argued that the researcher is likely to be unique or unusual, either in the nature of the problem addressed or in his skills and needs. I propose to the reader that the nature and structure of the problems I will discuss here are not unusual and the approaches should find application elsewhere. It may also be that the decision-related cognitive competencies, or the lack of them, that I attribute to myself, may be seen as reasonable to a reader, in that at least some others may share them. However, I nevertheless seek to avoid a self-indulgent specification, by triangulating introspection of my behaviour and perceived decision related competencies as a manager, against my informal observation of other managers over many years (though still a private test) and the declared touchstone developed from EP and other's empirical research (a viewable test). I seek to establish at least a plausible basis of assumption, that some others may be as I consider that I could be.

Another consequence of the approach I adopt is that it deviates from one of the traditions of the OR paradigm; the separation of the roles of analyst-consultant from the decision-owner-client. This owes its origins and justification to history and the economic use of skills rather than the needs of academic validity. In wartime the decision intensity of decision makers allowed no possibility that that they could do their own staff work, and their backgrounds were of necessity such that they would not have the skills necessary to build or process mathematical models. Sixty years later some of this argument remains but with diminished force. Managers have increasing familiarity with the process of model building and off-the-shelf tools are available to assist their solution; though it remains the case that the work load of the makers of important decisions will still not usually allow them the luxury of working up the analysis. However, in this respect OR is no different from any other specialisation. A Chief Executive drawn from a financial background will likely employ accountants to analyse situations on which he seeks to formulate views, but this in no sense disqualifies him from reading a balance sheet or understanding a proposed brand's costings. On the contrary someone from outside that background would usually under-perform were they not to acquire such skills. Separation is a convenience.

Nor should the familiar use of the "third party case", be seen as an important objective indicator of general utility. True, it constitutes a form of testimonial but it is usually just a sample of one and general applicability cannot thereby be demonstrated. The principal value of a case is to illustrate and to stimulate the imagination of the reader that wider benefits should exist. I suggest that although OR is rooted in science and embraces positivist assumptions, the reality of general practical validity of the prescriptions that it develops, are not tested within the paradigm. I suggest that the presenter effectively invites the reader or potential user to test the potential applicability of a proposed method against the totality of his or her own problem experience, and the advantages and disadvantages of whatever alternatives they are familiar with, however the technique derived or is illustrated. They are perforce party to the validation of the potential applicability of a proposal in another situation. The approach I adopt suffers no philosophic disadvantage in this respect, though I am quite clear that I ask the reader to a be a party to the process.

There is however one strong advantage that the approach I adopt here offers. OR has not only traditionally been performed by one party for another, but the language of argument of the modeller has usually been different from that of the decision maker. Poor communication has often inhibited exposition and implementation of good ideas. In consequence, the OR community has been concerned over many years with the interface of the adviser/decision maker relationship; that is to say understanding a decision maker's perception of a problem and translating it into the language of analysis and, in the other direction, communicating a justification of the analyst's conclusions in terms that the decision maker understands and, ergo, can adopt as his or her own. Addressing this is often, properly, a major component of studies and reports. If it is not explicit, it frequently lurks in the sub-text. Important though this is, it is not necessary that it should be always examined, or that all innovating cases adopt an outside-in perspective. I wish to examine the practicality of analytic technique, as technique, and the removal of the distinction between purveyor and beneficiary facilitates this.

A second element of personal motivation was curiosity concerning the building block that forms the basis of the approach here. In the years that separated my original training in OR in the sixties, as I moved from analyst to hands-on manager, until my renewed interest in modelling, a more formal recognition of the softer aspects of modelled domains had emerged and computational power had radically altered the speed and tractability of problem solution. However, there was little that was fundamentally new about the model analysing tools that were available for deployment. An exception was Data Envelopment Analysis (DEA) which was introduced in mathematical programming form by Charnes, Cooper and Rhodes in 1978 in their seminal paper of that year. This adopted the unfamiliar perspective of measuring performance in several distinct dimensions, in single statistics, without prejudging how factors should be combined. I was struck by the *retrospective* orientation and its usefulness from that view. However, it seemed that value independent analysis and the notion of inducing a value or objective statement that causes or would cause a particular entity (Decision Making Unit (DMU) within DEA, or a decision option in a decision analysis) to be favoured, had something to offer *prospective* decision making. The prospective opportunity was subsequently recognised (eg Stewart, 1996; Belton 1992; Doyle and Green, 1993; Cook and Green, 2000) but there remained an important distinction. Value free measurement may be reasonable in retrospective assessment, but value remains at the very core of decision making. As Simon (1965, p45) remarks "Decisions are more than factual propositions. To be sure, they are descriptive of a future state of affairs, and this statement can be true or false in a strictly empirical sense; but they select one future state of affairs in preference to another and direct behavior toward the chosen alternative. In short, they have ethical as well as factual content." Although DEA majors on the processing of *factual content*, it appeared to have within it the mechanisms for implication testing and those could be used to inform the elicitation of ethical content, in other words, to articulate with greater precision a decision maker's objectives. This opportunity had been underplayed.

It appeared that not only was the framework useful but, that the framework could constitute a natural approach with a synthetic ("bottom-up") outlook towards the way which objectives might be formed.

1.2 The Generic Problem and Approach in Outline

It is a viewpoint of this work that people may have clear, comprehensive, and sophisticated understandings of what they wish to achieve in qualitative terms, but these may not be articulated (even internally to a decision maker) in terms that can be readily operationalised. It is not that they are merely unquantified, nor given relative weight, but in some sense they have not been completely formed. Yet the issues at stake and the factors relevant to the decision may be quite clear and if not already quantified, frequently quantifiable, by the decision maker, or by analysts in terms which the decision maker may readily own. He or she is *vague* (a term which I later explain should not be misconstrued as connoting indecision or confusion), in the way such factors may be prioritised or compounded. We might say that objectives are understood but not known.

DEA has many features which are consonant with this outlook:

(a) It draws conclusions from unspecified objectives, making few prior centralised declarations of purpose.

(b) It identifies "situations" which cannot be "best" under any circumstances.

(c) It identifies the conditions under which a particular situation which could be best, is best. Or, alternatively, it establishes the valuation implications of asserting that a potentially optimal solution is optimal.

Much of this is achieved through the concept of looking at the valuation of a situation in its Best Possible Light. (I mention for completeness that this metaphor corresponds to what Charnes, Cooper, Lewin and Seiford (1994, p26) call the *multiplier form.* This is the dual of the *envelopment form* of DEA models. Whilst the latter has descriptive prominence in DEA, it is the multiplier orientation and its associated metaphor which seems most valuable here).

There are nevertheless profound differences, and this should not be seen merely as an exercise in applying DEA to prospective decision problems. Whilst the inspiration is clear, the structural similarity of the Basic Model, which will be examined in due course, arises as a consequence of a willingness to tolerate, at least at the beginning, incompletely specified objectives.

Later, I enumerate a taxonomy of decision and decision analysis structure, to clarify the domain of applicability of the approach developed, within the context of decision problems as a whole. In summary, the basis is that many decisions can be conceptualised, at least as an idealised approximation, as a selection of a single option from a set of many (or an infinite set), where associated with the options there are attributes, which in the construct are discrete, bounded, and identifiable (or can be approximated as such), and which are reflective of the decision maker's qualitatively well-understood objectives. There can be redundancy with respect to those objectives but in their totality they should adequately embrace the objectives. To these attributes can be attached cardinal or ordinal magnitude or logical indicators. The magnitudes of each attribute can be specified for each option. The quality of a decision is enhanced or diminished monotonically with the magnitude of the attributes which are preferentially independent. Thus the *facts* of the problem can be represented by an n x k matrix (n =options, k =attributes). It is further perceived that value can be attached to options by forming a value function (in the Basic Method, an additive linear function) of the attribute variables, but that the decision maker's vagueness militates against its full specification, initially.

On the basis of a linear mathematical formulation, which expresses the valuation of options and explicit constraints on preference, potentially optimal options are progressively reduced in an iterative loop. In the basic method, each option is examined in each loop, to determine the circumstances which most favour it and whether it is or remains potentially optimum (efficient). Each reduction provides data which can be used to facilitate the expression of preference, which provides value information, by constraining the value function flexibility, to the next cycle of the calculation routine. This would proceed iteratively, preferably, but not always, to a final single option. For convenience I refer to the basic approach and its derivatives by the appellation DORA-D (see Glossary).

The calculational processing tool within it, is flexible in the mechanics that can be employed to reflect the expression of preference. It can work from value functions generated within the process, preferences expressed between options or decomposed choices based on generated shortlists, or with preferences elicited by independent procedures.

Indeed, the facility to process such expressions is probably more flexible than the facility of a decision maker meaningfully to express preference, in a way which allows value to be reliably inferred. This thesis explores the implications of this possibility. I later seek to classify the complexity of choices in a manner which suggests their potential cognitive difficulty. I argue that one perspective of *all* decision analysis is that it constitutes a decomposition of decision situations which cannot be readily comprehended in totality, into a series of subordinate choices which the human mind is better equipped to process. To this I end, I introduce the idea of Fundamentally Decomposed Preference in which specific decision choices, or selections from groups of options, are broken down, insofar as it is possible to do so, into the most elementary non-trivial case in the classification I suggest. I will call this a Fundamentally Decomposed Choice. I discuss and illustrate the way in which these ideas can be incorporated in the framework.

But if human cognitive limitations introduce the need for analytic caution in one area, they might ameliorate the need for excessive sophistication elsewhere. Two aspects are particularly examined. One is the possibility, and implications, of labile value- the ability of the human mind to discriminate and sustain within itself the finer features of relative worth. The other is the significance of value as a conscious process. I will argue that both of these make excessive complexity of the operational models of our decision value systems redundant and, possibly, misleading. Whilst configural issues appropriately add to complexity, they can be accommodated within an essentially linear structure, with little or no loss of effectiveness.

However, complexity is forced upon us in the structure of problems. We are inevitably forced to impose structure and bounding in models of systems which are loose and unbounded. However, even within the limitations of hard structured models, not all decisions are made between options which are specifically enumerable. Many are selections of interdependent combinations of decision components or sub-options. We may describe these as portfolio decisions. Two main practical polarisations of a large array of theoretical possibilities are perceived. One involves the selection on a binary "in-or-out" basis from a list of sub-options. The value of the aggregate may be an additive sum of the values of the individual constituents. However, the components interact through limitations on one or more resources. This might be described as the Project Portfolio Problem (although this is a major exemplar of a structure that might be used elsewhere, in timetabling for example). The other is where the components can be selected in continuous proportions, and, whilst there may be a resource limitation, it is implicit in the structure (ie the variable defining inclusion is a proportion of resource). However, there is an additional attribute of aggregation which gives value to the collection which is distinct from the sum of its parts. Such a selection might be a portfolio of investments (or Financial Portfolio as will be used hereafter). As will be seen, such problems do not fit comfortably within the Basic Dora-D structure. This would, in principle, require evaluating each potential optimum explicitly with reference to each other portfolio, a formidable combinatorial problem. Two factors help us. It is possible to find the efficient equivalent or Peer of any pre-specified portfolio. A method called Frontier Probing is introduced to enable this in the case of both Financial Portfolios and Project Portfolios. A "pet" or attractive project can be used as an initial input and improved upon if not already efficient. Moreover, such solutions and other similarly generated possibilities can be used to generate preferences to progressively reduce valuation latitude, just as in the Basic Method. Financial portfolios are the subject of the personal decision problem that I sought to address. General and particular issues will be discussed further in this Chapter, and in detail later. Project Portfolios will be considered in the context of revisiting a published problem addressed by other researchers.

The versatility of the technique will also be discussed in the context of improving solutions, on the basis that some decision makers may make reliable statements of value in the way they favour particular options. It is arguable that we may be endowed with some innate capability to make intuitive leaps, to integrate holistically, even when we cannot synthesise piece-wise in a reliable manner. If so, we could usefully seek to infer value (or to elicit policy) from suggestions made. The technique used in policy elicitation mode, may also assist in finding better solutions, in situations that Lindblom (1959) characterised as being dealt with by Successive Limited Comparison. Multiple constraint decision problems that have traditionally been tackled using Multiple Objective Linear Programming, will also be discussed in the context of the approach here. The approach offers no advantages in terms of computational efficiency over traditional MOLP. However, it offers advantages in being readily implementable with standard LP software and, with attention to the ideas of Fundamentally Decomposed Preference, perhaps advantages of psychological reliability as well.

1.3 The Core Problem

The core personal problem I address here is how to select a portfolio of shares as a private, non-institutional investor. The problem will be introduced in some detail later but, having introduced a distinction between myself and a professional fund manager, I should make the point that, at least in intent, I have not sought a model which is naï ve or simplistic, relative to the real needs of the "serious" operators. Rather, it is the data I can use, the resources that I can bring to bear, and the detail of background knowledge that is more restricted. In the decision models I employ, I believe there is greater sophistication than might typically be employed by an investment house and in part this derives from the scale issue. Specifically, professional optimisation seems largely based on developments from a two dimensional, (return and risk of return) objective construction originally introduced by Markowitz (1952), which is the cornerstone of Modern Portfolio Theory (MPT). The approach I adopt here is many dimensional, and options are reduced and objectives refined within the valuation process. The experienced fund manager may more readily relate, or be able to pre-process, distant fundamentals to the two dimensional form for the traditional model. Nevertheless, it is possible that the concepts within the approach here could be usefully exploited by professionals.

The concept, that is expanded later in the thesis, is that a share is not a simple commodity but a package of properties which affect its value to an investor in various ways, available for purchase as a job lot. These properties may be hard facts, for example, financial performance measures, or the business sector of the underlying company; or soft, for example the opinion of the decision maker or of other parties on the quality or integrity of the management, or expected sales growth. Value attaches to these properties, which are not necessarily objective, are certainly individual to the decision maker and not determinable in advance. These will be reflective of the decision maker's objectives either directly ("I put high value on a share with high net cash."), or indirectly ("I value a high expected Net Present Value of future cash flows and *believe* that the mean level of profit over the past five years is an indicator of this."). These are attributes of the type referred to previously, though it should be noted, even at this stage, that the attribute which is conceptually for sale is a fact or conclusion to which a subjective valuation of worth is attached. Thus an attribute of future profit (or even expected future profit as an objective concept), cannot be for sale; but the attributes of last years profits, the statistics department's regression based forecast of next year's profits, the Investor's Chronicle or Aunt Annie's opinion as to those profits, may be. Most of such attributes are, or can be translated, into an additive scale of value (I will later suggest a linear scale); thus one purchases such attributes in proportion to the fraction of purchase price that a share has in a mix of shares. But there exists some attributes which are defined interdependently, where value is a more complex function of the proportions. These are most usually related to the value of diversity, or rather the attrition of value arising from the risk of concentration. Indeed if there were no diversifiable risk, all eggs could be put in the single most attractive basket. The problem thus extends from the single option selection problem outlined above, to generating a list with quanta.

1.4 Layout of this Thesis

For the convenience of the reader, I reproduce in this section the introductions and summaries of the succeeding chapters.

1.4.1 Chapter 2. Some issues of Decision Analysis

In this chapter, I explore some philosophical and psychological issues of decision making and analysis to serve as a context for the author's viewpoint and analytic predilections. This serves as a backdrop to assumptions that are built into the approach to decision aid that is developed in this thesis.

I start by considering the purpose of decision analysis and conclude, unexceptionably, that it is to ameliorate human cognitive impairment. I suggest, however, that the mathematically-based models and methodologies intended to assist, may have paid insufficient attention to this, either taking the nature of impairment as read, or not considering it at all. But consideration of, and assumptions about, the competencies we may have, and may not have, should be prominent in methodological design. This is as important as the validity of the mathematical devices that we make use of.

I then discuss the concept of decision maker's Objectives (later pursued in an EP context) and suggest that the solidity that seems to attach to the concept in organisational situations may be illusory and of limited help in seeking to assign weights in many factor situations. In qualitative terms Objectives may be well understood but Vague, in the sense that weight or priority cannot easily be attached to them. I suggest, and the methodology developed here assumes, that a decision maker, nevertheless, will often have a clear and potentially quantifiable understanding of the Attributes of a potential decision to which he or she attaches value.

I then discuss the value orientation of the method explored here. This is largely a question of personal appeal and I do not seek to suggest that orientations that others find useful are wrong. My main rationale is that, at root, multiple attribute decision problems are concerned with not being able to achieve everything one wants at the same time. Trade-off between one desirable outcome and another becomes the central issue, and this concept is the basis of value. Other mechanisms may confuse this.

This is followed by a consideration of whether a cardinal yardstick of value can be retained in the mind, without reference to an external standard. I conclude that it probably cannot; the properties required of interval, far less ratio, scales are not such that we are likely to have innate skill, in the absence of an external standard. This is particularly so, as an infinity of other "strategically equivalent" scales can indicate the same ranking of options. Nevertheless, even if the notion of objective internal cardinal value is suspect, it is a valuable fiction that can be used to render statements of preference free from contradictions.

The issue of value, and the associated concept of preference, is returned to in order to examine its potential stability or lability. Whilst stable values would not seem to be precluded philosophically, the mental capacity limitations associated with establishing ordinal standards with fine discrimination, provisionally at least, militates against this. I again prelude an EP consideration in which I can suggest that no adaptive purpose would have been served by stable values. I mention empirical conclusions that expressed values are labile, speculating that we might properly adopt the stronger conclusion that it is the values themselves that are labile. I buttress this by a discussion of two types of information process where the psychological foundations are more solid- the limited ability of people to make fine distinctions in observing sensory phenomenon, and the lability of judgement, which is evidenced by the success of bootstrapping models of judges.

The sovereignty of people's present values over future values is addressed, questioning the view of some that a decision maker should be attentive not just to present preferences but future values of future consequences. I suggest that this is perhaps misconceived: a decision maker only has a duty to his or her present values whilst having a clear duty to recognise, within present values, the future consequences of present actions. This would resolve a paradox in which Elster suggests Ulysses might be considered irrational in taking what most people would feel was a sensible precaution. This has pragmatic importance and not just philosophic interest.

I move on to discuss the concept of Rationality, finding it a more meaningful in a normative rather than a descriptive use, and I attempt a definition consistent with my personal viewpoint, exploitable within decision aid design, based on the ideas of valuation. A keystone of this is that decision maker controls the criteria of value. Recognising that, even with a definition with its central features controlled by the decision maker, there will be failures of execution, I put emphasis on intended rationality. Nevertheless, I suggest that he or she cannot intend rationality, if he or she wilfully ignores material violations in certain areas. I mention criteria which I suggest as tests of intended rationality of a decision maker. Some bear similarity to mathematical axioms of rationality, but others are included, eg the role of Conscious Process.

Prior to the consideration of a framework which operationalises these ideas, I consider the concepts of Efficiency and Dominance in multiple attribute decision situations. I mention the traditional definition of dominance, but prefer a value reformulation which, in most respects, is equivalent, but allows more simply for domination of otherwise non-dominated options by combinations of options. I also suggest that the view of normal dominance is satisfactory, provided we suppose that the set of attributes considered is both comprehensive and relevant. But in practice one considers an arbitrary selection of potentially relevant factors. I introduce the ideas of strong domination and weak efficiency.

The importance of sustaining a distinction between Indifference (one of the relations of classical decision theory) and Not Knowing (a common condition in the expression of preference by real human beings) is then briefly raised. I make the point that practical expressions of preference by people are usually statements of Strict Preference (doubt, I suggest, is Fuzzy Strict Preference not Weak Preference). However, the practicalities of analytic techniques, notably LP, which is used in the methodologies expounded here, and the requirements for conservative treatment, make it convenient, and minimally inhibiting, to treat such Strict Preference as if it were Weak.

I then discuss a model structure to operationalise normative rationality. Later I concentrate on models of deterministic structure, including risk as a determined property described by parameters. However, I start with a model relating a value of a decision, to measures of worth of the decision under particular states of the world and weights related to the comparative likelihood of those states of the world. The simple linear model which bears (only) a structural similarity to an SEU model allows the decision maker sovereignty over his rationality, within the constraint of intending a rational model, and allows minimalist assumptions regarding cognition. In particular, it depends only on a decision maker's ability to assess comparative likelihood (not objective probability) and comparative worth (not cardinal value). The decision maker is permitted to generate a measure of value using any parameter and designations wished, requiring only avoidance of comparative worth and comparative likelihood violations. This generalised structure seems to accommodate familiar approaches as special cases.

Turning to the measurement of the outcome desirability (either assuming a single state of the world, as later I effectively do, or as prelude to the application of a multi-state model), I introduce the concept of attributes and set down some of the principles governing how these might be evaluated and incorporated in a value model. These reprise some of the principles of rationality but introduce others, including what I call Qualified Self Awareness, on which I expand. This is the concept that a decision maker's values cannot be in a black box secret from himself. I argue that this ultimately enables an analyst, on behalf of a decision maker, to derive, if they do not already exist, sets of attributes which are mutually preferentially independent and can be incorporated in models which are additive and, ultimately, linear. Recognising the possibility of controversy of what may be a novel position, I underwrite the pragmatic safety of value linearisation (which I will use extensively) using other arguments.

I then give special consideration to the issue of configurality, which I suggest as a special and potentially difficult non-linear case. I suggest that disjunctive and conjunctive valuation can nevertheless be linearised using what I term the General Configural Model; a simple transformation of the Minkowski metric.

I conclude by attempting a generic classification or taxonomy of decision problems and approaches to their analysis. This is a prelude to the discussion of the structure of the proposed methodologies in Chapters 5 and 6.

1.4.2 Chapter 3. Generating assumptions of cognitive facility in decision making: An Evolutionary Psychology touchstone.

In this chapter I build a set of assumptions which I use to underpin the methodology described in this thesis. I start by reminding the reader that the reason for formal decision analysis is because we suffer from some form of impairment in our mental process of decision information. It is important therefore that one makes clear, plausible and balanced assumptions of what mental facility we have and do not have, which we exploit or seek to exploit in decision making and decision analysis. Unfortunately, a convenient digest of appropriate assumptions is not available "off the shelf" and accordingly I try to develop a "balanced" check list here. I attempt to use some of the ideas of Evolutionary Psychology (EP) as a patterning method and debate what capabilities and concepts would accord adaptive advantage in the environment of human evolutionary development in the light of issues which impact decision making. Although only reasonable assumptions, and not research conclusions, are sought this, is subject to broad triangulation, in particular by reference to empirical work.

In section 3.3, I outline a basic description of the EP concept and follow this with a brief description of how EP has been used to inform issues of psychology and to develop hypotheses for examination. I go on to describe how I seek to exploit it here. I make use of the test that if one cannot postulate a mechanism by which a mental capability could have secured at least a distal impact on reproduction in the ancestral environment, it is not reasonable to assume its existence. As minimalist assumptions are sought, the criterion is safe and secures balance.

I then address a number of decision-related issues from this perspective. I start with the more general issue of Reason itself; our unique capability to draw conclusions, action related conclusions, by connecting thought. I suggest that the adaptation is a powerful one but that the adaptive advantages accorded to survive in our difficult marginal niches only required moderately short-chain connections of thought, not the long near-infinite chains that artefacts of civilisation, which did not exist in the environment of evolutionary adaptation, now allows. I suggest that long chain reasoning arose from a purely serendipitous property of Reason, its capability of bootstrapping itself. Accordingly, we should be cautious in attributing to the mind powers which are indirectly dependent on those artefacts, or assumes that we possess mental systems which are analogues of sophisticated long-chain processing computers.

I then consider the nature of decision in the ancestral environment, contrasting it with modern decisions. Our ancestors would have adaptively applied their intelligence to toolmaking, organisation, relations within the group, and to the means of exploiting the environment for food, often involving issues of intellectual discrimination and judgement. Many would relate to a single clear oft repeated purpose for which learning, from both one's own experience and communicated vicarious experience, would be more useful than fundamental examination that characterises many modern problems. Single purpose allows simple "hillclimb" to be an adequate control heuristic for securing desirable parameters in the type of "design" problems that existed in our primeval world.

This presages a discussion of objectives and optimality. I suggest that no adaptive advantage attaches to articulated concepts of Strategy and Objectives in the sense in which these would be understood today. Goals, probably implicit, would be binary, and multiple objectives would be lexicographic or involve serial switching between single preoccupations. However, a considerable benefit would attach to weighing a multiplicity of factors related to a single goal. Optimisation, however, was not a concept that would have been needed to have been understood, nor would it have secured adaptive advantage. Optimal behaviour can be achieved through non-intellectual mechanisms and, indeed, is, even by animals and simpler organisms. The concept of relative improvement and the application of intellect to achieve this is, by contrast, fundamentally adaptive.

To illustrate possible differences in adaptive thinking from classically logical thought I dissect an empirically examined stylised problem, the Wason Selection Task, where the classically correct conclusion is not intuitive. However, the intuitive responses do seem to relate to the information discovery processes that might have led to adaptive decisions. Whilst this is a very specific example it serves to illustrate that an adapted mind is not a classically logical mind and the example serves as a prelude to the logically related issues of probability and cardinality.

Uncertainty is usually treated as a parametrically defined attribute within the methodology reported in this thesis, but its role in decision making generally is central. Moreover, the treatment adopted is aimed to be within the general concept and criteria of intended rationality explored in Chapter 2. For completeness, I therefore consider the adaptive implications of uncertainty. It is apparent that the mental notion of uncertain alternative futures, and the influencing of uncertain alternative futures by action are concomitants of Reason. It is also a sine qua non that a sense of comparative likelihood, including equal likelihood, and cognition of broad degrees of likelihood is adaptive. But it is difficult to go further and embrace any form of probabilistic cardinality as adaptive and therefore intuitive. Comparative likelihood allows the ordinal ranking of disjoint events which might be turned into scales akin to probability which might be suitable to a modern analyst for some purposes. Innate understanding of a probability of 0.5 is possible. However, we should otherwise be dubious about attributing cardinal probabilities to elicited subjective responses, from statistically untrained subjects, or which cannot be determined from objective considerations.

I go on to question innate comprehension of cardinal measurement and the ability to process number and quantity generally beyond that required for count and organisational arithmetic in the countable range. This leads into issues of concepts of value. Whilst the idea of ordinal value and compensation would seem to be entrenched, the concept of a scale of value would not appear to have adaptive advantage and it is difficult anyhow to see how a stable yardstick could be held in the mind. Nor would stable value and preference seem to confer adaptive advantage and this includes the weighting of factors. On these grounds one should expect value to be imprecise and labile, as expressed value seems to be. This does not of course invalidate value as a useful fiction for summarising preferences in a way which as far as possible renders them free from contradictions. I also debate the seeming facility for people most easily to trade-off only two factors, whilst also having an ostensibly polarised facility for considering a mass of factors holistically.

Finally I tabulate the cognition assumptions that I believe can reasonably be made and which act as a backdrop for the rest of the work. In essence these emphasise human abilities as a "comparator"; to make one-by-one binary comparisons and to order, rather than to assess cardinal degree.

1.4.3 Chapter 4. The investment portfolio decision

At this point I digress from discussion of general decision analysis considerations, to introduce the problem to which the approach developed within this thesis will be applied. I also summarise and comment on features of Modern Portfolio Theory which is the basis of existing "Quant" decision analysis in this area, where it is applied. There is a parallel thread in this thesis and I will return in Chapter 5 to further consider methodology without special reference to this application area. A reader who wishes it could therefore alternatively address this chapter prior to Chapter 8 which places the developed approach within the context of this problem.

In this Chapter I start with a consideration of the nature of objectives and approaches to the investment portfolio decision problem, considering the extent to which private and professional needs and capabilities correspond. I introduce my personal tastes as a decision maker. I mention that amongst established professional analysis there are two major strands of approach, Quantitative and Qualitative,

I then discuss Modern Portfolio Theory (MPT) which provides the theoretical glue underlying more formal quantitative professional analysis. I introduce the concept of systematic risk, or Beta, and discuss aspects of the Capital Asset Pricing Model. I will later borrow from these concepts. I argue that MPT has limitations as an exclusive normative methodology being very data intensive and, in essence, only a two-dimension model for which questions are begged. Its use by private investors is effectively precluded by cost. I conclude by suggesting that the traditional approaches can be considered as polarised paradigms. Each of these viewpoints do not need to exclude the application of the other, but the model based optimisation approaches, which occupy middle ground by characterising portfolio formation as a many dimensional problem are rare. Hallerbach, however, pursued the issue in his PhD thesis (1994) and the general arguments for such a framework were developed by Spronk and Hallerbach (1997) whose suggestion is referred to. They have subsequently continued to develop multiple dimensional analysis methods for the financial area.

1.4.4 Chapter 5. The Basic Technique

The following sections seek to introduce the basic Dora-D technique (originally Decision Option Reduction Analysis using concepts from DEA), placing it in the context of some of the decision analysis and cognitive assumptions already discussed.

It starts by relating the structure to the taxonomy outlined in Chapter 2. In essence it can be stylised as the selection of a single decision from mutually exclusive options characterised in terms of the magnitudes of a bounded set of attributes. The selection is informed by qualitatively well understood but quantitatively vague objectives. Each attribute is related monotonically to the goodness or badness of the decision. The data would normally be able to be represented in a complete matrix.

The inspirational origins of the technique in, and its connection to, Data Envelopment Analysis, is discussed, highlighting important distinctions. First amongst these is that the approach here is centred on decision makers' values.

Next the approach is structured, highlighting the central analytic objective, the formation of an additive linear value function. The concepts of assessing each potential decision in terms of a value function, which shows it in the best possible light, is explained and Maximal Comparative Advantage (MCA) and Comparative Advantage Function (CAF) are introduced. Initial Option Reduction is described, in which no decision maker's "values" (or just the most certain pre-emptive constraints) are specified. The concept of reducing the Latitude of the value function by seeking statements of preference is introduced, a process which is effected during Subsequent Option Reduction.

An LP formulation for effecting the process is outlined. Alternative methods or Mechanics for structuring elicitation and representing preference within the LP structure are outlined. The need for "breaking ties" is raised and approaches to this in Final Reduction are suggested. The approach is illustrated by an example.

Observations are then made concerning ancillary technical matters that might of interest to a reader or analyst.

Potential applications are mentioned, and the Chapter concludes with a brief discussion of the method in the context of the decision cognition assumptions the writer has made.

Sutton and Green (2002) constitutes an anticipation of this part of this thesis, although this Chapter amplifies some points. The structure was originally described in Sutton (1999), a transfer paper associated with this work, which suggested it as a concept for developing firm decisions from vague objectives.

1.4.5 Chapter 6. Extending Dora-D to portfolios by Frontier Probing

In this chapter the ideas introduced in Chapter 5 are extended to embrace portfolios. I start by reprising the characteristics of problems that can be handled using Basic Dora-D, noting that it requires explicit designation of options, but that there are significant examples of problems for which the combinatorial magnitudes or the definition of decisions in terms of continuous variables rule this out. Many of these can be described as portfolio problems.

I defer consideration of Project Portfolios but consider the handling of portfolios having the same structure as financial portfolios and use the term Financial Portfolios to embrace the generic class as well as the specific problem. I then structure this problem.

Such problems are characterised by combinatorially large or infinite numbers of options *and* lesser, but still unmanageably large, numbers of efficient options.

Frontier probing, a concept used in the attack of the core problem, is then conceptualised. This involves the insertion of explicit Frontier Constraints only when a violation of implicit constraints is observed. The method is illustrated with a worked example.

The chapter concludes by a discussion of the need for concavity in the function defining the interdependent portfolio attributes.

1.4.6 Chapter 7. Other methodological extensions

In this chapter I examine further features which can be used in association with Dora-D to ameliorate the problems of the impaired decision maker, to cope with more complex valuation or to simplify analysis. I also discuss further extensions to cover additional problem structures.

I start by highlighting some alternative methods for handling complex decision problems. I highlight two, Decomposition and Holistic Integration, for further exploration within the chapter. Looking first at decomposition, I observe that first one can break down decision selections into pairwise choices. Such binary choices themselves represent situations of varying complication and, as a prelude to their simplification, suggest a method of classifying them using an "[m,n] Complexity Indicator".

I move on to discuss how choices can be partitioned into groups of sub-choices of reduced complexity depending on mutual preferential independence. I mention the limits to simplification in partitioning problems and discuss the circumstances in which expressions of preference, relating to sub-choices, can imply a preference for one of the two options in an undecomposed pair.

Using the defined Complexity Indicator I refer to the most structurally simple case, classified [1,1], which I refer to as a Fundamentally Decomposed Choice. This figures in the discussion in a variety of ways later in the chapter.

I then discuss Franklin's Prudential Algebra as an example of decomposition. Based on his straightforward conceptualisation, I develop a modernised algorithm. Here I attempt to partition the problem of selection between a binary pair of options into series of partitions, limited to three of the most structurally simple choice types, for which I suggest we are most likely to be able to express reliable preference. I do this in a way which is designed to maximise the prospects of drawing a firm conclusion regarding the whole, from views expressed about the partitions. This I call Franklin Decomposition.

I also mention an approach which I call Larichev Decomposition, which only makes use of [1,1] choices. These are developed for a particular decision but are derived from the decomposition of sets of efficient options rather than individual options. I go on to describe a methodology for doing this in a way which allows the preferences expressed to be converted into value constraints in Dora-D. I also describe how the information declared in expressing preferences between Franklin Decompositions, can be used to reduce value Latitude and other potential optima, not just the options subject to the decomposition.

I then take the alternative perspective and show how holistic selections might be improved in a Dora-D framework.

I also talk about how the scale of a selection problem could possibly be reduced by only considering options which an impaired decision maker might reliably discriminate in value terms. The Representative Efficient Set is introduced. This concept reflects the ideas of Principal Components Analysis (though depending on completely different mechanisms).

I then conceptually consider how four further types of problem structure can be accommodated within the approach being presented. First is the problem of multi-attribute decisions under constraints, the type of problem that might otherwise be formulated in MOLP terms. An illustrative example is presented.

The second is a consideration of how configural valuation can be brought within the ambit of the Dora-D structure. Particular consideration is given to the treatment of the Modified Minkowski Metric introduced in Chapter 2.

I then consider the analysis of project portfolio selection problems, using as an example a problem already examined by other authors. I finally examine the translation of "voting" data of the type generated in group decision making or social choice into Dora-D structure. Cook and Kress (1990) have tackled this problem with a data envelopment approach. The formulation suggested is little different, but is somewhat closer to the principles of Chapter 5, and offers alternative insights.

1.4.7 Chapter 8 Using Dora-D in developing a personal financial portfolio

In this chapter I seek to illustrate the use of the approach in a practical application- the Core Problem. This is investment decision making- specifically my share decision making. I start by explaining my attitude to the problem and my approach to explaining this as, simultaneously, decision maker, analyst and researcher.

I describe my data sources and my Vague Objectives as decision maker. I then describe in detail the application of the Basic method to a share purchase analysis conducted in 1998. This involves an explanation of the Attributes I used and how I derived them, indicating problems I perceived and how I attempted to address them. I then describe a series of runs from Initial Option Reduction and subsequent reductions in which I employ a number of elicitation and representation mechanics, eventually homing in on a single CAF. I discuss the methodological conclusions I drew at the time.

I remind the reader of the limitations of the basic method in a portfolio situation and go on to discuss a more recent analysis based on May 2002 data using the Extended Model. I discuss the nature of the risk I am seeking to ameliorate and my tastes and attitudes concerning the valuation and representation of risk. I outline the risk measure incorporated.

I also describe how I use Beta, and a simplification that the Capital Asset Pricing Model allows me to make. I describe modifications to Attribute definitions relative to the 1998 analysis. I also discuss the issues of using a static model for sequential decision making and describe the approach that I chose to take in my role as analyst. I also discuss formulation short-cuts that can help to speed the NLP if it is taking too long.

Before going on to the actual analysis, I describe my initial share portfolio and some practical aspects which need to be acknowledged and taken account of.

I outline the issues involved in the selection of a Reference Portfolio and the choice made.

In this analysis I make particular use of the Larichev Decomposition and Attribute Weight Capping mechanics. My preferences between decomposed choices and their representation within the MP are discussed. The many analysis cycles involving capping adjustments are then outlined. This led to a single CAF which could be said to be my value function as decision maker. It also defined a theoretical portfolio. However practical problems had to be addressed and a series of other analyses were performed before an implementable plan was created. I describe these.

1.4.8 Chapter 9. Testing the approach using simulation

In this chapter, I review the performance of the methodology. I consider only issues of the mechanical efficacy of Dora-D to convert statements of preference, which simulate a variety of elicitation devices, into consistent and, for the expressed values, optimum decisions. Accordingly, I ignore issues of the psychological reliability of the elicitation device. Indeed, I assume that the simulated decision maker is totally reliable and consistent in the weights he or she attaches to attributes and, in binary preference situations, he or she can declare an accurate strong preference however slight the value advantage. The main purpose of the simulations was to:

(a) Pragmatically demonstrate that Dora-D will progressively reduce the potential optima and find an optimum.

(b) Indicate the relative speed of convergence for the mechanics used.

Most of the simulations test variations in elicitation mechanics for the discrete decision linear model (ie the basic model), though a configural discrete model and a portfolio model are also demonstrated.

I start by outlining the data used. In order to facilitate comparison between mechanics, ten standard sets of data are predominantly used. These serve to represent problems of moderate size and complexity.

An encapsulation of the multiple attribute decision analysis is to find the weights that attach to attributes. In the simulations I use a concept of revealing "Hidden Weights". Both the concept and values used are explained to the reader. The simulated decision maker is assumed to express preferences exactly and consistently, in accordance with these values, but the simulated decision maker is deemed unaware of the weights, which are only revealed to a simulated analyst through the expression of preference, within the analysis process being considered.

Two criteria of analysis performance are then discussed. One, the Information View, focuses on the number of options that remain potentially optimal. The other, termed the Performance View, concentrates on the average value, assessed using the "hidden weights" of the options, that at a particular stage remain potentially optimal, given the preferences declared.

The methodology used is briefly discussed which is centred on the Basic Methodology using the Andersen-Petersen variant. An issue arises here and the use of a different option elimination criterion in these simulations, from that commended for practical situations, is explained.

I then relate the various experiments undertaken. The first is by way of context setting. A decision by analyst and decision maker of how many attributes to embrace in an analysis, is itself an expression of value that influences both the number and identity of potential optima, just as expressions of preference do. At extremes, it either completely determines the optimum, or fails to eliminate any options, without the need for further analysis. Taking the Information view only, I examine how the number of potential optima is influenced by the number of attributes considered for inclusion in an analysis, and seek to establish relationships.

Simulations 2 to 6 consider the impact of reductions of value function latitude, secured by expressions of preference between options and different mechanics for identifying options for comparison. I address the extent to which reduction is achieved by Dora-D, if such expressions are reliable. Different methods of identifying options for comparison or prioritisation are examined.

Simulation 7 assesses the reduction achieved by the ranking of attribute weights.

I go on to consider the mechanical efficacy of Weight Capping, before examining two approaches to [1,1] decomposition.

Simulations 11 to 13 examine the effects of mis-specifying a non-linear value mechanism as a linear one. Two true underlying value structures are investigated. In one of these no major problems emerge. However, LP infeasibilities were caused in the other. Alternative methods of proceeding are investigated, one appearing to be more effective.

I then move to a situation in which non-linearity is considered possible, by assuming, *within the simulated analysis,* that the decision maker's values can be represented by the Modified Minkowski Metric. However, we still test this concept on a mis-specified model, by making the simulated decision maker's "actual" value function correspond to a multiplicative model. A model in which we simultaneously seek both the arithmetic weight and power parameters is considered first. This is unsuccessful. In Simulation 15 we consider the Conservative Fixed Parameter methodology discussed in Chapter 7, finding more encouraging results. Finally, within this group, the relationship between the configural parameter and the number of efficient options at Initial Option Reduction is assessed, demonstrating relatively low variation.

1.4.9 Chapter 10. Round-up

I finally seek to round-up the work, hesitating to refer to this as Conclusions. In the first section of the chapter, I make some general observations concerning what the author has learnt and aspects that would be useful to others. In the final sections, I list concepts and ideas developed in this thesis which I suggest are potentially publishable and highlight areas raised which would benefit from further research.

Chapter 2 Some issues of Decision Analysis

2.1 Introduction

In this chapter, I explore some philosophical and psychological issues of decision making and analysis to serve as a context for the author's viewpoint and analytic predilections. This serves as a backdrop to assumptions that are built into the approach to decision aid that is developed in this thesis.

I start by considering the purpose of decision analysis and conclude, unexceptionably, that it is to ameliorate human cognitive impairment. I suggest, however, that the mathematically-based models and methodologies intended to assist, may have paid insufficient attention to this, either taking the nature of impairment as read, or not considering it at all. But consideration of, and assumptions about, the competencies we may have, and may not have, should be prominent in methodological design. This is as important as the validity of the mathematical devices that we make use of.

I then discuss the concept of decision maker's Objectives (later pursued in an EP context) and suggest that the solidity that seems to attach to the concept in organisational situations may be illusory and of limited help in seeking to assign weights in many factor situations. In qualitative terms Objectives may be well understood but Vague, in the sense that weight or priority cannot easily be attached to them. I suggest, and the methodology developed here assumes, that a decision maker, nevertheless, will often have a clear and potentially quantifiable understanding of the Attributes of a potential decision to which he or she attaches value.

I then discuss the value orientation of the method explored here. This is largely a question of personal appeal and I do not seek to suggest that orientations that others find useful are wrong. My main rationale is that, at root, multiple attribute decision problems are concerned with not being able to achieve everything one wants at the same time. Trade-off between one desirable outcome and another becomes the central issue, and this concept is the basis of value. Other mechanisms may confuse this.

This is followed by a consideration of whether a cardinal yardstick of value can be retained in the mind, without reference to an external standard. I conclude that it probably cannot; the properties required of interval, far less ratio, scales are not such that we are likely to have innate skill, in the absence of an external standard. This is particularly so, as an infinity of other "strategically equivalent" scales can indicate the same ranking of options. Nevertheless, even if the notion of objective internal cardinal value is suspect, it is a valuable fiction that can be used to render statements of preference free from contradictions.

The issue of value, and the associated concept of preference, is returned to in order to examine its potential stability or lability. Whilst stable values would not seem to be precluded philosophically, the mental capacity limitations associated with establishing ordinal standards with fine discrimination, provisionally at least, militates against this. I again prelude an EP consideration in which I can suggest that no adaptive purpose would have been served by stable values. I mention empirical conclusions that expressed values are labile, speculating that we might properly adopt the stronger conclusion that it is the values themselves that are labile. I buttress this by a discussion of two types of information process where the psychological foundations are more solid- the limited ability of people to make fine distinctions in observing sensory phenomenon, and the lability of judgement, which is evidenced by the success of bootstrapping models of judges.

The sovereignty of people's present values over future values is addressed, questioning the view of some that a decision maker should be attentive not just to present preferences but future values of future consequences. I suggest that this is perhaps misconceived: a decision maker only has a duty to his or her present values whilst having a clear duty to recognise, within present values, the future consequences of present actions. This would resolve a paradox in which Elster suggests Ulysses might be considered irrational in taking what most people would feel was a sensible precaution. This has pragmatic importance and not just philosophic interest.

I move on to discuss the concept of Rationality, finding it a more meaningful in a normative rather than a descriptive use, and I attempt a definition consistent with my personal viewpoint, exploitable within decision aid design, based on the ideas of valuation. A keystone of this is that decision maker controls the criteria of value.

Recognising that, even with a definition with its central features controlled by the decision maker, there will be failures of execution, I put emphasis on intended rationality. Nevertheless, I suggest that he or she cannot intend rationality, if he or she wilfully ignores material violations in certain areas. I mention criteria which I suggest as tests of intended rationality of a decision maker. Some bear similarity to mathematical axioms of rationality, but others are included eg the role of Conscious Process.

Prior to the consideration of a framework which operationalises these ideas, I consider the concepts of Efficiency and Dominance in multiple attribute decision situations. I mention the traditional definition of dominance, but prefer a value reformulation which, in most respects, is equivalent, but allows more simply for domination of otherwise non-dominated options by combinations of options. I also suggest that the view of normal dominance is satisfactory, provided we suppose that the set of attributes considered is both comprehensive and relevant. But in practice one considers an arbitrary selection of potentially relevant factors. I introduce the ideas of strong domination and weak efficiency.

The importance of sustaining a distinction between Indifference (one of the relations of classical decision theory) and Not Knowing (a common condition in the expression of preference by real human beings) is then briefly raised. I make the point that practical expressions of preference by people are usually statements of Strict Preference (doubt, I suggest, is Fuzzy Strict Preference not Weak Preference). However, the practicalities of analytic techniques, notably LP, which is used in the methodologies expounded here, and the requirements for conservative treatment, make it convenient, and minimally inhibiting, to treat such Strict Preference as if it were Weak.

I then discuss a model structure to operationalise normative rationality. Later I concentrate on models of deterministic structure, including risk as a determined property described by parameters. However, I start with a model relating a value of a decision, to measures of worth of the decision under particular states of the world and weights related to the comparative likelihood of those states of the world. The simple linear model which bears (only) a structural similarity to an SEU model allows the decision maker sovereignty over his rationality, within the constraint of intending a rational model, and allows minimalist assumptions regarding cognition.

In particular, it depends only on a decision maker's ability to assess comparative likelihood (not objective probability) and comparative worth (not cardinal value). The decision maker is permitted to generate a measure of value using any parameter and designations wished, requiring only avoidance of comparative worth and comparative likelihood violations. This generalised structure seems to accommodate familiar approaches as special cases.

Turning to the measurement of the outcome desirability (either assuming a single state of the world, as later I effectively do, or as prelude to the application of a multi-state model), I introduce the concept of attributes and set down some of the principles governing how these might be evaluated and incorporated in a value model. These reprise some of the principles of rationality but introduce others, including what I call Qualified Self Awareness, on which I expand. This is the concept that a decision maker's values cannot be in a black box secret from himself. I argue that this ultimately enables an analyst, on behalf of a decision maker, to derive, if they do not already exist, sets of attributes which are mutually preferentially independent and can be incorporated in models which are additive and, ultimately, linear. Recognising the possibility of controversy of what may be a novel position, I underwrite the pragmatic safety of value linearisation (which I will use extensively) using other arguments.

I then give special consideration to the issue of configurality, which I suggest as a special and potentially difficult non-linear case. I suggest that disjunctive and conjunctive valuation can nevertheless be linearised using what I term the General Configural Model; a simple transformation of the Minkowski metric.

I conclude by attempting a generic classification or taxonomy of decision problems and approaches to their analysis. This is a prelude to the discussion of the structure of the proposed methodologies in Chapters 5 and 6.

2.2 Why analyse decisions? The impaired decision maker.

Curiously, despite the many words written on how to analyse decisions, there is relatively little on why it is necessary. We might expect early words of any treatise to consider the issue and the following are amongst the early lines of a few works that address normative decision making.

> "Decisions permeate life. Indeed, many would argue that it is the ability to choose, to express free will, that distinguishes intelligent life from lower forms. However, we shall not rehearse that argument here. Instead we shall accept as a matter of empirical fact that each of us has the power of choice. Each day we make many decisions. Most are so unimportant that they can be left to whim: for example, whether or not to put salt on a meal. But some, particularly those we encounter in our professional lives, are sufficiently important that we undertake a careful analysis before deciding on a course of action." (French, 1986, p13).

> "In an uncertain world the responsible decision maker must balance judgements with his or her preferences for possible consequences or outcomes. It is not easy to do and, even though we have a lot of practice we are not very good at it." (Keeney and Raiffa, 1976, p1).

> "One of the most important tasks faced by decision makers in business and government is that of selection. Selection problems are challenging, because they require the balancing of multiple, often conflicting objectives, criteria or attributes." (Olson, 1996, vii).

> "Personal and management decision making can be complicated and confusing. The future of your organization and the progress of your career can be profoundly affected by what you decide, and yet most people receive little instruction in decision making." (Kirkwood, 1997, xi).

> "The key word is *analysis*, which refers to the process of breaking something down into its constituent parts. Decision analysis therefore involves the decomposition of decision problems into a set of smaller (and, hopefully, easier to handle) problems." (Goodwin and Wright,1991, p3).

Neither these works, nor others, address themselves at great length to why we ought to analyse decisions; though perhaps in these sentences we may find the main components of an explanation. It is apparent that we make very many decisions throughout our daily lives, with or without decision analysis. Later I shall suggest that the very "purpose" of our rationality, the adaptive advantage that it gives us, is the analysis of decisions. Contrary to Keeney and Raiffa, I suggest that we are in fact quite good at making decisions, or at least at making those types of decision that mankind already familiar with before the advent of our modern technological society. French suggests that decisions that we choose not to analyse are essentially trivial. Whilst I agree with him that professional life provides a particularly fertile ground for decision analysis, we might argue that the most important decisions in our personal life are the ones to which we actually pay little analytic attention. We may analyse the selection of a computer, car or hi-fi, yet, although the more important selection of our homes or jobs may be partially evaluated, these seem largely to be left to instinctual "holistic integration". The choice of our partners, arguably the most important personal decision, I suspect is only superficially analysed, even by the most model oriented amongst us.

I suggest that we specifically analyse decisions that have both the quality of importance *and* which we believe that we are not very good at. Our belief in our decision making ability may relate to our actual capabilities, or to our perception of them. Both of these may be influenced by specific experience (our direct familiarity with like decisions) and our adaptive experience (the environment of critical problems in the era in which our reasoning and other skills evolved). Modern economic life both intensifies the consequences of deviations from optimum choice and places most non-personal problems outside our adaptive experience, adversely affecting both our decision making abilities and our perception of them. We are, in short, impaired with regard to these problems and indeed we should analyse them, although often this impairment of perception, of cognition, also impairs our ability to recognise this. If there were no impairment there would be no decision problems. Optimal solutions would be intuitively obvious, and wisdom would be a universal quality.

I will later attempt to formulate a checklist of those elements of decision making and analysis that people might be good or bad at. However, one of our key impairments lies in the limits in our mental capacity to see to the limits of interacting factors, which for *all* real decisions are probably unbounded. As Simon (1957, p196) observes in defining his concept of Bounded Rationality, "The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real-world - or even for reasonable approximation to such an objective rationality". Simon (p199) goes on to suggest that someone making decisions nevertheless intends rationality and that this "requires him to construct a simplified model of the real situation in order to deal with it. He behaves rationally with respect to this model, and such behaviour is not even approximately optimal with respect to the real world." It is implicit that this process requires neither analysis nor conscious scrutiny, we just do it. Nevertheless, it also conditions all forms of argued decision making, including normative decision analysis. But in addition to simplification, decomposition of problems into separable parts of importance but of tractable size (the feature of the Goodwin and Wright quote), and the prioritisation of factors or attribution of weights, are important mechanisms for placing problems within the bounds of our rationality. It is the basis of Benjamin Franklin's "moral or prudential algebra" in 1772 (quoted in Appendix 1, and examined later in this thesis), and, even further back in history,of Cecil's sixteenth century assessment of options concerning Mary Queen of Scots. As DN Kleinmuntz (1990) also remarks, "Decision Analysis relies on the general principles of problem decomposition: a large and complex decision problem is broken down again to representations consisting of alternatives, beliefs, and preferences... An advantage of this approach is the reduction of information-processing demands since the decision maker can focus sequentially on simpler individual components of the problem."

If the fact of cognition impairment necessitating analysis seems unexceptionable, it is curious that decision tool designers seem to prescribe (the need for decomposition apart), either on the assumption that the nature of our impairment is self-evident, or without specific consideration of it at all, with less safe consequences. An example of this might be Multiple Objective Linear Programming (MOLP). This is predicated on a host of declared arithmetic assumptions and I suggest two implicit cognition assumptions. First, that a decision maker *cannot*, prior to the analysis, readily integrate the multiple objectives which conflict in their prescriptive implications, into a meaningful single overall measure of preference. Second, the same decision maker *can* nevertheless make a meaningful selection between the decisions identified as efficient, or at least of comparative preference between decisions represented by extreme points of the feasible space. If the latter is not true or if this comparison is not easier than prior integration of objectives, then the concept may not be helpful. There are grounds for supposing that some binary comparisons may be cognitively difficult so there seems a legitimate question here. The same issue arises, for example, in the Zionts (1981) method for choosing between discrete alternatives.

If it is a sound argument that the reason for decision analysis is human cognitive impairment, it follows that the nature of the decision maker's lack of facility must be at the root of the methods and models used to mitigate it. Such considerations may, indeed, be more important than the mathematical validity of methods and theorems which receive emphasis in Management Science literature. Some aspects of this are pursued in general terms in this Chapter and, from the perspective of evolutionary psychology, again in Chapter 3.

2.3 Objectives and Vague objectives

Objectives (including goals, targets, missions, etc.; which some distinguish) are central to our working lives, but also affect our private lives. We want to maximise profits, maximise our salary, win an Olympic Gold Medal, run a mile in under four minutes, run as fast as possible, visit China, have a nice garden, the best garden in the neighbourhood, maximise consumer satisfaction, be a good parent, pass our exams, maximise return, get home as fast as possible, minimise formulation costs, maximise growth, minimise risk, minimise rail fatalities, minimise rail fares, maximise punctuality, get a First, annoy Dad, achieve £100m of profit, achieve consistent profit growth, and win the Cup.

We feel that we understand these matters and to some extent we do. Certain lexicographic and binary objectives provide a clear guide to action, and are readily testable. We know whether we went to China, ran a mile in under four minutes, or achieved a First. But, thereafter, difficulties of one sort or another enter the concept. Objectives enjoining optimisation may provide firm guides to action and may be testable ("minimise formulation cost"), or their achievement may be subject to debate ("maximise profit"). But invariably qualifications enter the arena. These may be of the "but also ..." or "subject to ..." types. Sometimes these will be implicit ("get home as fast as possible *without* causing an accident") others be can readily clarified ("minimise formulation cost *without* detriment to product performance") but others require much more convoluted expression. Thus "maximise profit" may evidence a priority but cannot be actioned without constraints or amplification. These, will often imply the lower weighted inclusion of non profit variables, which conflict with the main achievement variable. Other objectives may provide perfectly adequate motivation and guide prescriptive action, whilst leaving open the testable meaning of the objective ("have the best garden in the neighbourhood"). Yet others may only reveal dilemmas without providing meaningful guidance. Thus "safety is paramount" may show political concern but ultimately it is unsustainable as a decision objective and has to be balanced with fares, profits, passenger amenity, and service reliability, if the existence of the operation is to be sustained. Even then the problems do not stop, simple statements, unexceptionable at root, may act as a coverall for a host of complex issues. Fares and profits may be manageable concepts (though accountants may warn us against hidden complexities) but what actually is safety, amenity and reliability?

Our facility with objectives, the ability to formulate and respond to them, seems to receive little research attention. Indeed, many works that descriptively address decision making or judgement do not include the word "objectives" in their indices. My own professional, but non-academic, experience of them could be said to be extensive. I have interpreted them as an OR analyst and I have optimised them in models. I have written them in Annual Operating Plans, Five Year Plans, Mission Statements, Job Descriptions, Managers' Work Plans for sections of business and for businesses, as responsible manager and as a "staffer". From both perspectives, I have analysed and criticised such statements of objectives of others, and been required to operate under them. In response to them, I have watched the performance of myself, and other people, in a business under my stewardship, and in other situations. I have been assessed and assessed others in the light of them; or *ostensibly* so. The greater my familiarity, the more sceptical I have become of the apparently well understood idea of "objectives" as an intuitively easy concept, in all but the simplest of idealisations.

I see the concept as capable of working effectively at two ends of the spectrum of their representation. The first, might be called "Work Plan" mode, "Reduce number of staff to 35, Launch brand X by 30 May, Work within budget of £1.5m, Research entry to Latvian market, etc". These provide for specific testable action. Whilst derivative decisions follow from these, such statements have many of the qualities of decision promulgation and are not simply or mainly value statements.

The second we could call "Mission Statement" mode; "Seek to become the leading supplier in Europe of electronics widgets, offering products at the cutting edge of reliable technology. Build close, trusting relationships with our customers, so that they see us, not merely as partners, but as extensions of their own businesses identified with their business ambitions. In our industrial operations, ensure that we do not merely respond to pressures for environmental good citizenship, but lead the business communities in the countries in which we operate towards providing an environment in which our grandchildren would wish to live. Provide opportunities for a fulfilling career for life for those of our people that wish it, whilst ensuring that their skills are always being extended for their benefit, not just ours. Ensure a profitable long-term investment for our shareholders by being careful with the resources they have placed at our disposal, without compromising our long-term ambitions for short-term expediency".

Such statements probably do not provide direct actionable guidance but provide a context, an intention of business style and climate, for people to develop actionable objectives for themselves. Despite ends and means being confused, the risk of utopian ambition, and lack of information on resolution of conflicts, they provide a background of considerations. Their merit lies in their qualitative articulation of factors.

In between, in the area of projects and in the resolution of specific alternative decision choices, Objectives seem, at least to me, to be less easily operationalised than the analytic models of OR appear to imply. Whilst documentation of cognition of objectives is sparse, the suggestion is not unique. Lindblom (1959, p82) notes that people can more readily agree about policy (decisions) than ends (objectives). He writes, "Except roughly and vaguely, I know no way to describe- *or even understand-* what my relative valuations are for, say, freedom and security, speed and accuracy in government decisions, or low taxes and better schools than to describe my preferences among specific policy preferences that might be made between the alternatives in each of the pairs." [My emphasis]. He makes a more radical point than mine, questioning the suitability of what he calls the Rational-Comprehensive method, which involves isolation of "ends" as a prelude to seeking the means of achieving them. He contrasts this with Successive Limited Comparisons where the selection of values/goals and the analysis of needed action, are intertwined, and the test of a good policy is that people find themselves agreeing on the policy, without agreeing that it is the appropriate means to an agreed objective. He also remarks, "The idea that values should be clarified, and in advance of the examination of alternative policies is appealing. But what happens when we attempt it for complex social problems". I would exclude the word "social". March (1978) too observes, "Human beings have unstable, inconsistent, incompletely evoked goals at least in part because human abilities limit preference orderliness."

This limitation in comprehension of what we seek, may be thought of as another manifestation of Simon's Bounded Rationality, already mentioned. (Though he perhaps paid less attention to the complexity of values than of the complexity of decision consequences). Nevertheless, the full expression of objectives can be seen as similarly expanding from bald, simple statements, into a mass of qualification, amplification, and supplementation, of unbounded specificity, which we find it difficult to get our minds around to secure precision of consequential action. I later suggest that we lack innate capabilities of multiple objective juggling because our ancestors' problems were of a simpler type, governed by *roles* not long-term *objectives*. Shorter term aims of harsh simplicity were handled as serial single objectives and decision situations (of adaptive consequence) generally involved simple trade-offs.

One of the approaches of the "Rational-Comprehensive" method has been to choose from qualitative statements of objective, a primary quantifiable measure and then to include "subject to ..." qualifications, in effect optimising in one dimension and satisficing in all others. It could be coincidental that this structure finds a ready analogue within the tools of Management Science such as Linear Programming (though one might wonder whether ease of computability of such a structure is the thread linking the human mind and the software driven machine). The other approach commended for the professional sphere, is the progressive decomposition of a few unquantified complex strategic objectives, into derivative sub-objectives in a hierarchical manner, until these can simply quantified or classified, or a suitable proxy can be identified for the objective. The relative value of these low level objectives is then elicited by a mechanic which satisfies analyst and decision maker (eg French, *op cit,* p105; Keeney and Raiffa, *op cit,* p41; Kirkwood, 1997, p41, Olson, 1996, p9; Saaty, 1980).

Underlying this, is the notion that the sub-objectives and the values relating to them have an existence, *ab initio,* of the same type as the primary objectives; that is that they are, in some sense, already in the mind of the decision maker (or in the corporate mind) and we merely need to elicit them. Merely to state this in such bald terms, exposes that this can rarely be so. Following Keeney and Raiffa's example of a postal service; suppose the overall objective is "to provide efficient, dependable service to the users of the system and to government". They suggest one sub-objective might be to "minimize the total cost of handling the mail". A derivative of this (mine not theirs) could be "minimise the perceived value to the customer of the lost time in posting letters". A further derivative could be "minimise the time spent by customers looking for post boxes". Would the decision maker have the latter two in mind when he or she started? Unlikely; but even were this so, he or she would not be cognisant within his or her bounded rationality of the myriad of similar possibilities. It is more realistic to see the process as sub-objective *creation* rather than elicitation. Quantification follows identification of, hitherto, unidentified issues. Such an identification is important but suggests that, whilst objectives may exist in the mind in a firm qualitative sense, they may not exist with actionable precision in the values attached to them. A guide to the identification of options is not the same as statements of value that determine selection between them.

But, in any case, is this professional model the way we naturally think about decision problems? In our personal lives, we might have the objective of finding a new home "a nice place, suitable for the kids we are planning to have, within suitable reach of work". We might then identify factors; the size of the main bedroom, the kitchen equipment, the friendliness of the neighbours, the proximity of the nuclear power station, the distance to the railway station and to the shops, the size of the garden, the availability of good schools in the locality, the rail journey time to work, the state of the decoration, the existence of garaging, the distance to our parents, nearness to the golf club, the pleasantness of the neighbourhood, and so on. We can get to such lists independently of a tree of sub-objectives. Of course, such a tree can prevent one missing relevant *factors* and it is useful for that purpose, but a top down approach is not insurance against omissions, anyway. Proximity to our parents, retail shops, recreational facilities and friendly neighbours were not within our original objectives. Generally, I suggest we can find contributors to the goodness and badness of a decision, and can have a strong belief about the relative importance of such contributions, without firmly or precisely articulating our main objectives. Indeed, it is the integration of such valuations which can constitute the articulation of those objectives.

In this work I take the position that a decision maker often has a good qualitative understanding of what he or she wishes, which is a composite of many objectives. However, he or she has not articulated these quantitatively nor translated them into a hierarchy of consequential sub-objectives. Moreover, whilst every decision has quantified or quantifiable defining Attributes associated with it, which reflect progressions in decision quality, its goodness or badness, related to fundamental objectives, they will often be highly derivative and not direct quantitative equivalents or branches of the objectives. (This is different definition of Attribute from that of Keeney and Raiffa (1976, p34) who see an *attribute* as a measurement used to represent an *objective*). Nevertheless, I also take the position that a decision maker may well be able to articulate many of the pertinent attributes in exploitable terms. Even if these are not necessarily initially exhaustive, they may often cover the ground adequately or can be made to do so, with little sophisticated intervention by the analyst.

It is in this sense that I describe a decision maker's objectives as "Vague". I ascribe this specialist meaning within the thesis. It is defined here as "incompletely articulated" or "inexactly or only partially expressed in quantitative terms". The term, however, excludes elements of the dictionary definition. It does not embrace "inexact in thought", and, particularly, does not imply indecisiveness or woolliness. In using "vague", I try to avoid confusion with the specialist connotations of words such as "uncertain" and "fuzzy".

Vagueness might arise from two mechanisms. The first is an unwillingness or inability of a decision maker to commit, or to commit yet, to a firm quantification. The second is a more speculative assumption, arising out of the nature of objectives, which prevents certainty in some more fundamental sense.

I suggest that many objectives that can be simply and clearly expressed in qualitative terms and are in principle quantifiable, cannot be expressed by a single quantitative measure and are likely to be describable only by an undetermined complex of variables. Moreover, it is likely that only one or two of a set of multiple objectives will be easily expressible as individual measures. Usually one quantitative objective can be defined relatively straightforwardly. In business, money-value objective scales such as Net Profit, Opportunity Cost, DCF, Marginal Contribution etc. are frequently invoked and are readily manageable. Additional objectives may be less simple to express eg Increase market standing (market share, market penetration, or consumer loyalty?); maximise plant safety (how defined?); maximise profit growth (what period?)). However, objectives become quantitatively very much less specifiable, as more are included in the objective mix. Rather, the objectives become concepts. These give rise to sub-objectives in the traditional approach but, in the view here, provide context in which value can be accorded to the effects of decisions. Examples might be "Financial Risk", "Environmental Friendliness", or "Customer Satisfaction".

I do not suggest that this is cannot be handled within existing approaches. Top-down hierarchic generation of sub-objectives is a viable route. However, here, to deal with both the difficulty of succinctly quantifying many types of objectives, and a decision maker's likely vagueness in articulating individual objectives and their interact, I adopt an outlook in which the focus is removed from starkly stated hard Objectives. I propose to focus on the softer representation of a decision maker's ambitions, the decision Attributes and their desirability. Attributes must be reflective of issues of importance and possible importance to the decision maker and, as such, they will be objective-related, but the decision maker's commitment to them may be of a lesser order. Attribute measures should relate monotonically to desirability, that is either more should be better to the decision maker, or more should be worse, throughout the scale range. Notwithstanding, *concrete* Objective measures if articulated, would sit compatibly within this framework. Objective measures are also Attributes.

With this outlook, the Decision Analysis problem might be looked-upon as the progressive *bottom-up* synthesis of a unique quantitative objective, expressed in terms of the attribute variables, which metrically reflects a decision maker's preferences between packages of attributes corresponding to alternative decisions. It is a synthesis process designed to operationalise the decision maker's vague qualitative objectives, converting them by stages to a hard quantified one, or at least converging towards one, enabling the identification of a preferred option and/or option ranking.

2.4 The concept of Value in decision analysis.

The method explored here, and my personal disposition, reflects a Value frame of reference. It is what Stewart (1992) characterises as a "Value or Utility Based Approach" in distinction to "Goal or Reference Point" oriented approaches, or the "Outranking Concept" of Roy and others (eg Roy, 1968), for dealing with multiple factor problems.

Whilst I recognise that some may find the alternative approaches instructive, they have little appeal to me. The issue relates to the idea of Compensation. If all indicators of decision quality point to the same conclusion, no problem arises, the desirable conclusion is the single non-dominated option. If this does not pertain, we are forced to choose between options which are better in some respects and worse in another. We are forced to express a preference between a relative gain in one or more dimensions and a relative loss of others. "I prefer the combination A, B and C to D, E, F" or, equivalently, "I place a higher value on A, B and C". This may be symbolised (as per Keeney and Raiffa, 1976, p68 or French, 1986, p75; though formal consideration of the existence of utility functions which map preference pre-ordering, is attributable to Debreu (1959) :-

$$\boxed{\begin{array}{l} \mathbf{x}_1 \succ \mathbf{x}_2 \Leftrightarrow v(\mathbf{x}_1) > v(\mathbf{x}_2),\ \ \mathbf{x}_1 \sim \mathbf{x}_2 \Leftrightarrow v(\mathbf{x}_1) = v(\mathbf{x}_2) \\ \text{Where } \ \mathbf{x}_n \text{ specify the magnitudes of a "bundle" of} \\ \qquad\qquad\quad \text{attributes defining option n} \\ \quad v(\mathbf{x}_n) = \text{value of attributes } \mathbf{x}_n \end{array}} \tag{2.1}$$

Trade-off or Compensation, whether a gain on some measure is *more valuable* than a gain on another, seems to be at the root of multi variable decision making and would seem most fruitfully to be tackled directly. If I may not have both what is the most valuable? To take refuge in approaches which, by some procedure, avoid the decision maker having to face specific relative value problems seems at best to be generating indirect (and, ergo, likely inferior) expressions of value, for example, in deviation metrics. More dangerous is that one hides the generation of indicators behind complex arithmetic manipulation, which makes the attachment of intuitive understanding to levels of indicators and parameters difficult (for example, to concordance and discordance).

I except from this, lexicographic Goal Programming (GP) (eg Lee,1972; Zeleney, 1981), in certain situations. This might indeed be an appropriate technique, for example, if there are marked non-linearities of desirability over the domain of hypothesised decisions, which might be adequately reflected in pre-emptive step movements. This could define a useful heuristic to guide the user to the vicinity of an optimum. However, in the region close to a practical solution, I would expect a more subtle trade-off of factors, and therefore a different mechanic, to be appropriate. Stewart notes that GP can be seen as an operationalisation of Simon's satisficing principle (eg 1965,p xxiv), but satisficing is a forced consequence of man's Bounded Rationality. Whilst this may be a good mental heuristic rendering difficult problems tractable *in the absence of other decision aids*, it is not incumbent on the normative modeller to apply a corresponding principle. On the contrary, he or she works to extend our rational bounds and must not be restricted to elaborated replicas of the mental process. This risks missing an opportunity.

2.5 The internalisation of value

In the world of tangible measurement we adopt standards, until relatively recently, often physical objects, which we can touch and hold, that enable us to extrapolate one unit of magnitude to 2, 1000, or 0.38276. With such standards we can adopt cardinal measurement.

In adopting a Value conceptualisation of preference I make no assumption that I, or anyone else, carries around a cardinal yardstick of value (as von Winterfeldt and Edwards (1986, p 353) express it) "in his or her head waiting to be elicited". I will discuss the lability of preference shortly and this would seem to rule out an internal reference standard concept, but there are philosophic objections in any case. Although I equate preference with value, statements of preference constitute statements of *ordinal* value only. Whilst a value structure uniquely specifies a preference structure: a preference structure does not specify a value structure (Keeney and Raiffa, 1976, p81). A value function, which satisfies an ordering of option preferences, will be no better nor worse than any strictly increasing transformation of the same function (the square of it, for example). It is only when one seeks to introduce first order Value Difference Measurement (French, p 82), to prefer the gain in benefit between two options to the gain between two others, that any internal basis within the mind exists for converting preference information to an interval scale.

This involves exchange statements of the type, "I would prefer someone to swap a vanilla ice cream for a coffee ice cream more than I would prefer someone to swap a strawberry ice cream for a raspberry ice cream". Although von Winterfeldt and Edwards suggest, "Ordinal judgements of differences on genuine subjective continua can also be constructed by a little introspection", I wonder whether this can be done, sufficiently consistently and reliably, to yield useful cardinal scales for many categories of preference. I am not aware of empirical evidence but would remark that, if first order preference is labile (which I will consider), second order effects, on which internalised interval scales depend, will be much more so.

Moreover, applying an Evolutionary Psychology test, such exchanges would have not arisen as adaptive, or even practical, choices in the environment in which we evolved. It is therefore unlikely that we would have evolved innate mechanisms for dealing with them. They are indeed rare as concrete propositions in the stated form in modern life. (We might, as per French (p 85), construct a scale of value for quantities by asking a respondent if the exchange $a_0 \rightarrow a_1$ is of equal value to an exchange $a_1 \rightarrow a_2$. But this choice is always hypothetical as a_0 and a_1 cannot simultaneously be available for exchange). Comparing $a_0 \rightarrow a_1$ with an alternative independent exchange $a_2 \rightarrow a_3$ is also unreal as perhaps the ice-cream example illustrates. Of course one might be able to equate preferences to objective yardsticks of value which are external to the decision maker, such as money; and if we allow comprehension of cardinal probability we can construct scales of utility. However, an innate comprehension of probability may also be suspect, as will be discussed later.

As we cannot readily define internal units of equal value, a precondition of the construction of a mental standard, I reject the ability of people uniquely to cardinalise value on an interval scale as an *internal* concept, that is independently of external standards, and of probability. Moreover, as will be discussed, the ability to develop additive scales of value depends on value being a conservable quantity. In the presence of mental lability this also seems doubtful as a strict proposition. Nevertheless, the assumption, in analysis, of such a structure for inferential purposes appears useful and legitimate. I see the notion that the decision maker carries a value yardstick within him or her as a Fictional Construct of the type described by Vaihinger (1935) (p15): not a "copy of reality" but, along with all constructs in the world of ideas, an "instrument for finding our way about more easily in this world". One might envisage the process as a condensation of preference to get an assumed measure of relative worth of options, one of many which might do the task, which summarises the decision maker's purposes. Essentially, the scale of value expressed should render his or her declared preferences (expressed in a reliable way at a particular moment) free from contradictions, or as free from them as alternative candidates. Provided the decision maker behaves "as if " he or she had the valuation mechanisms embedded within him or her, he or she endorses the measure and its implications, and is an adequate representative of other stakeholders in the decision, then the model has met the requirements of validity. It does not need to be "true" in some absolute sense, it just needs to be one of the many strategically equivalent scales that reflect explicit preference, from which identical conclusions would be drawn.

To illustrate, we might consider Saaty's Analytic Hierarchy Process (Saaty,1980). This, *inter alia*, generates from comparisons of qualitative importance between pairs of characteristics, a ratio scale, which we might call "value", of the various characteristics. This is based on finding the eigenvector of a matrix representing the relative "importance" of each pair of characteristics. To construct this, Saaty assigns a numeric ratio of relative importance to each semantic statement, for example 5 to "Essential or Strong Importance". These assignments are arbitrary, and a user might assign different numbers to the same statements; for example, the square roots of Saaty's indices. Should he or she do so, each element of the eigenvector would be scaled by approximately the square root of each element of the original eigenvector (this can be empirically illustrated and can be shown to be precisely true for fully consistent matrices). The two scales would be strategically equivalent leading to the same ranking of preferences (except where two characteristics are close in value *and* the subject had also made material mutually inconsistent responses). What represents the true scale of importance or value, is whatever quantitative operation the decision maker feels should be attached to his semantic statements.

2.6 The stability of preference and value

If we cannot have a cardinal scale of value within us, can we have stable values, preferences and tastes. This is not necessarily precluded philosophically. One can note that, in the physical world, many of the methods that give rise to standard cardinal measurement are in reality ordinal operations. For example, a balance does no more than compare the relative weight of what is in one pan to that in the other. However, cardinal measurement allows one to be highly economic in the number of reference standards kept; indeed, just a single standard of weight allows one to construct a cardinal scale of weight of infinite extent and subdivision using only the simple, "ordinal", balance. Ordinal scales, however, do not have this property of economy. For example the Moh scale of hardness depends on the ability of a sample mineral to scratch or be scratched by specifically named minerals. A standard must therefore be held representing hardness for each point in the scale. Thus to have stability of values, preferences and tastes within an ordinal system, we must hold multiple reference points within us.

This is not a formidable problem when we seek to compare an option in a familiar domain in a single dimension (or a standard composite in multiple dimensions). Thus, for example, we could hold "within our mind" an ordinal scale of preference of motor cars, based on familiar marques of car. We might be able to "slot" a new "option" into this scale, dealing with perturbations from the most similar standard in the normal mix of attributes, in a trade off process. If such differences were relatively small, I suggest we could expect stability of preference, that is, if a similar choice was presented at a different time, a similar selection would be made. If the choice is multidimensional, and not within a familiar domain, this would not necessarily apply, nor would it apply within a familiar domain if attribute variation differed considerably from our references. But there is also an information issue. 3 bits provides for an ordinal scale of 8 references in a single dimension (or correlated multi-attribute) scale, allowing quite subtle discrimination. The same 3 bits provides for only binary classification in 3 dimensions allowing for little trade-off discrimination. Accordingly, we need a considerably greater number of ordinal scale standards for precision of definition, and, thus, preference stability, in a multi-dimensional situation.

I later examine the issue of preference stability from an EP viewpoint. For the moment I briefly summarise my conclusion that it is difficult to perceive an adaptive problem that stability of preference in multi-objective trade-off would solve. In general, ancestral problems would have been disparate, often not repeated and, for the most part, single objective at the time a decision was made. I will argue that we were most likely endowed with evolved mental and biochemical abilities to select a priority single objective giving rise to an innate *serial* single objective "heuristic". Given such a mechanism, there would be considerable adaptive advantage in being able to exploit experience to improve accuracy of *assessment* of factors related to a current single objective, even when assessing the impact of many inputs; and we can expect innate competence in this area. But that is not a value problem. We are unlikely to have been endowed with a stable mental yardstick of value, or an ability to reduce multiple attribute magnitudes into a finely differentiated structure of consequential values.

Fischoff, Slovic and Lichenstein (1980, p137) write, "Expressed values seem to be highly labile. Subtle changes in elicitation mode can have marked effects on what people express as their preferences. Some of these effects are reversible, others not; some deepen the respondent's perspective, others do not; some are induced deliberately, others are not; some are specific to questions of value, others affect judgments of all kinds; some are well documented, others are mere speculation": but it seems that more than mere *expression* is labile. Within our language is the understanding that preference is mobile and ambiguous. We "are in two minds", we "change our mind" , we do things "against our better judgement". Are the values our own or assumed for others for whom we must act? Do they reflect our tastes and desires unmodified by conscience or strength of will? We know that they are affected by extremes of mental state (Alloy and Abramson, 1979) and that our assessment of the likelihood of positive and negative events also seems affected by induced mood (Wright and Bower, 1992). But are not values also likely to be significantly altered by subtle changes in emotional influences? A bright spring day puts a bounce in our step, makes us more optimistic than yesterday, and alters our values. Our values change after a drink, after sex, after we have "slept on it", and, perhaps, after a few minutes. "Positive or negative moods often seem to influence our attitudes and values, the judgments we form .., the way we speak.., the way we plan .., and even the way we relate to well-known others." (Forgas and George, 2001, p4). Additionally, as March (1978, p155) observes," We avoid our preferences. Our actions and our preferences are only partly linked. We are prepared to say that we want something, yet should not want it, or wish we did not want it." He also considers (p154) that we construct our preferences in association with our actions that "we elaborate our tastes as interpretations of our behavior": this would be impossible in a mind of stable values. Wright and Goodwin (1999) similarly emphasise that options inform values, as well as vice versa, and distinguish between unformed and labile values. They also debate the methodological implications that the prior intellectual consideration of values associated with a decision in prospect, may be different from our feelings, and hence values, for it once made. They suggest simulation as a means of looking beyond the decision. But whilst this may diminish uncertainty, lability is likely to remain.

This instability adds to the philosophic problem of what a person's values actually are. Are they as just expressed? Probably not if the person cannot explain his change of view. They are likely to be snapshots, not of a materially moving target, but a momentary expression of equal validity to those coming immediately before or afterwards. This would imply that the conclusion based on any such momentary value may be perfectly satisfactory but that the value frame of the person still remains unseen, hidden in a never expressed average, or even with no average at all.

Of course, the unstable nature of preference might not in principle always preclude the statistical discrimination of fine structure, *should* a model of preference be fitted over enough observations. However, the nature of the decision problem, in contrast to a diagnostic one, is that it is essentially one-off. Meaningful judgements can be expressed between relatively few expressions of preference, and most decision problem structures do not lend themselves to increasing the number of "experimental observations". (Indeed, multiplication of elucidation tests, risks loss of validity by compromising the sustainability of the interest of the decision maker). I suggest that, similarly to the Heisenberg Uncertainty Principle of physics, there is an inherent unimprovable uncertainty in the expression of preference. Moreover, attempts to discern fine or complex value structure by seeking to examine preference, as if from an unconscious black box for which all options are open, will in most instances be spurious and misleading. It is doubtful that such derived values, unless they are consciously endorsed, can be considered rational. I will return to this point.

Whilst one must remain open to the possibility of mis-specification, this feature alters the "burden of proof". The task of the modeller is not to define a comprehensive "best fit" representation of the mind of the decision maker, independently of that mind; in any case, as futile a task in a jelly world of labile preference as taking a photograph of an electron. Rather it is to capture, as simply as possible, all well-considered expressed preferences of a decision maker in a transparent statement of value that he can examine, modify and use to produce potential solutions and which can enhance (though not fix) the solidity of his value and preference base.

One should note that lability of Value does *not* imply either, that people find it difficult to identify issues or factors of importance in a reliable and stable way, or, that their structuring is unstable. In fact, the contrary seems to be the case, as suggested, for example, by B Kleinmuntz (1990) in distinguishing the relative competencies of men and machines, and Axelrod (1976, p14) in commenting on the apparent stability of cognitive maps.

2.7 Capacity as an insight into preference stability

I have suggested that we are unlikely to have been endowed with a stable mental yardstick of preference, enabling translation of attribute magnitudes into a finely differentiated scale of consequential values. Can this view be triangulated by other evidence?

Certainly, if we have a restricted ability to discriminate in other areas of importance to our survival, then it would not be reasonable to expect our minds to be capable of reducing multi-dimensional attributes into a more finely graduated scale of value. Without that, stability of value is impossible.

Miller (1956) collated work by others from which he assessed channel capacity to discriminate and report psychological (mainly sensory) phenomena differing in degree on a single dimension to approximately 2.5 bits (the "magical number seven" as he approximated it). He amplified this by explaining, "First, note that 2.5 bits corresponds to about six equally likely alternatives. The result means that we cannot pick more than six different pitches that the listener will never confuse. Or, stated slightly differently, no matter how many alternative tones we are asking him to judge, the best we can expect him to do is to assign them to about six different classes without error. Or, again, if we know that there were N alternative stimuli, then his judgment enables us to narrow down the particular stimulus to one out of N/6". There were some phenomena for which at least a 3 bit capability was recorded but none as high as four. He noted that where the number of dimensions in which a phenomenon was represented increased, so did the aggregate information discriminated, though the information capacity in each dimension diminished. He reached no firm conclusion on what the aggregate limit of multidimensional information was possible for people to discriminate, nor the number of dimensions in which it was possible to make simultaneous meaningful distinctions. However, he noted an experiment by Pollack and Ficks who varied six different acoustic variables (frequency, intensity, rate of interruption, on-time fraction, total duration and spatial location) and allowed each variable to take one of 5 levels, ie 15625 different possibilities. Under these circumstances transmitted information rose to 7.2 bits corresponding to 150 different categories or just over 2 sub-divisions per dimension.

Relative to the variety of variation of any phenomenon in the environment (nearly always infinite), this degree of discriminatory power (more rigorously, the ability to report discrimination) seems paltry. I will suggest later that this apparently limited capability is commensurate with the primeval "need" to discriminate.

However, if this capacity limitation is reasonable it is an important indicator of potential value precision. If we cannot hold an ordinal standard of many bits for well practised and manifestly important sensory phenomena, it is unreasonable to expect that we could discriminate multivariable contributions to value with greater finesse. If we can store less than 2^3 ordinal scale standards for single dimensional, and less than 2^8 standards for multidimensional sensory phenomena, we cannot expect contributors to value (of far less adaptive importance) to have more "in the mind" standards.

However, value integration over multiple dimensions is likely to be cruder than this. In the single dimension task phenomena were discriminated *and* ordered, but in the multiple dimension tasks they were discriminated but only ordered in each dimension (eg quite loud, half left and far away). They were not reduced to a single dimension, such as, for the Pollack and Ficks experiment, to a measure of noticeability. Reduction to a single dimension will itself reduce precision and information, and we might expect a single dimension measure *derived* from multiple attributes to be reported with less reliability than an unmodified important phenomenon measured directly. If we cannot postulate a mechanism, or an adaptive advantage to the species, should we not expect a weight defining a value trade-off to be discriminated with less reliability than the 2.5 bit average that Miller suggests for specific sensory systems?

I am not aware of any empirical research explicitly on this. However, Hayes (1962) conducted experiments on a simulated military problem for which multi-dimensional information on factors relevant to the decision were presented to suitable subjects. *Inter alia*, he examined the amount of information transmitted from available data to the solution of the problem. This was measured in a manner which reflected the reduction in the variety of decisions made as a result of the information. For example, if data reduced eight *a priori* equally likely options to two selections, each chosen with equal probability, there would be 2 bit transmission. Information transmission increased with a higher number of options (8 rather than 4), but there was no evidence that the provision of extra information, in the form of data on other relevant characteristics, resulted in more information being transmitted to the decision, nor were the decisions of better quality. Also, remarkably (although Hayes makes no comment), the information extracted is low. In an experiment where particular attention was paid to the training of subjects and there was improvement in transmitted information and decision quality, transmission was, in the 8 alternative case, typically 1.5 bits, a considerable reduction on the Miller discrimination. 1.5 bits constitutes an ability to classify reliably into only three classes. Even if the Hayes problem was argued to be difficult (the genre may be unfamiliar to many of us, but there should is no reason why the problem should have been difficult for the subjects he used), it is hard to believe that better than 2 bit extraction is possible.

The failure to extract extra information by adding variables, adds support to the view of Larichev (1992) that people cannot reliably express preference between binary choices that simultaneously vary in more than two dimensions. When data on additional factors does not result in extra transmitted information, it is an indication that we cannot readily digest the implications of multi-dimensionality in establishing our preferences. (Though, we should not ignore that a decision maker might be attaching zero value to the additional dimensions). Capacity constitutes a "bound to rationality", as Taylor (1975) points out when describing "cognitive strain". This bound appears quite limiting to our value discriminatory competence in multiple dimension situations.

In one matter, however, the translation from an attribute measure to value is clear enough. Favourable variations however small, if in the same direction, will always be perceived as more valuable, and this conclusion may be argued to be finely discriminated. But the source of this ability is apparent and distinct from multi-dimensional integration. It does not depend on an in-the-mind standard, it arises from properties of the objects not of the mind. Decisions in this case are literally "no brain" decisions.

2.8 Judgement as an insight to preference stability

Judgement in a multiple factor situation has much in common with choice in a multiple attribute situation. Indeed, we often say of decision makers that they require the quality of "judgement"; by this we mean that they should be good at translating multi-variable data into decisions. Unfortunately, the quality of decision making is usually uncheckable by examining outcomes, except in the long run and then only poorly. Values which underlay the decision can rarely be checked and, because often the circumstances were unique, the maintenance of those values and their "definiteness" cannot readily be examined.

On the other hand, Judgement, which we might define as the process by which information is processed to form a conclusion, is checkable against outcome, in some circumstances. Judgements may be of the types: What is the best city for our

Head Office? Is John more handsome than Peter? Is Manchester United the best soccer team this season? Is the accused Guilty or Not Guilty? Will Brave Warrior win the 3.15pm at Chepstow? Has the patient got Chicken Pox? How long will the patient live? Some of these are multi-variable decisions of the type we are used to analysing (the first, for example): they involve value as well as factual assessment but are not checkable, unless best is pre-defined. Evaluation is also are confused by environmental circumstances differing from the assumptions which underlay the decision. Whether John is more handsome, or MU the best, are essentially decision-less expressions of opinion dependent on private value judgements. "Is the accused Guilty?", is a question of fact not of value, but is not checkable outside a process which gives rise to the same question. Brave Warrior's success in *that* race can be checked but the experiment is not repeatable and the unbounded number of the factors militates against generalisation. But whether the patient has Chicken Pox and how long the patient will live, do not involve value or preference, though the questions do involve *valuation*, and at least at some point the correctness of the conclusion becomes clear. Moreover, the judgement of facts relating to the diagnosis and prognosis can be, and is, repeated many times, in sufficiently identical circumstances, for only variation in a bounded set of clinical indicators to be relevant. It is for this reason that the quality of clinical judgement can be judged, and, very importantly, assessed in terms of how information is processed to draw conclusions on the consistency of the valuation of factors bearing on the problem.

We can argue that values, preferences, or tastes are the judgement determined "weights" from which knowledge of magnitude of factors is converted into subjective measures of "goodness" of each decision option. This is very similar to a clinical judgement task of valuation of factors. The consultant must form, from a weighted composite of cues, a subjective measure (in this case true/false) of whether the condition is present. However, the differences (a well-bounded unambiguous problem, a clear goal, repetitiveness, an objective, if unrevealed, association between clearly observable factors, and an observable end allowing retrospective testability), permits scrutiny of the valuation process which is not present in most decision processes. It is also a problem where consultants can learn and improve with their own direct and vicarious experience of other specific cases or research conclusions from them. In short it is an examinable problem, which reasoning players should do particularly well at, but is sufficiently similar in other respects, for broad conclusions to be translatable to decision analysis value and preference formation. Indeed, consistency of application in cue weights in such a judgement process, is likely to be of greater order than the consistency of the components of a value objective or preference. Work on clinical judgement is likely to indicate an upper bound of capability in value definition in more ambiguous circumstances.

A significant amount of work has been done on Judgement and consolidated into general findings (Goldberg, 1968, 1970; Dawes and Corrigan, 1974; Dawes, Faust, and Meehl, 1989; B Kleinmuntz, 1990). The most intriguing of these follow the development of models which relate factors, not to proved medical conditions, but, to clinicians' diagnosis of medical conditions. Hoffman (1960) described such valuation simulating models as *paramorphic* representations of the decision maker's judgement; a word he coined to reflect that such models did not even attempt to mimic the actual evaluative mechanism but merely the results of such processes. Hoffman's linear models captured a considerable proportion of the variability of his subject judges. He also observed that increasing the complexity of the models to incorporate the configurality (interaction effects) of assessment, appeared not to add to the explanatory power of the model.

Hoffman himself made no use of the models he developed to assist diagnosis but Yntema and Torgeson (1961), investigated both the mathematical adequacy of linear models of value and the linearisation of value that subjects seemed to use to value non-linear phenomena. They suggested that such models of judges could be more effective future predictors than the actual judgements, "The improvement achieved by averaging a number of responses suggests an intriguing possibility. Artificial, precomputed judgments may in some cases be better than those the man could make himself if he dealt with each situation as it arose.". Bowman (1963) drew a similar conclusion in the production management area; "a decision rule with mean coefficients estimated from management behaviour should be better than actual performance" (p321). The conclusion that the model of the judge appears to be better than the judge appears to be robust, and Baron (1988, p406) suggests, " ..even for the cases that the judge has already judged." The methodology of capturing the policy of a decision maker/s in a model, often a simple linear model, and then using the model rather that the judge is termed Bootstrapping. Hoffman

(1968) pointed out the methodological difficulty of establishing the existence of configurality in judgement processes, observing, for cases examined, that significant interaction effects were small relative to linear contributions. Slovic (1969) also challenged the suggestion that judges do not introduce configurality into their processes, looking in detail at two stockbrokers. However, the central issue, for the purpose of this thesis, is not that judges are configural (which I am willing to take as read), but that unsophisticated linear models of judges outperform the actual judges, notwithstanding the additional factors such as configurality, special circumstance treatment, access to additional information, and whatever other intelligent use of information they bring to the assessment.

Goldberg (1970) introduces an interesting qualification. Whilst models outperform single judges on which they are based, models of the composite or average views of several judges looking at the same evidence, may not outperform the composite judgements of the group of judges. Blattberg and Hoch (1990) also suggest that whilst a statistical (non-bootstrap) model may out-perform an expert, a composite of statistical model and judge outperforms the model.

Dawes and Corrigan (1974) go further, questioning best fit representations and demonstrating that in cases that they examine, equal weight and random weight linear models also outperform the judges on which they are based. Dawes (1979) subsequently argued that such "improper models" chosen by nonoptimal methods have practical validity. Wainer (1976) further suggests that equal weight models often differ little in predictive power from optimally weighted models, have greater robustness, and are more appropriate. A similar view is also taken by Einhorn and Hogarth (1975) outside the Bootstrapping context. However, whilst I am sympathetic to the concept of avoiding gratuitous precision and the concomitant danger of self-delusion that precise parameters imply a precise model, it is illusory that such an approach removes view-forming on appropriate weights from decision maker/ analyst. The selection of variables, and the choice of scale units on which an attribute is measured, may be viewed as implicit weight selection. The fact that very simple paramorphic models outperform the judges on which they are based is not of itself a reason to eschew a more complete, but still paramorphic model, unless user accessibility compensates imprecision. Wainer's view appears somewhat extreme; it is not difficult to synthesise counter examples. [In an illustrative test I undertook, one could continue to get very good answers (ie explain a comparable percentage of variance), if equal weights were assumed when optimum weights were, say, .6 and .4, but not when true weights were .8 and .2. Then it appears better to ignore the second factor altogether. Moreover, there is a mid-ground where it appears that neither simplified heuristic is adequate. It is possible that a composite, in which equal weight, 2:1 weight, or 1:0 weight, are alternatively assigned, might have explanatory power close to an optimum least squares model.]

The success of paramorphic models relative to the judges on which they are based does not reduce the importance of humans in being able to recognise patterns and to identify factors of importance. However as Dawes, Faust and Meehl observe (1989, p1671), "The possession of unique observational capacities clearly implies that human input or interaction is often needed to achieve maximal predictive accuracy (or to uncover potentially useful variables) but tempts us to draw an additional, dubious inference. A unique capacity to observe is not the same as a unique capacity to predict on the basis integration of observations. ...greater accuracy may be achieved if the skilled observer performs this function and then steps aside, leaving the interpretation of observational and other data to the actuarial method." Ebert and Kruse (1978) provided specific evidence of the applicability of bootstrapping in the investment environment. They too suggest that analysts should concern themselves with the search for new cues and development of search procedures rather than making judgements about future returns.

How might these findings be interpreted in a process model?

Let v = true value of condition being assessed
v_J = judged value of condition being assessed
ε_J = random variable representing overall error of judge's estimate
x_n = magnitude of factor n (x_n may embrace a constant term)
a_n = true weight of factor n
e_n = judge's systematic error in weight for factor n
ε_n = judge's random error in weight for factor n
S = contribution to value of factors not in paramorphic model (perceived as random by the paramorphic model but discerned by the judge subject to error).
ε_S = judge's error in estimating contribution of factors not in paramorphic model

S and ε_S subsume all factors not incorporated in the paramorphic model of the judge, including unincorporated variables (including random effects) and configural modifications. We can thus write the true relationship describing the link between a judged phenomenon and the weight of indicative factors as

$$\boxed{v = \sum_{\text{all } n} \mathrm{a}_n x_n + S} \tag{2.2}$$

... and the judged value according to the judge as

$$\boxed{v_J = \sum_{\text{all } n} (\mathrm{a}_n + \mathrm{e}_n + \varepsilon_n) x_n + S + \varepsilon_S} \tag{2.3}$$

... and the paramorphic estimate as

$$\boxed{\begin{array}{l} v_M = \sum_{\text{all } n} \bar{\mathrm{b}}_n x_n = \sum_{\text{all } n} (\mathrm{a}_n + \mathrm{e}_n + \delta_n) x_n \\ \text{Where additionally} \\ \qquad \bar{\mathrm{b}}_n = \text{paramorphic model estimate of } \mathrm{a}_n + \mathrm{e}_n \\ \qquad \delta_n = \text{error in estimate of } \mathrm{a}_n + \mathrm{e}_n \end{array}} \tag{2.4}$$

What then becomes the issue is the accuracy of the Judge relative to the paramorphic model. Elements common to both are irrelevant, this includes both the actual weights *and* the systematic error of the judge's error in assessing those weights which are embraced by the paramorphic model. The elements of variance of the paramorphic model from true, which are not also included in the model of the judge, are the variances of $S, \delta_n.x_n$: Those of the judge are $\varepsilon_S, \varepsilon_n.x_n$. This implies (assuming error independence) that, for paramorphic models to be better than the judge:

> The variance of the paramorphic estimation error of the judge's weights (multiplied by the factor magnitudes)
>
> +The variance of configural and other effects discerned by the judge but not in the paramorphic model

must be less than

> The variance of the judge's *random* error in estimating weights from occasion to occasion (multiplied by the factor magnitudes)
>
> +The variance of the error the judge makes in estimating configural and other effects not in the paramorphic model

We might, arguably, assume that the variance of the error that the judge introduces in considering special effects, is small compared with the error made in the paramorphic model as a result of excluding them (allowing that an expert has the ability to discern significantly useful additional factors). We are left with the conclusion that paramorphic model estimation errors *plus* the variance of special effects considered by the judge but excluded by the model, is outweighed (and if the phenomenon is as robust as it appears, considerably outweighed) by the variance of random errors of weight estimation by the judge. In short, for bootstrapping to be viable implies that the judge *must be highly inconsistent in the weights he or she adopts* from one occasion to another. (At the time of preparing the above argument, I was not aware of Camerer (1981) who examines statistical conditions for the superiority of bootstrapping, based on correlation coefficients. However, the view here might provide a more revealing illustrative perspective for this purpose).

This adds further weight to the suggestion that people's inherent cardinal weight stability is low on matters of value. Despite the judge having a clear, defined and testable objective on a repeated task fine-tuned experience, he or she is inconsistent in assessing factors. A decision maker has no such solidity of purpose with respect to a non-repetitive task. It does not seem reasonable that the decision maker's value weights should be intrinsically more stable. However there is a further issue: in the judgement task, whilst weights may vary from case to case, there is a central tendency, a long-run adherence to a stable cause-effect relationship. This is not present in unrepeated circumstances against a background of Vague objectives that characterises general decision making. We might even reasonably argue that for this situation the concept of a central tendency is not meaningful, it might be better imagined as a random walk. We are left with lability.

Simon (1955, p246) observed, "My first empirical proposition is that there is a complete lack of evidence that, in actual human choice situations of any complexity, these computations [of trade-offs] can be, or are in fact, performed. The introspective evidence is certainly clear enough, but we cannot, of course, rule out the possibility the unconscious is a better decision maker than the conscious". Hayes, Miller, Bootstrapping, the EP perspective, the characteristics of ordinal scale standards, and the meaninglessness of unconscious value, triangulate the view; they make Simon's qualification seem unlikely.

Fischoff (1991) suggests research paradigms based on underlying assumptions about how people may assess value. At one extreme is "Articulated values; People know what they want about all possible questions". At the other is "Basic values; People lack articulated values on specific topic". The concept of labile values within a context of Vague objectives would seem broadly compatible with the second of these.

2.9 The sovereignty of a person's present values

March (1978, p144) observes, " Rational choice involves two kinds of guesses: guesses about future consequences of current actions and guesses about future preferences for those consequences." Elster (1979), considered that Ulysses, in seeking to avoid the seduction of the Sirens, and to compensate for weakness of will, was not fully rational in pre-committing himself to an irrevocable action, thereby foreclosing options which he would otherwise have had. I suggest that both make an assumption concerning the rational actor which is unnecessary; that is a persons future values, as values independent of the present, are relevant to current rational choice. I suggest that this distinction is redundant and that a decision maker has no duty to his or her future values. Moreover, the concept is essentially meaningless as the extent to which a decision maker anticipates them and makes a present choice between sets of tastes or values, whether tagged temporally or not, he or she actually incorporates them in his or her present values.

Is such a view morally sustainable? Should we not allow, for example, that our grand children will have different values from our own and have a right to exercise them?

The issue here is that we might (and, I consider, as a matter of personal ethics, should) have regard in our *present* valuation and preference mechanisms to the *future consequences* of our decisions and, for example, place value *now* on the future condition of the environment; or we might, as a matter of policy, avoid pre-commitment of action of little direct relevance to this generation, leaving appropriate decision making to succeeding generations in the light of their values. But this is a statement about our *current* values. It is also reasonable to imagine oneself in the future and, by attempting to anticipate "feelings" beyond the decision taking and the decision consequences, modify our present values. But we should only act on the considered values and consequential preferences, labile though they may be, that we have ourselves at the time we commit. When the present gives way to the future then, and only then, are new values relevant.

Our expectations of the future values of ourselves or other people, inasmuch as they are not incorporated in current values, are also relevant to current decisions as *factors*, in the same way that future consequences are. It is in this manner that we must judge Ulysses's rationality. If Ulysses had expected that his values would be unchanged, indeed, it would have been irrational to have pre-committed action. His future self would be a faithful agent of his present self, and to deny the opportunity of changing or fine-tuning the decision, in the light of unexpected changes in circumstances, would have been foolish. However, he saw his expected future values as contrary and inimical to his present values. It would have been irrational had he not sought to prevent the potential corruption of his present decision by the action of a person (his future self) whose values he repudiated. He was perfectly able to judge an inferior outcome, given his present values, had he allowed his contradictory future values to have a vote in the disposal of the matter.

The ethical position of trustee, agent, or manager making decisions on behalf of others, is similar. We could argue that he or she should put aside his or her own values for the purpose of the decision. But what if he or she has more than one constituency affected by the decision and those interests conflict, or considers any of them offensive to his or her ethics? Ultimately, the decision maker must resolve such considerations within his or her personal value system which cannot be set aside (those values may be the very reason why they have been trusted with authority). The decision maker adopts the values he or she chooses. This may be in good or bad faith, altruistically or selfishly; but adopt values he must.

2.10 Rationality

Most decision aids suppose a rational decision maker, desirous of making a decision based on rational principles; the aids themselves supposedly incorporate within their model structures rational procedures. What does the concept of rationality involve and how do I use this term?

My dictionary says, "of or based on reasoning or reason" or "sensible, sane and moderate; not foolish absurd or extreme". Baron (1988, p3) suggests, "The best kind of thinking, which we shall call *rational* thinking, is whatever helps us fulfil our personal goals... When I argue that certain kinds of thinking are most rational, I mean that these help people fulfil their goals", and (p32) "Rationality concerns the methods of thinking we use, not the conclusions of our thinking. Rational methods are those that are generally best in achieving the thinker's goal". He goes on to point out that Rationality is not the same as correctness nor does it imply a selfishness of motive. One can subsume within oneself the goals of another or of society. Evans and Over (1996, p25) point out "We have a goal when we aim to reach some state of affairs - ie to make some proposition true - by means of our actions." These imply a process by which purposeful action is determined.

Elster (1986, p12-13) expands this idea. He sees rational choice as an intentional process involving a 3 piece relation between behaviour (B), a set of cognitions or beliefs (C), and a set of desires (D). First the desires and beliefs should be *reasons* for the behaviour in the sense that given C, B is the best means to achieve D. However, C and D must cause B and do so "*qua* reasons", that is intentionally and not incidentally. He also suggests that rationality also requires that the "belief has a maximal degree of inductive plausibility given the evidence". The belief must also be caused by the evidence and do so "in the right way" eg not as a result of faulty reasoning.

However, Lee (1971, p15) suggests, "A fundamental assertion that "man chooses rationally (optimally) may be taken to be true by tautology ..". and as Baron (p289) observes, "Many scholars (especially economists, but also some psychologists and philosophers) have been reluctant to admit that people are sometimes irrational, so they have tried to develop criteria of rationality that are consistent with our behavior."

Rationality connotes both issues of process and criteria and seems to be used differently in descriptive and prescriptive contexts. Von Winderfeldt and Edwards (1986, p2) however suggest that, "The notion of rationality is clearly prescriptive: in any version, it explicitly says some thoughts and actions are appropriate and others are not. But one can easily distinguish two kinds of prescriptions. One has to do with ends or goals or moral imperatives ....A quite different kind of prescription has to do with selecting ways of thinking and acting to serve your ends or goals or moral imperatives."

Yet another view is provided by French (1986, p28) who suggests "Very roughly a rational decision rule is compatible with certain principles of good decision making. Thus the meaning of 'rational' is context dependent; it depends on the principles of good decision making being discussed." He goes on to advise his readers to distrust the word. Although he does not explicitly say, we might infer, given that his subtitle is "An introduction to the mathematics of rationality", that the principles of good decision making embrace at least to some degree, conformity with axioms of rationality. Lee (1971, pp7-9), however, whilst noting that rational decisions depends on the decision principle employed, that these may differ for different people, and are dependent on the relevant information available, comments "The basic idea of a rational decision is that it is in some sense a "Best" or "optimal" decision". Sutherland (1992, p4) makes the similar point, " ...a rational action is the one that, given the person's knowledge, is most likely to achieve his end. Rationality can only be assessed in the light of what a person knows ..."

It is this concept, the notion of rationality as comment on the testable goodness of the result, as an indicator of what one ought to do, as a normative concept relating to the decision, that most appeals to me.

But even within a normative view, rationality would seem to be something requiring a qualification: within what constraints? As Simon (1957, p182) comments in introducing his principle of Bounded Rationality "The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real-world - or even for reasonable approximation to such an objective rationality." Rationality, to be useful, must not be dependent on absolute assumptions of Mr Spock-like omniscience, but must deal with our information, ourselves, and our aids, as they are, within a bounded domain. Because of this Simon (1978) distinguishes Procedural rationality from what he calls Substantive rationality. We must accept that if our capacity to handle information were different, our rationality would be different too. Moreover, an absolute concept is not useful to a decision analyst whose purpose is to expand the bounds, as, even armed with his tools, his own capacity, like that of the decision maker, is still woefully limited and he cannot throw this off.

There is an additional problem which has already been mentioned; the nature and stability of our goals and values. I will propose that we may not have been endowed by the evolutionary process with a sophisticated intuitive capability to handle strategic objectives of any complexity and I have already suggested that our values are labile. Our goals, ambitions, desires, values, tastes and preferences would seem as March (1978) suggests, neither "absolute", "stable", "consistent", or "precise" but at any instant a pulsating mass of imprecision, instability, variability and non-specificity. Rationality, as I would wish to define it, in this situation might seem illusory and it does require sleight of hand. However, it can be secured operationally by an element of imposed stability, not unchanging nor even slow-changing, but for an instant steady.

Such steadying can be achieved by definition or at least self-declaration of the decision maker's ambitions or desires, even if incomplete. It should be noted that this requires no more than a temporary conscious recognition, a commitment to memory of sufficient duration to allow reflection or subsequent recall and conscious scrutiny, a check to fluidity. Such a process can set the ever-shifting sands of our tastes, at least to the extent that is necessary for the demands of traditional rational choice theory (as described by March) to apply. It comes at a price; possible self-delusion (that our true values are *permanently* as we assessed them at that moment) or poor sampling from the extremities of the haze of the person's desires. Nor does this, of itself, extend the bounds of rationality of the decision maker. Indeed, it imposes artificial structure on the problem, additional bounding.

However, I suggest it is necessary for the normal restrictions of rationality to be extended by supplementary means.

2.11 Towards a normative rationality and material optimality

Once a person seeks to extend the normal bounds of his rationality by some aided process (including self-aid, such as implied by a pen and paper analysis of a problem), his rationality ceases to be entirely internal. He debates with himself. Just as desires and goals must in some sense be declared, so also must the concept of rationality which he will seek to impose on himself. Normative considerations then must apply. Whereas before I might have goals and cognitions and I decide, now the element of "ought" is clear. The concept of criterion enters the arena. This is especially so as soon as the problem is to be shared between more than one mind, through discussion with other people or through an adviser-decision maker relationship. However, it is also true as soon as the extensions of the bounds of rationality is to be achieved by decision modelling, and the extension of the computing powers of the mind by electronic computing. At this point Lee's notion of positively seeking to be "Best" *must* enter the concept of rationality. Only when the problem is externalised or taken out of its box to be subjected to some private but conscious process that we avoid the tautological quality (that he warns us of). Otherwise, if the decision was not the best within our "rational" facility we would simply have made another.

Now we must declare our purpose (at least implicitly) in advance of our decision and test against it. We may not be rational, we probably will not be rational, but we do not seek to be capricious (except possibly as political tactic outside the ambit of our declared problem); we seek to be rational. Simon(1957 pp196-199) makes the distinction between "intended rationality" and "objective rationality" and it is a useful one for normative decision analysis. We might say that the posture of an objectively rational decision maker is to intend to be rational in his decision making. Once, employing Procedural rationality, we decide to analyse normatively in order to extend bounds, we must adopt a formal or Substantive rationality to take the matter further.

What then are the properties of normative rationality? I suggest as a basis that:

> A decision maker makes a rational decision if he can find no decision which is materially superior to it, using such criteria as he judges best reflects the value to him of the decision, given his perception of the possible outcomes in terms of the factors that he considers relevant, and his perceptions of their likelihood. Where a decision maker believes that alternative criteria may be applied, there shall be at least one criterion under which this condition is met. He intends rationality if he seeks a rational decision.

This definition in effect allows a decision maker to make the rules (*any* rules which he intends should be rational), which define the value to him of a decision. Moreover, given that rationality does not require a person to be of fixed mind, he or she may change them flexibly. Rational decisions must then conform to those rules; irrational decisions will not. However, a critical issue is that a third party can never gainsay the subject on the basis of the *decision* alone, though it is permissible to challenge or recommend declared criteria of intention. For clarity it should be made clear that such rules might not be accurately articulated or communicable. Indeed, as they are likely to be soft and qualified in subtle ways, they are unlikely to be. Thus they may not, indeed cannot be, perfectly reflected in the models used to assist the decision maker. These models are further downstream. These are simplifications of reality and *include* simplification of the decision maker's rules.

A lack of conformity either indicates a failure to adequately reflect the rules *or* irrationality. Such a failure may not indicate invalidity of the model. A decision maker may reject a specific "suggestion" of a formal model, adopting instead, say, the second option. Such decision may be rational and be so without impugning the integrity of the model, which could have been instrumental in highlighting the option. Moreover, the rationality or irrationality of the decision could still have been clarified by aid of the model.

We might also note that whilst a derivative model, seeking to reflect a decision maker's rules, may involve searches for what the analyst may describe as an "optimum", the rationally intentioned decision maker will merely be seeking approximate or material optimality. We should allow that any concept of optimality must also be owned by the decision maker and must embrace deviations that he or she may tolerate. There is no "ought" about implementing modelled or even real optima, but there is rational imperative in implementing approximate optima.

This idea of rationality does not, of itself, require uniqueness of rational decisions, as real rules will not be determinate even if their mathematical approximations are. There may be multiple rules (which should not be confused with multiple objectives, as the former can embrace methods for resolving conflicts of objectives). Use of satisficing, for example, might also allow multiple rational solutions. However, the rationally intentioned decision maker must ultimately produce unique decisions and must resolve non-uniqueness (see below).

This basic concept of rationality might be usefully supplemented by more traditional mathematical axioms of rationality (eg French, 1986, p39). However, I prefer to weaken slightly these supplements by considering these as tests of the intended rationality of the decision maker, or recommendations to him or her, rather than as tests of the rationality of the decision itself. I suggest that a decision maker who intends to be rational should, given evidence that he has not done so *in a material way*, modify his criteria to comply with:

1. Samuelson's (1938) Weak Axiom of Revealed Preference. This states that if a person reveals a preference for A over B he may not also reveal a preference for B over A. A may not be judged more valuable than B and also be less valuable.

2. Transitivity. A rationally intended decision maker will not sustain any criterion that causes him to conclude that A is more valuable than B, which is more valuable than C, and that C is more valuable than A, in the face of evidence that a material contrary effect has or could be produced. (Transitivity is argued by Sen (1986) to be implied by 1 above).

3. Independence of Irrelevant Alternatives. A decision maker may properly re-examine his criteria for any reason, and this can be stimulated by the introduction of new alternatives which may not, in the end, be candidates for selection. In this sense, an irrelevant alternative may legitimately influence choice. However, a rationally intended decision maker will not sustain a criterion in the face of evidence that it has a *material* inherent property, causing the dependence of a selected option on the alternatives available, when these are not themselves suggested for selection under the criterion.

Nor will he reject a criteria *solely* on the grounds that it rules-out a particular option as a candidate for selection.

4. Domination. A rationally intended decision maker will not sustain a criterion which causes him to a prefer A to B if for every factor which the decision maker considers to be relevant to the valuation, B is superior to A.

5. Resolution of Equivocation. A rationally intended decision maker may apply criteria which do not reduce options to a single choice. When this is not so he will resolve the ambiguity by

   - supplementing, condensing, combining, or replacing criteria.

   - choosing between them by an arbitrary process. (Random selection is a valid rule).

   In modifying criteria he will sustain his rationality with respect to the original options.

6. Independence of Value from Prior Circumstances. A rationally intentioned decision maker should seek to place the same value on identical circumstances defining a decision outcome whatever gave rise to them, whilst recognising that the value consequences of a decision may be influenced by the State of the World that is coincident with them.

   (Within this rule, criteria such as Minimax Regret are acceptable, although they do not appeal to me. At first sight this does not meet the "Independence from Prior Circumstances" requirement. However, knowledge of the characteristics of rejected options, which seem attractive in hindsight, will exist after a decision. Such knowledge is a one of the consequences of a decision and can be anticipated. It is an issue of ethics, not of rationality, whether one seeks to avoid the discomfort of knowing a better result could have been obtained, or to avoid hindsight criticism.)

7. Conscious Process. A rationally intentioned decision maker may express preferences arising from conscious or unconscious processes. However, if they arise from unconscious processes, he or she will consciously review them. Moreover, he or she will not sustain preferences which are

inconsistent with a value mechanism which he or she consciously holds or adopts, though this may be reviewed at any time.

A rationally intended decision maker may properly tolerate immaterial deviations from the purer forms of these statements. For example, the scores generated by Saaty's Analytic Hierarchy Process (Saaty, 1980) can be influenced by the inclusion of irrelevant alternatives. A rationally intended decision maker can ignore such objections if the conclusion is robust to the incorporation, or produces a solution judged to be close in value to what might otherwise be obtained. He ceases to be rationally intentioned if he recklessly ignores evidence that a particular result could be misleading for this reason.

2.12 Efficiency and dominance

Based on Keeney and Raiffa (1976, p68) we can define dominance mentioned under 4 above as follows:

Let decision options a' and a'' have attributes
$\mathbf{x}' = (x'_1, \ldots, x'_i, \ldots, x'_n)$ and $\mathbf{x}'' = (x''_1, \ldots, x''_i, \ldots, x''_n)$
where
$X_i(a') \equiv x'_i$ and $X_i(a'') \equiv x''_i \quad i \in (1, \ldots, n)$
Assume that X_i are mutually preferentially independent and that preferences increase in each X_i
Then $\mathbf{x}'$ *dominates* $\mathbf{x}''$ if
$x'_i \geq x''_i, \forall i$ and $\exists i \; x'_i > x''_i$ (2.5)

Under such circumstances a' *may* be a candidate for best but a'' cannot be, as it is inferior in at least one attribute and superior in none. All non-dominated options are said to be *efficient, pareto-optimal,* or members of the *efficient set.*

If we assign *any* positive valuation weights to the x_i so that

$v(\mathbf{x}') = \sum_{i=1 \text{ to n}} b_i . x'_i$ and $v(\mathbf{x}'') = \sum_{i=1 \text{ to n}} b_i . x''_i$
Then if $\mathbf{x}'$ dominates $\mathbf{x}''$
$v(\mathbf{x}') > v(\mathbf{x}'') \; \forall b_i \in \{\mathbb{R} : b_i > 0\}$ (2.6)
Where $v(\mathbf{x})$ = value of bundle of attributes, $\mathbf{x}$

This arises as the value contributions must be equal in all cases where the x_i are equal and must be greater in the dominating option for all the other attributes for which, perforce, the x_i are greater. In short, it is impossible to find an additive value function with positive coefficients, which gives the dominated option a greater value. This opens up the possibility of redefining dominance and efficiency directly by reference to valuation, as is done shortly. This is useful as the attachment of value to decisions is part of the perspective this work, but it importantly deals with a problem not accommodated in the basic definition. That is the case where a number of options are not dominated but some within them *jointly* dominate others; that is they are dominated by a convex combination of efficient options.

For example, the options a, b, and c described by attributes (10, 0), (0,10), and (1, 1) are all non-dominated according to the basic definition. However, if the structure allowed us to choose combinations of a and b in our decision (as we could, for example if a and b represented shares), we can say that a 50:50 or even a 10:90 admixture of a and b would dominate c. But even if we are not allowed to "mix" solutions, value may allow us to draw conclusions, provided we can make additional assumptions concerning the form with which attributes 1 and 2 relate to value. If we assume that value, is linearly related to the magnitude, we cannot be certain whether a is better than c, or b is better than c. However, we can be sure that c is inferior to one or the other as *no* attribute weight assignment renders it *simultaneously* superior, that is of higher value, to both. c is therefore inefficient. This would not be the case if, for example, the value to the decision maker was the sum of the magnitudes of the attributes to the root of 4. An equal weighting of attributes would then render c superior to both a and b.

There is one other issue that makes the unmodified Keeney and Raiffa definition more complicated to use within the attribute oriented methodology proposed here. We may properly say that any real decision has an open-ended number of attributes and those under consideration are selections from an infinite super-set. Over the infinite super-set no option will be dominated. (After we have considered all the usual grounds for choosing a computer printer, there will always be excluded considerations. An otherwise dominated selection may have a nicer colour, for instance).

The implicit assumption of the basic definition is that *all* attributes relevant to the decision, and *no* irrelevant attributes, are included. The exclusion of relevant factors is a continuous risk of all management decision and analyst support and there is nothing, in respect of theory, to resolve a mis-specified model of which the parties are ignorant. However, I suggest that the inclusion of attributes does not necessarily assert their relevance but only the possibility of their relevance. We should, and in the method developed here I do, allow a decision maker to assert irrelevance of a provisionally included attribute, that is to assign zero (and not just infinitesimal) weight to an included factor. This is particularly so where attributes are not objectives per se, but measures associated with objectives. It may also be that some attributes can be considered as *alternative* proxies for a vague objective, not all of which should be given non-zero weight.

Given this view we can only assert absolute superiority:

$$\boxed{\begin{array}{l}\text{if} \\ x'_i > x''_i,\ \forall i\end{array}} \qquad (2.7)$$

We can use the expression $\mathbf{x}'$ strongly dominates $\mathbf{x}''$ if this condition is met. I describe an option that is not strongly dominated by an option or a combination of options as *weakly efficient.* In the value orientation, an option is strongly dominated if no valuation of factors can be found which results in the assessed value of an option being absolutely greater than all other options. It is weakly efficient if a valuation of attributes can be found such that it is at least equal to all options, but no better than equal for at least one of them. In this thesis I will also include weakly efficient options in the *efficient set* unless otherwise mentioned. The rationale is that such an option should remain under consideration until a positive weight is established for at least one attribute for which the condition $x'_i > x''_i$ applies; for if zero weight is eventually assigned to all such attributes the option becomes co-optimal with another. Such an option should not be eliminated prematurely. It might in due course be necessary to break ties but this may be just as properly be done on the basis of attributes outwith the analysed set as those within it that are assigned zero weight.

Though this may be a fine point unlikely to be material with real data, it has a practical advantage. As will be seen later, it usually obviates the need for "epsilon" in the LP formulation which will be used, and which complicates computation in DEA. (If one allows that at least infinitesimal weight attaches to *all* attributes then a weakly efficient solution would cease to be optimal, though then one might ask whether there are excluded factors of comparable importance).

2.13 Indifference and Not Knowing

Classical preference theory, as for example described by French (1986, pp61-96), assumes three relations *Strict Preference* ($\succ$), *Indifference* ($\sim$), and *Weak Preference* ($\succsim$), the last implying either Strict Preference or Indifference. Indifference is an equivalence relation implying exactly equal value. In the world of decision making theory, one may be obliged to accept an archimedean view of Indifference. For example, if there exists a sum of money M_1 which is strictly preferred to having a pound weight of tomatoes, and having one pound weight of tomatoes is strictly preferred to having a lesser sum M_3, there will exist a third sum M_2, $M_1>M_2>M_3$ such that one is Indifferent between having a pound of tomatoes and M_2.. But actual Indifference is, I suggest, rare in selection of prior determined real life options and practical preference is expressed (even if not processed) in two different relations, Strict Preference, and Cannot Say.

True, in many situations, Don't Know may indicate closeness in value (we might say, a "Close To" relationship) and this may be exploitable, but our differing discriminatory competence in varying elicitation situations will mean that in many situations small differences in value could be discerned in some preference comparisons, whilst larger true differences will be unidentified in others. ("Close To" is much the same as Intransitive Indifference and leads to semi-orders and interval orders; though the relational rules that attach to these, may not help us much in practice).

The formulations I discuss are expressed in a value rather than in the relation orientation, and in this frame Indifference implies Equivalence of value. However, for the reasons above, I have been reluctant to invoke the concept of Equivalence, except when no further information is available. Then, treating Close To as Identical may often be better than sustaining "No view".

Equally, the mechanics of linear programming which I use, forces (in straightforward mode) the use of the operators $\geq$ and $\leq$ for indicating the value of a preferred option relative to another; this corresponds to Weak Preference in the relation orientation. As far as possible, however, I seek the most certain statements of preference, that is statements of Strict Preference. Although one could say that one intends elicitation of Strict Preference, their purpose is to minimise errors of transitivity in a Weak Preference formulation. However, it is reasonable to say that this issue is of negligible practical importance, as a technically Weakly Preferred option could be converted into one that would be Strictly Preferred, by an increment of infinitesimal value to one or more attributes.

2.14 An operationalisation of normative rationality under conditions of impairment

How can the idea of rationality, and efficiency, be translated into quantitatively succinct equivalent representations that can actually be used downstream in analytic aids, in models? My aim here is to seek an operationalisation which retains, as far as possible, both the concept that the decision maker maintains ownership of the rules by which a decision is considered rational, and at the same time makes no heroic assumptions regarding the cognitive facility of decision makers. I do not insist that a rationally intentioned decision maker should make use of explicit models, but I do suggest that, should he or she do so, they would wish to incorporate normatively rational model rules paralleling their internal criteria.

Although I will later concentrate on deterministic representations, at this stage it is appropriate to consider uncertainty. The minimalist representation I suggest assumes that the decision maker can discern and act on comparative likelihood and comparative value but does not depend on innate concept of quantitative probability or an internal yardstick of cardinal value. However, I will allow in the operationalisation that decision makers might be able to assess quantitative probability or to discern a cardinal scale of value. (Later, I go further and suggest that comparative likelihood may actually be the limit of our innate competence). For simplicity I break the description into two parts. First, (taking into account uncertain futures) given that a decision maker is able to assess comparative desirability for the outcomes for all decision options under alternative states of nature, what overall preference order may be considered to be rationally modelled? Second, given that desirability under any given state of nature is manifest from the magnitudes of attributes, what valuations of attributes may be considered rationally modelled?

I here put aside for the moment the issue of how the attributes of decision outcomes determine decision value, considering initially the impact of assessed likelihood of alternative states of nature, for the moment, taking the valuation of a particular decision under a particular state of nature as read. The first of these issues is ultimately more central to this thesis but I consider briefly the impact of uncertain states of nature within the rationality framework. I propose first that a preference order may be considered rationally modelled if there exists a valuation for all decision options i such that the value V_i of that decision shall be greater or equal to the value of every less preferred option, and less than or equal to every more preferred option. This value measure shall be defined by :

$$\boxed{V_i=\sum_{\text{all } j} q_j . v_{ij}} \tag{2.8}$$

where q_j (>0) are prospectively determined weights for unknown states of nature, states of nature being mutually exclusive and where v_{ij} is a measure of desirability of decision i, under state of nature j, assessed prospectively in whatever rational manner the decision maker shall desire; *provided that* if the outcome of decision g, under state h, is preferred to outcome of decision k under state l, then v_{gh} is greater than v_{kl} and provided also that if state of nature m is more likely than state n, then q_m is greater or equal to q_n. In summary, any weighting of value related to likelihood is considered rational, provided that more likely events are given higher weight than less likely events and any valuation of outcomes may be considered rational, provided a more preferred outcome is given a higher value than a less preferred one.

This structure can represent or simulate other approaches. eg

Subjectively Expected Utility. q_j become the subjective probabilities of each event j, and v_{ij} $(= v_i$ for all $j)$ is derived by a utility transformation of an expression of money value or another primary yardstick. More complex transformations of probability could also be accommodated, perhaps including those used in Prospect Theory (Kahneman and Tversky, 1979).

Maximin Return. This could be simulated by specifying first that all q_j are equal whatever the likelihood of each state j. Moreover the v_{ij} shall be derived by a transformation of primary value, such that each increment of primary value shall have a very large actual value, compared with any corresponding increment from a higher primary value base.

Laplace's Principle of Insufficient Reason. All q_j are equal.

Minimax Regret. Provided Regret can be prospectively determined, it is in principle open to the decision maker to use his "anticipated regret" as a measure of value, in effect saying that he values freedom from thoughts of what could have been.

Equation (2.8) ensures that an absolutely dominated option, ranks below an option which dominates (ie is superior to it under all states of nature), but it also prohibits as rational choices situations in which weaker forms of probabilistic domination apply. To illustrate, amongst alternative states of nature let two options, 1 and 2, have identical v_{ij}, except for $v_{11}, v_{12}, v_{21}, v_{22}$. Further, let option 1 have the same value result under state 1 as option 2 does under state 2 and vice versa ie $v_{11} = v_{22}$ and $v_{12} = v_{21}$, but let state 1 be more likely than state 2 $q_1 > q_2$. The two options are only differentiated by the relative likelihood of the two states. It is rational to prefer the option that is superior in the most frequent circumstance, ie if $v_{11} = v_{22} > v_{21} = v_{12}$, to prefer option 1.

The construct also ensures transitivity and independence from irrelevant alternatives.

Under the construct, a decision maker might rationally base his conclusions, for example, treating the most likely state of nature as if it were absolutely certain, or averaging the four most likely, or giving the first 4 most likely, relative weights of

10, 9.5, 2, 1 (in order of their likelihoods) regardless of their objective probabilities. It would not be permitted, under this model, to make the selection exclusively on the basis of the second most likely state of the world alone, or to use the weights 10, 9.5, 1, 2, if state 3 is more likely than state 4.

I now turn to consideration of the measures of outcome desirability that a decision maker, or analyst on the decision maker's behalf, might develop, to which Equation (2.8) can be applied (and on which this thesis has greater concentration). These desirability outcomes will be defined by the magnitudes of discernible factors or attributes consequential on the decision. I propose the following characteristics of a normative model, within the context of the operationalisation of principles introduced, and, in part, reflecting the tests of rational intention of the decision maker:

1. Fixed scalar value. A model of decision desirability or value should relate anticipated outcomes or attributes to a one-dimensional measure which is fixed, however temporarily. It should be noted that this automatically secures compliance with Samuelson's Weak Preference Axiom, and Transitivity.

2. Dominance. A decision maker may discern for any set of attribute magnitudes consequential on a particular decision, whether an increase in the magnitude of each of them is desirable or undesirable, or may be undecided. A model of the decision maker's values will not assign a lower measure of value to one decision outcome relative to another otherwise identical outcome, if the direction of differences in magnitude of all differing attributes is favourable.

3. Independence of value from prior circumstances. A model should place the same value on identical attribute magnitudes defining a decision outcome, whatever gives rise to them .

4. Scales of desirability monotonic with objective measures of value. Some attributes of decision outcomes may be measured on scales which are objective measures of value, outwith the value frame of the decision maker, eg attributes specified in money value. It is open to the decision maker, perhaps with the assistance of an analyst, to transform them to conform to a personal measure of value or desirability. However, he or she should

nevertheless build a scale which is monotonic increasing with the objective scale. Thus, for example, a rationally intentioned decision maker may assign different marginal values for equal increases in profit, dependent on the base level of profit, but should not at any point accept a negative value on such an increase.

5. Qualified self awareness. A rationally intentioned decision maker will not impute to himself, or accept as a result of third party analysis of his preferences, a more complex model of value and preference than he can justify in terms of his conscious value intentions. For example:

    - Form of relationship of attribute magnitudes to value. A decision maker should be able to define, in concept if not in mathematical detail, how attributes relate to value, or how they might. An analyst may test that this is well considered, but a model should not reflect a more complex structure than that declared. In the face of contradictions between declared intention and observed behaviour, the decision maker should be able to redefine the form of any relationship, but declared intention is sovereign.

    - Personal utility of money value, or other scales of worth. A rationally intentioned decision maker, in transforming such scales to those of personal desirability, should be able to justify the form of transformation in qualitative terms. Thus he or she would not sustain points of inflection, lack of smoothness, or convexity or concavity in the transformation, without conscious acknowledgement that it reflected his or her intentions in principle.

    - Mutual preferential independence (see for example, Keeney and Raiffa, 1976, p101). A decision maker should consider whether his preference structure might include preference switches inconsistent with preferential independence. If it does not, he or she should not entertain options which could not be optimal without such a breakdown, or allow a model which suggested such options. Where mutual preferential independence does not apply, a decision maker should, with the help of an analyst, redefine attributes in a way such that it does.

- Configural preference. A rationally intentioned decision maker may wish to be attentive to configural issues, for example, whether the potency of one attribute is dependent on the magnitude of another, or whether his value frame is conjunctive (favouring general performance over many attributes) or disjunctive (favouring good performance on any). However, a model of his behaviour should not reflect them if he does not affirm them.

With regard to point 5, I intend a philosophically distinct point from the justification of parsimony in other OR modelling. In the latter the OR analyst is seeking to model a system which is at least partly external to his client. It is a system that is rarely completely known. We might in these circumstances use simple models as adequate approximations, to ensure tractability or communicability, or, because, lacking sufficient reason to assume a more complex form, we invoke Occam's Razor. We remain entitled to use these justifications in decision modelling. For example, we might approximate what we *know* to be a non-linear relationship by a linear one. However, I intend a more powerful obligation and I return to this issue shortly

Finally, within this section, I consider forms of representation of the value of decision outcomes. Where preference is dependent on more than two attributes, which are all mutually preferentially independent, then a decision maker's values can be represented by an additive value function of the form:

$$v_i = \phi\left(\sum_{\text{all } k} w_k . g_k(a_{ki})\right) \qquad (2.9)$$

Where a_{ki} = the magnitude of attribute k for option i

w_k = a weight factor

ϕ, g_k indicate functions defining positive transformations

Where preferential independence cannot be assumed, preferentially interdependent attributes can be grouped into functions of multiple variables, effectively converting them into mutually preferentially independent variables. eg

$$v_i = \phi\left(\sum_{\text{all } k} w_k \cdot g_k(a_{k1,i}, \ldots, a_{km,i})\right) \tag{2.10}$$

Where $a_{kr,i}$ = the magnitude of attribute r, of group k, for option i.

Such functions can be inserted into probabilistic representations of the type of equation (2.8) or used directly, as is the predominant approach of this work. It is an assumption of this work that the transformation implied by the functions g_k can be pre-determined, or adequately approximated, outwith the process of converting multiple dimension value into a single measure. They can thus be converted into linear form and this feature is an important part of the approach developed here.

2.15 Implications of Qualified Self Awareness

In the last section I introduced the idea of Qualified Self-Awareness and I amplify the concept and highlight some implications.

Importantly, the term is not intended to imply "knowing one's own mind". This would be inconsistent with the premise of a decision maker impaired by cognition difficulties. However, the decision maker is not looking externally. He is not even looking introspectively at his own suspected mental condition, about which he might be mistaken. He is declaring values *which cannot be secret from himself.*

Of course a decision maker may exhibit unconscious *behaviour* which is inconsistent with his asserted *values.* He or she may also base behaviour on "information that is not well represented in consciousness", as Bowers (1981) suggests. But we must not conclude that the decision maker therefore has unconscious values. For example a manager may exhibit disjunctive behaviour in the way he selects staff and this could be pointed out to him, but this says nothing about his preferences or values, unless he asserts that it this is indeed what he *wants* to do.

Nor when the decision maker acts as an agent for an organisation or another party, rather than on his or her own behalf, is the situation different. For example, faced with a hitherto unrecognised but now understood issue of whether the business should use disjunctive, neutral, or conjunctive scoring in staff selection tests, the decision maker might indicate that he does not know. However, there is no answer to be found in a hidden store of value within him that has not been accessed.

Anomalies in his or her preference structure are that and no more, unless they trigger a conscious examination of the conclusion. In the circumstance suggested, he should now articulate a position by thinking about the issues, or consulting colleagues. (Configural value, however, does involve some special issues and these will be discussed shortly).

This argument, of course, allows a decision maker to change his *conscious* mind in the light of the implications of his behaviour. Indeed a review of intention is important whenever behaviour is inconsistent with assertion.

Qualified self-awareness is intended to imply that the decision maker knows what attributes contributes to the *value* of a decision, and in broad terms the form (though not necessarily magnitude) in which they contribute to that value. Thus, he or she should be able to nominate value-indicating attribute measures and units and, indeed, transformations to those units, at least to the extent of correcting the compression or expansion of a scale relative to another attribute under consideration. That is to say, he or she can, with technical help, establish a linear measure, at least to the extent that no other expression remains obvious, more likely, or more appealing.

As discussed previously, a decision maker probably does not possess an internal standard of cardinality against which to linearise, however he or she can do this *relative* to the scale of another attribute pertinent to the problem (or an objective scale outside the problem which can be related to those within it, eg money value). Alternatively, a decision maker may be able to assume a linear scale, if he or she is able to say that the intervals of a scale, or a translation of it, can be *looked upon* as having equal worth, or if it cannot be said that they do not.

Of course, some attributes may not relate directly to an issue that the decision maker should value and there may need to be an analytic link. For example, a decision maker may not know the way in which environmental lead concentrations affect human health. This is a factual not a value matter, and he or she will depend on technical advice to make the link; but, having done so, he or she should transform a lead concentration measure to something approximately linear with his or her value set.

If, as a result of conscious thought with help and advice, the decision maker has done the best he or she can, even if they don't know whether they have succeeded, then nothing more can be done by plumbing the covert corners of their mind through indirect processes. What they are then consciously aware of, is *all* on this matter that they can by themselves be aware of.

If this view is mistaken, and we might have pertinent values of which we are unaware and cannot reveal by conscious thought, does this form of linearisation matter? I suggest not, because:

(a) The expression of preference and value seems unstable. First order parameters of a linear model will themselves be subject to considerable uncertainty and this uncertainty would swamp all but extreme second order effects. Extreme effects are likely to be conscious.

(b) Should the expression of preference indicate the possibility of a second order effect, the phenomenon can be subjected to conscious scrutiny.

(c) The success of linear bootstrapping models in judgement exercises, suggests that a decision maker is likely to benefit from a linear model rather than none at all, and that in judgement situations his or her ability to use interaction information is limited, even when he or she is overtly aware of it.

(d) Even if unconscious value non-linearity exists and could be measured through behaviour, it may not be material to the decision. First, the linear scale may be strategically equivalent, or sufficiently so, to the "true" scale. Second, only material non-linearities in the decision space embracing potential optima (ie efficient options) are relevant. In the vicinity of the efficient frontier, a non-linear function might be approximately and sufficiently linear. In the simulations to be discussed in Chapter 9, I tested sensitivity using 10 similarly constructed data sets each consisting 50 options and 5 variables. A "True" non-linear value function was assumed for each variable based on fairly extreme transformations as follows (one variable, squared; one, to the power of 1.5; one, unmodified; one to the power of 2/3; one, square rooted: all negative values unmodified). Amongst the 500 options, 134 were efficient in the "true" models. Of these 126 were also replicated in the linear equivalent models. Other simulations, explored the effects of quite radical mis-specification and, with appropriate process methodology, these do not appear to cause severe problems even when a false linear assumption is obstinately maintained.

(e) Finally, it is arguable that implicitly revealed unconscious preference not endorsed after conscious scrutiny, has less authority as a basis for decision making.

In summary, Qualified Self Awareness allows one to work with linear models, either by allowing a conversion to linear form, or considered acceptance that the form is as good as another. This view can radically simplify decision analysis, but in an important sense it is less permissive. It allows fewer options to be potentially optimal than with a complex model for which we allow a greater number of parameters to be fixed on a black box basis.

2.16 Disjunctive and Conjunctive value

Configural value is a form of non-linear valuation where the value potency of one attribute is heightened, or diminished, by the magnitudes of one or more others. Conscious and specific interactions between particular variables should be capable of being handled within the above approach.

However, there is a form of configural decision evaluation warranting special attention. These are decision evaluations in which the decision maker seeks, on the one hand, general complementary performance over a variety of performance measures, or, on the other, where exceptional performance is sought for one or few criteria rather than average performance over many. These are, respectively, Conjunctive and Disjunctive evaluation (Dawes, 1964). We may look upon linear models as being configurally neutral; that is, neither Conjunctive nor Disjunctive. Both Conjunctive and Disjunctive evaluation are important in decision making. For example, we may wish to recruit an excellent all-round General Manager, whom we might select using conjunctive methods; or an outstanding designer who has demonstrated talent in a specialist area, but not necessarily across the board, using a disjunctive approach. Can we accommodate these important forms of configural valuation within an additive and, preferably, linear framework?

Einhorn (1970) examines use of a generalised form of valuation that accommodates any degree of polarisation in either direction, based on the Minkowski metric, which he attributes to Dawes:

$$V(X) = \left(\sum_{\text{all } i} a_i x_i^r\right)^{1/r}$$

Where x_i = variable defining attribute magnitude for attribute i
a_i = constant
r is a parameter defining configurality;
> 0 for disjunctive valuation;
$= \infty$ for fully disjunctive, maximise best, valuation;
$= 0$ for linear valuation;
< 0 for conjunctive valuation;
$= -\infty$ for fully conjunctive, maxi-min, valuation.

(2.11)

At first sight this seems a model of exceptional complexity. So it is, if it is the measure of value, rather than the conclusions that are drawn from it, that is important. However, one should recognise that there are many valuation functions that are strategically equivalent to the above (ie that rank all options in an identical order). Moreover, one such is the above equation raised to the power of r; that is:-

$$\boxed{V(X) = \sum_{\text{all } i} a_i x_i^r} \qquad (2.12)$$

I refer to this as the General Configural Model. The reader will note that this may now be considered as an equation of additive and linear form, where the original variables have been transformed by a power transformation. The implication is that disjunctive and conjunctive decision making can be dealt with in a linear framework by exaggerating or diminishing the impact of attribute differences at higher parts of the scale relative to those in lower regions. If configural preference of this sort is considered to be at work, data could be pre-processed using such a transformation or r could remain undetermined in the analysis. A variation on the basic methodology explored, making use of this form of function is explained later.

Multiplicative valuation (if it is consciously articulated) can be similarly treated, as a logarithmic transformation also generates a scale which is strategically equivalent to the (positive) scale from which it is derived.

The notion of using transformations to reduce the impact of interaction terms was remarked on, incidentally to their main purpose, by Yntema and Torgerson (1961), "Another way of dealing with interactions was suggested by one of the subjects in our experiment on ellipses. He complained that we had stretched one end of the worth scale, the scale on which the marker moved. In a sense he was right: transforming our arbitrary scale so as to shrink the end to which he objected would have reduced the amount of interaction. Perhaps this is what people do when they learn good judgment about practical matters. Perhaps they tend to define scales of worth in such a way as to minimize interactions.". They did not employ the expression "strategically equivalent", but the notion that in some sense people might establish internal linear measures of worth, fixed by choosing that strategically equivalent scale that minimises interaction, is an interesting one.

I have empirically illustrated that the power transformation $V(X) = ax_1^r + ax_2^r$ can reliably approximate a value function strategically equivalent to a cross product value model $V(X) = b.x_1.x_2$ over a very large range, perhaps indicating that we can assume this restricted family with tolerable safety, when the specific nature of the interaction is not known and cannot be elicited.

Certainly, the General Configural Model allows treatment where a self-aware decision maker seeks or endorses configural assessment, which he or she does not attribute to specific interactions. As will be seen later, there are also interesting and exploitable relationships between options that are efficient with linear models and those which are efficient under corresponding configural assumptions.

2.17 A taxonomy

It is useful to characterise the alternative structures of decision and approaches to decision analysis, to place specific aids to decision within a wider context.

Stewart (1992) subdivides approaches within MCDM as "Value or Utility Based Approaches", "Goal and Reference Point" methods, "The Outranking Concept", "Fuzzy Set Theory" and "Descriptive Methods". He, essentially, subdivides the first of these into (i) those methods designed to help a decision maker to explicitly articulate a value function from the defined objectives, (ii) Saaty's "Analytic Hierarchy Method", which develops a metric scale of value from qualitative statements of relative importance, and (iii) interactive methods in which the decision maker is asked to articulate preferences between possible trade-offs in objectives in the vicinity of particular feasible solutions. What Stewart describes as Descriptive Methods, I perceive as methods of condensation of a large range of criteria into smaller more manageable sets.

Korhonen, Moskowitz, and Wallenius (1992) also sought to classify analysis approach, in a review of methodology. Borrowing from their ideas as well as those of Stewart, I attempt here a more detailed classification. However, I do not attempt to structure "unstructured" problems as examined in Mintzberg, Raisingham, and Théorêt (1976).

We might accordingly describe approaches to decision analysis in terms of problem structure or, more correctly, the structures decision makers and analysts choose to approximate unbounded decision domains. We can conceive such a classification as a many dimensional specification of decision and methodology properties with a categoric "scale" within them:

Thus, we may have problems involving:

1. a. Single objectives/criterion including well understood multiple objectives with pre-determined compensation between them

   b. Two objectives/criteria

   c. Several objectives/criteria

d. Many objectives/criteria

e. One main objective with secondary objectives treated as constraints.

2. a. Well specified quantified objectives with defined relationships to factors

b. Well specified quantifiable objectives with unspecifed relationship to factors at the outset

c. Well understood qualitative or vague objectives with quantified attributes which are related to the objectives but, prior to analysis, in unspecified ways

d. Imprecisely understood objectives with explicit recognition of relevant factors

e. Potentially definable objectives but predominantly non-quantifiable in the decision time-scale

f. Poorly understood objectives but clear binary preference for at least some factor trade-offs

g. Poorly understood objectives; no clear binary preferences

h. Undeclared objectives

i. Intertwined means and ends

3. a. Deterministic outcomes

b. Uncertain outcomes -quantifiable uncertainty determined by

-independent conditions of the environment (alternative states of the world)

-quantifiable uncertainty dependent on the decision

-unquantifiable environmental risk (complete uncertainty)

-environment independent risk

-defined by deterministically treated factors

4. a. One factor

   b. Two factor

   c. Few factors

   d. Many factors

   e. Open-ended

5. a. Relationships between factors/attributes and objectives are linear

   b. Relationships between factors/attributes and objectives are non-linear

   c. Relationships between factors/attributes and objectives are lexicographic or Boolean

   d. Relationships between factors/attributes and objectives are unclear

6. a. Relationship between decision desirability and objective achievement is linear and non-configural

   b. Relationship between decision desirability and objective achievement is disjunctive

   c. Relationship between decision desirability and objective achievement is conjunctive

7. a. Decisions in which there is no or only one constraint on factors/resources to be present in the implemented solution

   b. Decisions where there is more than one constraint on factors/resources represented in the solution

8. a. Well defined metrically quantifiable factors

   b. Well defined ordinal or categorical factors

   c. Qualitative and loosely defined factors

9. a. Decisions where the objective related attributes are common and magnitudes can be traded-off in principle.

   b. Decisions with incomparable or non-compensating elements

   c. No objective related trade-offs necessary

10. a. Decisions having determined or definable options

    b. Decisions having undetermined or open-ended options

11. a. Two discrete options (including action and no action alternatives)

    b. Few discrete options

    c. Many discrete options

    d. One continuous decision defining variable

    e. Several continuous decision defining variables

    f. Portfolio decisions involving discrete selections of decision components

    g. Portfolio decisions involving continuous fractional selections of decision components

12. a. One-off single decisions

    b. One-off multiple decisions

    c. Repetitive independent decisions

    d. Multistage decisions, dependent on outcome of previous components and or improved information

e. Continuous time dynamic decision-making, dependent on outcome of previous decisions

f. Involving dependence on one or many other decisions, or potentially influencing one or more other decisions, not necessarily in the analysis ambit

13. a. Decisions made independently of other parties

b. Decisions involving co-operation with co-decision makers, with outcomes dependent on their actions

c. Decisions involving competition with other parties, with outcomes dependent on their actions

14. a. Unimportant or low impact or insensitive decisions

b. Important decisions having significant impact and conclusion sensitivity

15. a. Routine

b. Non-Routine

16. a. Urgent time-constrained decisions

b. Time non-critical decision

17. a. Decisions involving one decision maker or decision making entity or where one person or entity is "trustee" of the value frame to be adopted.

b. Decisions where the conflicting interests and values of two or more parties are to be accommodated

18. a. Decisions which are taken

b. Decisions which emerge

19. a. Decisions where analysis conclusions are dominated by models, analysis or computed assessment

b. Decisions where analysis informs a subjective conclusion

c. Decisions requiring cursory scrutiny

d. Decisions involving thought, consultation, the exercise of judgement or unstructured or qualitative analysis.

20. a. Analysis emphasis on making a single recommendation

b. Analysis emphasis on generating a rank ordering of options

c. Analysis emphasis on generating a short-list of high-scoring contenders

d. Analysis emphasis on generating a short-list of potentially optimal contenders (eg all pareto optimal options)

e. Analysis emphasis on eliminating no-hopers

f. Analysis emphasis on generating options/ identifying factors/ elucidating objectives or values, not conclusions

g. Analysis emphasis on identifying value free implications and illuminating the problem situation

21. a. Decisions using one analysis methodology

b. Decisions making use of more than one, either sequentially or concurrently

c. Decisions using no specific or definable methodology

22. a. Value or Utility based approaches

b. Goal, Reference Point, Lexicographic, Satisficing methods

c. Methods depending on properties of Binary Relations; the Outranking Concept; Linear order generation algorithms not dependent on proxies of value

d. Methods not involving the definition of formal decision criteria.

In a limited sense, problems and solution approaches (whether trivial or strategic) can be seen as cells within this multidimensional matrix. However, it is not suggested that the subdivisions represent all possible subdivisions, that they are always mutually exclusive, nor that all, or even a high proportion of the many millions of cells, define distinct problems. The hazard of classification is that no real decision can be neatly compartmentalised. A decision is inherently a function of its own qualities and those of the decision maker. Even that "reality" is subject to the perspective of the decision maker and analyst, as they extend their bounds of rationality by force fitting the situations they face into jackets of tractability which make sense to them. It may be useful to amplify some of the less obvious expressions above (where they have not been already discussed) and to illustrate them by seeking to classify some familiar problems and approaches within them.

I distinguish (1c and 1d) between several objectives and criteria and many objectives and criteria as, some multi dimensional techniques may become unmanageable if the number of criteria extends beyond a limited number. By the same token, in certain instances two objectives can be adequately managed by simple extensions to single objective techniques, when a greater number presents too great a difficulty, allowing 1b to be discriminated as a separate case. 3b, "Uncertain outcomes-defined by deterministically treated factors", refers to the use of statistical measures of uncertainty (eg "variance") as determinate qualities, such as the two dimensional mean return and return variance descriptions of portfolios used in Modern Portfolio Theory. Decisions with a single constraint eg a cost budget would be 7a. 11d may be exemplified by the setting of an interest rate by a Central Bank, 11e by the length, breadth, height, weight and thickness in a packaging design decision. 11f distinguishes portfolio problems where for example individual projects are selected for, say, a research programme from 11g , problems where the proportions of individual elements are to be selected. This is exemplified by share portfolio formation which is the practical problem which is given considerable attention within this thesis. 12a could be the selection of a new lecturer; 12b the selection of new intake of undergraduate students; 12c the repetitive reordering of stock items; 12d the selection of a medical diagnostic test following the results of a previous test; 12e might be control decisions for a chemical plant or the adoption of buy/sell rules for commodity purchases in the light of price, consumption and stock. 13b might be a bidding sequence of partners at Bridge or the interdependent decision making of production, sales and engineering managers; an example of 13c is a pricing decision in a competitive market.

Category 22 recasts Stewart (op cit). Methods seeking to generate value and any cardinal metric of decision desirability would be classified as 22a. Methods based on where one wishes to go to, improve upon, or what one wishes to achieve in respect of any criteria without using a compensation mechanic would be classified as 22b. Methods of moving from statements of preference to orders depending on the properties of relations are 22c. Electre (Roy, 1968) and Zapros (Larichev and Moshkovich, 1995) are of this type. AHP, as Saaty proposed it, is a method for translating semantic statements of importance into measures of value, ie is 22c. Were one to use the same information to develop a ranking which minimised transitivity violations, perhaps by settling the relative positions of outcomes involved in "absolute importance" parings before those with weaker statements with this would be a 22c method.

A decision on what drink to buy at a vending machine, might be characterised as; 1c, 2d, 3a, 4c, 5c or d, 6c, 7a or b, 8c, 9a,10a, 11b, 12a, 13a 14a, 15a,16a, 17a, 18a, 19c, 20a, 21c, 22d. The regular use of LP forming part of a materials blending computer system might be represented as; 1a, 2a, 3a, 4d, 5a, 6a, 7b, 8a, 9c, 10a, 11e, 12c, 13a, 14b, 15a, 16a, 17a, 18a, 19a, 20a, 21a, 22a; or a predominantly decision tree approach to a new market entry decision as 1e, 2a, 3b i/ii/iv, 4d, 5a/b/c, 6a, 7b, 8a/b/c, 9a, 10a, 11c, 12a, 13c, 14b, 15b, 16b, 17a, 18b, 19b, 20c/e, 21a, 22a

I will in Chapter 5 introduce the technique constituting the basis of the approach adopted in this work and I will describe its domain of applicability by reference to this taxonomy. Later I will characterise other problems that can be accommodated. However, the above serves to illustrate the massive range of structural possibilities, and that the alternatives that I attempt to structure will be only a small sub-set of them. It goes without saying that, in any case, they will be bounded simplifications of the scope of real, inevitably unbounded, problems.

Chapter 3 Generating assumptions of cognitive facility in decision making: An Evolutionary Psychology touchstone.

3.1 Introduction

In this chapter I build a set of assumptions which I use to underpin the methodology described in this thesis. I start by reminding the reader that the reason for formal decision analysis is because we suffer from some form of impairment in our mental process of decision information. It is important therefore that one makes clear, plausible and balanced assumptions of what mental facility we have and do not have, which we exploit or seek to exploit in decision making and decision analysis. Unfortunately, a convenient digest of appropriate assumptions is not available "off the shelf" and accordingly I try to develop a "balanced" check list here. I attempt to use some of the ideas of Evolutionary Psychology (EP) as a patterning method and debate what capabilities and concepts would accord adaptive advantage in the environment of human evolutionary development in the light of issues which impact decision making. Although only reasonable assumptions, and not research conclusions, are sought this, is subject to broad triangulation, in particular by reference to empirical work.

In section 3.3, I outline a basic description of the EP concept and follow this with a brief description of how EP has been used to inform issues of psychology and to develop hypotheses for examination. I go on to describe how I seek to exploit it here. I make use of the test that if one cannot postulate a mechanism by which a mental capability could have secured at least a distal impact on reproduction in the ancestral environment, it is not reasonable to assume its existence. As minimalist assumptions are sought, the criterion is safe and secures balance.

I then address a number of decision-related issues from this perspective. I start with the more general issue of Reason itself; our unique capability to draw conclusions, action related conclusions, by connecting thought. I suggest that the adaptation is a powerful one but that the adaptive advantages accorded to survive in our difficult marginal niches only required moderately short-chain connections of thought, not the long near-infinite chains that artefacts of civilisation, which did not exist in the environment of evolutionary adaptation, now allows. I suggest that long chain reasoning arose from a purely serendipitous property of Reason, its capability of bootstrapping itself. Accordingly, we should be cautious in attributing to the mind powers which are indirectly dependent on those artefacts, or assumes that we possess mental systems which are analogues of sophisticated long-chain processing computers.

I then consider the nature of decision in the ancestral environment, contrasting it with modern decisions. Our ancestors would have adaptively applied their intelligence to toolmaking, organisation, relations within the group, and to the means of exploiting the environment for food, often involving issues of intellectual discrimination and judgement. Many would relate to a single clear oft repeated purpose for which learning, from both one's own experience and communicated vicarious experience, would be more useful than fundamental examination that characterises many modern problems. Single purpose allows simple "hillclimb" to be an adequate control heuristic for securing desirable parameters in the type of "design" problems that existed in our primeval world.

This presages a discussion of objectives and optimality. I suggest that no adaptive advantage attaches to articulated concepts of Strategy and Objectives in the sense in which these would be understood today. Goals, probably implicit, would be binary, and multiple objectives would be lexicographic or involve serial switching between single preoccupations. However, a considerable benefit would attach to weighing a multiplicity of factors related to a single goal. Optimisation, however, was not a concept that would have been needed to have been understood, nor would it have secured adaptive advantage. Optimal behaviour can be achieved through non-intellectual mechanisms and, indeed, is, even by animals and simpler organisms. The concept of relative improvement and the application of intellect to achieve this is, by contrast, fundamentally adaptive.

To illustrate possible differences in adaptive thinking from classically logical thought I dissect an empirically examined stylised problem, the Wason Selection Task, where the classically correct conclusion is not intuitive. However, the intuitive responses do seem to relate to the information discovery processes that might have led to adaptive decisions. Whilst this is a very specific example it serves to illustrate that an adapted mind is not a classically logical mind and the example serves as a prelude to the logically related issues of probability and cardinality.

Uncertainty is usually treated as a parametrically defined attribute within the methodology reported in this thesis, but its role in decision making generally is central. Moreover, the treatment adopted is aimed to be within the general concept and criteria of intended rationality explored in Chapter 2. For completeness, I therefore consider the adaptive implications of uncertainty. It is apparent that the mental notion of uncertain alternative futures, and the influencing of uncertain alternative futures by action are concomitants of Reason. It is also a sine qua non that a sense of comparative likelihood, including equal likelihood, and cognition of broad degrees of likelihood is adaptive. But it is difficult to go further and embrace any form of probabilistic cardinality as adaptive and therefore intuitive. Comparative likelihood allows the ordinal ranking of disjoint events which might be turned into scales akin to probability which might be suitable to a modern analyst for some purposes. Innate understanding of a probability of 0.5 is possible. However, we should otherwise be dubious about attributing cardinal probabilities to elicited subjective responses, from statistically untrained subjects, or which cannot be determined from objective considerations.

I go on to question innate comprehension of cardinal measurement and the ability to process number and quantity generally beyond that required for count and organisational arithmetic in the countable range. This leads into issues of concepts of value. Whilst the idea of ordinal value and compensation would seem to be entrenched, the concept of a scale of value would not appear to have adaptive advantage and it is difficult anyhow to see how a stable yardstick could be held in the mind. Nor would stable value and preference seem to confer adaptive advantage and this includes the weighting of factors. On these grounds one should expect value to be imprecise and labile, as expressed value seems to be. This does not of course invalidate value as a useful fiction for summarising preferences in a way which as far as possible renders them free from contradictions. I also debate the seeming facility for people most easily to trade-off only two factors, whilst also having an ostensibly polarised facility for considering a mass of factors holistically.

Finally I tabulate the cognition assumptions that I believe can reasonably be made and which act as a backdrop for the rest of the work. In essence these emphasise human abilities as a "comparator"; to make one-by-one binary comparisons and to order, rather than to assess cardinal degree.

3.2 A basis for declaring cognition assumptions

I suggested in Chapter 2 that the reason we resort to explicit analysis of decisions (using the term in its broadest sense) is that we suffer, or believe we suffer, some form or degree of cognition impairment. Assumptions of cognitive facility, of impairment of facility, lie at the core of decision analysis. If our capacity were to be unimpaired there would be no need for it. A reliable intuitive appreciation would guide us inexorably to the best decision.

For some problems this may be possible, but implicit in the existence of the other techniques we adopt, is that we cannot reliably cope unaided with larger or structurally difficult problems. We are unable, or believe that we are unable, to extract and process information from the environment and generate the most effective action without ancillary assistance outside the exclusive process of our own minds. That assistance embraces the simple; we might ask another's opinion (expanding our own facility by the support of another mind), write down pros and cons, or resort to long division by pencil and paper as we find mental arithmetic difficult. Or, it may be more complex. We may never personally know, far less be capable of remembering, the complex array or factors that we might recognise as relevant to determining the optimal operation of a complex of oil refineries, and may require the assistance of computers to solve the differential equations that underlie the design of an engineering structure. We may add to the problems of discernment, remembrance and process of factors, the issue of understanding the nature of the criteria by which one seeks to judge effectiveness of the conclusions one might reach.

We practically deal with such problems by such devices as decomposition of large complex alternatives into smaller problem partitions, which can be assessed within our cognitive powers; the re-presentation of problems into simpler more restricted forms to which we can more clearly apply concepts of rationality; the synthesis of simple criteria within our bounded rationality from cognitively complex unmanageable ones outside it; the formal paper or computer calculation of value determining arithmetic which we cannot do in our heads nor estimate with adequate precision; the recasting the range of options and factors which we choose to consider so that they can be accommodated within our capacity for comprehension; the modelling of the physical relationships of decision situations and our value mechanisms which we can tractably "solve" outwith our own processing capability using computers, and then map to the real world, etc.

I would be surprised if many would find this remarkable. However, with impairment at the centre of the need for analysis it is incumbent on aid designers overtly to recognise it. An implicit assumption is often made. An OR analyst may commend the use of a linear programming formulation to solve a refinery scheduling and crude blending problem because he believes, though does not say, that a person will not be able to do the necessary optimisation calculations in his or her head. Given that I suffer strain with modest mental arithmetic, I am inclined to accept this conclusion. However, in other model process situations facility may be less clear cut and, unless explicit, may dangerously imply unexamined and unreasonable mental capability.

Whilst the mathematical assumptions and axioms on which our quantitative methods of assistance are based, tend to be clear and well justified, the undeclared assumptions of the skills of a decision maker may move to less sure ground. Similarly classical discussions of preference are based on premises that preference can be discerned and expressed as relations of strong preference, weak preference and indifference, the last of these being equivalent to equality of value between two options. It constitutes a powerful theoretical and useful construct but it is independent of considerations of how the mind can address these issues.

It is appropriate that similar or greater attention is paid to cognitive assumptions as to the mathematical assumptions that underlie a technique. However, ultimately, as in the case of mathematical assumptions, it is only necessary that such assumptions are overt, reasonable and balanced (in the sense that they should not be excessively fussy, demanding, and precise in one area, and excessively permissive and sweeping in another). It is not essential that they should be proved to be "true". It is against this background that I seek a basis for assumption.

There is a major body of empirical research governing the ability of people to make judgements. This particularly covers distortions or biases introduced into information assessment and the heuristics by which good, if not optimal, solutions can be found to decision problems given the complexity of information available and limitations in our ability to process it. However, the research, whilst constituting a substantial and rich mosaic of findings, is, from the perspective of decision analysis, a collection of largely independent conclusions without a comprehensive unifying theory which can be directly used by the developers of decision analysis techniques or the designers of decision aids. In normative application it serves principally, and very valuably, to improve decision making by increasing the awareness of decision makers of potential pit-falls.

This research also has varying depths of coverage. Thus, for example, the limitations of human rationality, the propensity to satisfice rather than optimise, and the impairment in our ability to assess probability, the dependence of value conclusions on question framing and the lability of expressed value, are well discussed. However, more incomplete are such questions as the capacity of the human mind to cope with multiple factors in decision situations, the ability of decision makers to compute value, and the ability of the mind to discriminate between or to express a preference between combinations of factors. Whilst goals and objectives are central to decision making and so familiar in business and institutional life, the literature on goal and objective formation and comprehension does not seem to have fully addressed issues in a form relevant to formal decision analysis. Do we have intuitive facility in the formation and execution of simultaneous multiple objectives beyond the rather trivial and undiscriminating wish list, "Maximise this and this and this; minimise this", for example?

At the commencement of this exploration I was content to declare my cognitive assumptions on empirically unclear issues, and to design an analysis aid based on an introspective view of my own needs and information processing limitations, moderated by my practical but subjective and informal observation of the behaviour of other people in business and elsewhere over my working career. This was after all a personal exploration. I argued (and still do) that the validation of a decision aid or *normative* business model in terms of its usefulness to another person rests primarily in that potential user applying the test of whether it make sense in his or her world view for his or her problem: Does it pass the utility test? Do the assumptions seem reasonable? Nevertheless, there is risk in over-dependence on introspection even when validated by the subjective scrutiny of others. I may delude myself concerning my own cognitive facility and the scrutineer may exhibit the same introspective flaw when judging reasonableness.

There is also merit in developing aids which are in some sense minimalist in terms of the cognitive assumptions made or which can accommodate conservative cognition assumptions, even if some decision makers are more able or more relaxed in their ability to process decision information. Some framework for assumption forming seemed appropriate.

Were the empirical findings balanced, tidy, and comprehensive for the purpose of aid design, it would be straightforward to base assumptions exclusively on these. As an alternative I attempted to suggest assumptions employing some of the ideas of Evolutionary Psychology. To construct a simple coherent structure of decision analysis related cognition, I make use of the following tests to a generate an assumption list.

1. Is the assumption consistent with what we might judge to be an innate cognitive facility based on Evolutionary Psychology considerations?
2. Is the suggestion consistent with basic anthropological evidence regarding cognition capabilities in contemporary but primitive societies? For this I rely principally on Hallpike (1979).
3. Is it consistent with the thrust of empirical psychological work related to judgement?
4. Is it adequately consistent with my introspective view of my own capabilities and my subjective view of other people's?
5. Is it conservative?

It is essential that my purpose here, decision model formulation, remains clear. my limited purpose is to use the ideas of EP to help to suggest a balanced set of minimalist assumptions. It is possible that EP may have a more significant contribution to make regarding aspects of cognition that affect judgement and decision making, possibly providing unifying theoretical glue. There would be substantial research simply to do this. Even if I were to wish to be more ambitious, lack of training and experience in these disciplines means I can merely scratch the surface in an elementary way. It is not intended to provide more than a skeletal theory; patterned but hopefully plausible assumptions not materially contradicted by a weight of contrary evidence.

3.3 The Evolutionary Psychology approach

Evolutionary psychology has developed as a discipline in the last two decades and owes much of its impetus to the ideas of Leda Cosmides and John Tooby who remain prominent publishers in the field. It owes its origin to the concept of the adaptation of biological organisms to their environments by natural selection developed principally by Charles Darwin (1859 and 1998). It "is psychology informed by the fact that the inherited architecture of the human mind is the product of the evolutionary process" (Cosmides, Tooby and Barkow, 1992, p7). It presently remains centred on informing mainstream psychological and sociological issues and has not significantly spilled out into related areas. Nigel Nicholson has recognised its importance as an aid to understanding behaviour in management and work situations but I am not aware of specific use to which it has been put in OR/MS.

At the core of the approach is the central assertion that "the brain is an adapted organ like any other" (Nicholson, 1997). "The mind is a systems of organs of computation, designed by natural selection to solve the kinds of problems our ancestors faced in their foraging way of life, in particular, understanding and out-manoeuvring objects, animals, plants, and other people." (Pinker, 1998, p21). Our interacting web of emotions, cognitive facility, intelligence, intuitions, instincts, reflexes, senses, the brain led control of other organs, and the brain involved biochemistry that encourages us to undertake particular courses of behaviour or better enables us to deal with certain situations, are all evolved through natural selection to cope with the environment in which they evolved.

The concept is elegantly simple and independent of the complexities of modern genetics and embryology, unknown to Darwin, which give it effect. For evolution to occur we require that an organism replicates itself, though with at least slight perturbations serving to impact the effectiveness of replication of its descendant, *and* a selection mechanism. (Darwin,1859, Dawkins 1986). Although plant and animal breeding by *human* selection had been going on for many centuries, it was Darwin's genius to recognise an auto-regulatory process, the ability of the environment to select superior perturbations and to accumulate small changes over hundreds of generations. He encapsulates this (p63) "Can it, then, be thought improbable, seeing that variations useful to man have undoubtedly occurred, that other variations useful in some way to each being in the great and complex battle of life should sometimes occur in the course of thousands of generations? If such do occur, can we doubt (remembering that many more individuals are born than can possibly survive) that individuals having any advantage, however slight, over others, would have the best chance of surviving and procreating their kind? On the other hand, we may feel sure that any variation in the least degree injurious would be rigidly destroyed. This preservation of favourable variations and the rejection of injurious variations, I call Natural Selection." If an organism has a trait that enables it relatively to better reproduce the trait in a succeeding generation then that trait will grow in succeeding generations.

We must nevertheless recognise that modern genetics can lead to unusual and "discontinuous" phenomena. For example pleiotropy, in which a single gene can influence more than one characteristic, one of which may be beneficial and the other not; or heterozygous characteristics (arising from mixed combinations of alternative alleles) which confer replicative advantage, whilst one or both homozygous forms are injurious. One should also note that complex systems can be converted to other complex systems by the alteration of a single gene, an insect antenna to a leg, for example (Shepard, 1987, p268), and I will return to this point. The central issue remains that the trait combination in the gene pool should be adaptive, that is serve to contribute to the reproduction of the organisms possessing them, and hence their own replication.

Along with the notion that the brain is an adapted organ, is the second pillar of the EP approach- it was not adapted to solve the problems we face today. It evolved to solve the adaptive problems (that is problems "whose solution can affect reproduction, however distally" (Cosmides, Tooby and Barkow, 1992, p8) in the environment in which we evolved. Man (from *Homo Habilis* to *Homo Sapiens*) has existed some 2m years, roughly corresponding to the Pleistocene era, and of course spent many millions of more years evolving to that state. Neanderthal man, exhibiting organisation, perhaps arrived some quarter million years ago. *Homo Sapiens (Cro-Magnon)* emerged within the last 100,000 years with other human species dying-out by 30,000 ya. Mithen (1996) argues from an archaeological perspective that man and its predecessors first developed General Intelligence and subsequently specialist modules including in broad order Social Intelligence, Natural

History Intelligence, and Technical Intelligence. Finally in modern *Homo Sapiens* "cognitive fluidity" developed promoted by the emergence of language. Nevertheless, over much of that evolutionary time, including the Neolithic period (up to approx 5000 years ago), the environment was relatively stable, that is, despite marked climatic movements, changing at a rate that could be tracked by evolution. Thus, "Our species spent over 99% of its evolutionary history as hunter-gatherers in Pleistocene environments. Human psychological mechanisms should be adapted to those environments, not necessarily to the twentieth-century world" (Cosmides and Tooby, 1987, p280). Pinker (1998) expands the same thought "Our brains are adapted to that long-vanished way of life, not to brand new industrial civilizations. They are not wired to cope with anonymous crowds, schooling, written language, government, police, courts, armies, modern medicine, formal social institutions, high technology, and other newcomers to the human experience.", as does Nicholson (1998, p420) "In the ancestral environment of uncertainty and danger we evolved cognitive systems which now fit uneasily within a world of complex problem solving, rational calculus and probabilistic reasoning."

To bring the point directly to the subject of this thesis, we know that modern organised life puts a premium on bringing to bear on decision problems such concepts as precise calculation, extended compound calculation, classical logic, cardinal quantification and metric measurement, cardinal probability, dexterity with multiple quantified objectives, optimisation, stable and precisely articulated values, simultaneous multi-attribute compensation etc. But our brains will have been adapted to solve the *reproduction affecting* decision problems of our ancestors. The extent to which modern economic man is also adept at solving his problems is determined by the characteristics of the mechanisms necessary for the solution of primitive problems and whether those mechanisms can be brought to bear on modern decisions. This, at least in part, is determined by the structural similarity of the decision problem domains.

3.4 How is EP normally used?

Evolutionary Psychology is used to examine how the selective pressures of the ancestral environment might work in order to "generate hypotheses about the design features of the human mind". In this way it is used to help discover previously unknown psychological mechanisms. (Cosmides, Tooby and Barkow, 1992). Cosmides, Tooby and Barker also observe that the flow from "Adaptive Problem" to "Psychological Mechanism" can be reversed to explain the adaptive function of observed phenotypic characteristics.

Tooby and Cosmides (1992, p75) outline what they call an evolutionary functional analysis as defining an adaptive target, describing the background conditions in terms of the recurrent structure of the ancestral world that is relevant to the adaptive problem, suggesting a design ("..features in the organism that comprise the adaptation or suggested adaptation"), and then examining and evaluating its performance both in achieving its ends in the ancestral environment and assessing its impact on behaviour in a modern environment. They summarise the process as, "... asking a series of engineering questions: Would the proposed design have interacted with properties of the ancestral world to produce target adaptive outcomes? Does the proposed design interact with properties of the modern world to produce outcomes than one actually observes in real organisms, whether these outcomes are adaptive or not? Is there an alternative design that is better able to generate adaptive targets under ancestral conditions?...".

It attempts to answer two types of question: "What is the explanation for psychological phenomena that we can observe?" and "What behaviours might we hypothesise which we should seek to observe?".

3.5 How I use EP.

Here I seek to parallel much of this process. For example, I seek to postulate decision making, judgmental, and evaluative competencies. I suggest the type of decision problems, adaptive problems, our ancestors might have had to face, notwithstanding that much the of our prehistory must be conjectural. From this I hypothesise entrenched mental mechanisms, either intuitive capabilities, intellectual facility, or other systems, that they should have developed to deal with them. In some instances. I also seek, paralleling the reverse flow methodology outlined above, to rationalise some aspects of empirical research, which are not already patterned within a coherent theory, by seeing how well they can be explained by evolutionary function. The objective is a list of competence assumptions, making visible those products of the evolutionary process which, unlike eyes and bones, cannot be seen.

However, to achieve that I must not only assume the existence of some competencies but also the absence of others for which there might not be direct evidence. Whilst such an extension requires extreme caution it can be justified within the principles of the approach. Dawkins (1986) makes use of the image of the watch drawn from William Paley's "Natural theology -or Evidences of the Existence and Attributes of the Deity collected from the Appearances of Nature". The watch had been used by Paley as an example of a system that demonstrates by its complexity that it had been designed by man, to draw the conclusion that biological complexity must have been similarly designed by God. Dawkins presents a statistical description of the evolutionary process. Complex systems in organisms can and do arise by cumulative selection of small changes. Evolution can accumulate extraordinary complexity by small degrees but the chances that such systems can arise by chance by spontaneous transformation are astronomically improbable.

The same argument also enables the assertion that no complex biological system should exist, including those relating human mental faculty, unless it served an adaptive purpose in the environment *at some point* in evolutionary history. In short unless this test can be met the system is not merely invisible and its existence unknown and unused, but it is astronomically improbable that it is there. Thus, for example, whilst we could have a sense of direction and facilities to navigate over short distances, we cannot have in-built systems for navigating over thousands of miles, as unlike birds, we never needed to.

If a system served an adaptive problem of the early evolutionary environment which ceased to be relevant later, the system might be selected out, it might erode by genetic drift or mutation (in the absence of any selection) or be adapted by natural selection to another purpose (as in the adaptation of penguin wings from flight to swimming). But it might continue to exist in adapted or vestigial form. One could in such circumstances envisage a progressive erosion of such a capability resulting, perhaps, in an incomplete system. However, we might speculate (whilst acknowledging the hazard of doing so) that wired-in cognitive "electronics", could erode faster than physical structure and would need to be sustained by the functional needs of man's later evolutionary history, though we should not expect this to extend beyond the Neolithic.

Three factors muddy interpretation. One, already alluded to, is the ability of profound phenotypic system change to be achieved by simple genetic changes. This opens-up the possibility that a mind feature can arise by chance ideally suited to modern life, for which our ancestors would have had no use. However, a new *complex* system cannot come into existence spontaneously as a new creation (for Dawkin's reasons) but only by a simple genetic change causing sea-change transformation of an existing comparably complex system (eg by mutation). I suggest that the chance that such a complex alternative system being viable is inversely related in some manner to the complexity of the system; there are compounding opportunities for there being an immediately fatal flaw. But an additional feature here is an immediate change of transforming significance would not only have to be immediately viable and potentially superior but *immediately superior.* A "new" system will not be sub-optimised though will be up against an "old" system that is. It may thus be selected out before any inherent superiority of the new system can be established, ensuring stability of the "inferior" configuration. Whilst such gross transformations may occasionally have been converted into sustained adaptations it seems probable that they were rare. Indeed, physical characteristics which we can examine would seem to have gradualist origin.

Of greater difficulty is the possibility that design of value in the ancestral environment is made use of in very different applications of recent origin. Modules of Pleistocene mental facility might be linked to create a competence apparently only relevant to modern needs. However, in such cases we should be able to see both the ancestral features employed and the "work-arounds" used to adapt them to modern problems.

Finally we must recognise that within our cognitive toolkit we have a vital multipurpose tool; the ability to think. This adaptation enables us to fine-tune our responses to unfamiliar situations and radically transforms the potency of a rather more common adaptation in the animal world, the ability to learn. But the ability to think, powerful though this is, should not blind us to the puny limitations of our unaided mind and that this skill still depends on innate attributes. I discuss this later in this chapter.

Notwithstanding these complications, the lemma I postulate for judging the decision and judgement skills is:

> The human mind has *no* mechanisms, architecture or facility except those evolved to solve adaptive problems present in the Pleistocene environment or before apart from those which are concomitant by-products (spandrels) of such features.

In generating assumptions of the innate competencies of the modern decision maker, I have sought to apply the test of whether a particular skill could have improved the survival of man in the environment of evolutionary adaptation (that is his capability to reproduce) or be related to it in a describable way. If not I assume that it was not, and *is* not, there. This is to a large extent an experiment of imagination, as perhaps EP more generally is, based on conjectured adaptive situations, and may seem ambitious. However, I remind the reader that I seek no more than to suggest plausible and minimalist assumptions. Thresholds can perhaps be more easily justified as they do not depend on knowing and understanding the full panoply of ancestral life. I have sought to check these in terms of whether other problems could be imagined for which extended facility would have had an adaptive function, indeed, whether they would even be practically useful for problems outside the modern era. I use EP as an alternative to an arbitrary check-list and a means of balancing it, not to imply that the arguments here prove their truth.

It is a list of such assumptions that is developed over succeeding sections. The list is summarised in Section 3.13 and the reader may wish to refer to this from time to time.

3.6 The reason for Reason

It is instructive to discuss the concept of reason within the framework of this investigation. As Evans (1983 p1) observes "No subject in psychology has a longer tradition of study than that of thinking, which goes back well before the separation of the disciplines of philosophy and psychology." However, this investigation is directed to practical matters and a simple definition of reason may suffice in this context. I suggest that reason is the process by which people assess information by connected thought, that is draw inferences by conscious deliberation. (The phrase "connected thought" is drawn from an OED definition and I have also used Evans (op cit p7). Following, Barsalou (1992 p275). I propose, for purposes here, that thought involves a series of transformations performed on the contents of working memory, where these transformations and contents are conscious at least to some degree. I accept that this is incomplete, dodging as it does the meaning of consciousness but suggest it is adequate for these modest purposes. (Hale (1999, p9-26) criticises various attempts to define it but does not come up with a succinct description. I ask the reader as a possessor of consciousness to recognise it).

Reason may in this model be looked at as a process with soft edges at one end of a continuous spectrum of activity primarily controlled by the brain (though supported and influenced by biochemistry) ranging through such processes as Intuition (also an aspect of intelligence), to the involuntary control of the physical systems of the body. In seeking to draw an arbitrary line in the spectrum between red (Reason) and orange (Intuition) we will allow that Reason embraces conscious internal scrutiny of unconscious thought. For example, some people may "see" an anagram without conscious process but will consciously confirm that it is correct. However, if there is scrutiny of the conclusion, that the combination recognised, is indeed an anagram, then it is reason for our purpose. A conclusion drawn without conscious scrutiny (perhaps, a judgement of an individual's character) I will describe as Intuition. We might also distinguish this from Instinct using this for a process in which environmental information is processed directly into action without conscious intervention.

Although reason is the basis of modern knowledge such as Astronomy and Philosophy, These and most other sophisticated knowledge cannot solve adaptive problems. Reason is too complex a system to be a spandrel. What then can we say about the adaptive problem that reason solved?

For there to be an opportunity for evolution to amplify a trait there has to be a reproduction affecting difference; in this case a difference in behaviour. But reason is a process of connected thought. Therefore, it is in *deliberate* actions we can take that we would not have otherwise taken in the ancestral environment, using the information we extract from the environment and process by such connected thought, that constitutes the basis for adaptation. We may call this the exercise of choice. Reason allowed considered choice leading to relatively superior action; that is superior by improving the reproduction of the organism possessing the genes encoding the trait, relative to that that could be achieved by acting only the basis of pure instinct, hormones, taste, smell and the other oriented adaptations operating within an integrated *action* system. Hale (1999, p143), in slightly different terms, sees "human understanding" as the evolved capacity to acquire and use knowledge of causal processes in the natural world. He identifies (p282) "faculty of intention" as being a necessary complement to thinking and understanding, "which would be impotent without it". This he perceives as the pre-disposition to the implementation of mentally conceived action plans. Damasio (1994, p165) also suggests " ..that the purpose of reasoning is deciding and that the essence of deciding is selecting a response option, that is, choosing a nonverbal action, a word, a sentence, or some combination thereof..". This capacity for deliberate action enabled us to inhabit environments to which we would otherwise be unsuited and to handle the complexity of multiple, interface, or fast changing environments.

But we should look at this system and its importance within the context of other systems we possess and other organisms possess. It is perhaps worth reminding ourselves that we are possibly the only organism capable of comprehensive reasoning. Other organisms are highly successful and live complex social lives, and in the animal world in particular, solve adaptively and practically similar problems to those of our *Homo Sapiens* ancestors from whom we are indistinguishable. These species manage very well without reason. We also remain dependent on a variety of action oriented systems which are arguably far more immediately critical for species survival. If we lost our sex drives, child nurturing instincts, recognition of pain, or immune systems, reason would not prevent the gradual loss of our species. As so many other species manage to solve the complex problems of survival and reproduction without our special system, and we critically need many of the systems that we share with them, we might argue that reason is merely a marginally useful adaptation within the total scheme of evolutionary importance. This is notwithstanding the high standing that we choose to attach to ourselves as a result of having it.

This adaptive solution is nevertheless an extremely elegant one because of its multi-purpose versatility, it enabled us to occupy the "versatile environment" and what Hale calls "artefactual" niches. It is capable of flexibly contributing to a variety of adaptive problems; navigation, catching, picking, transporting, keeping, and processing food; keeping warm and safe, identifying, keeping and protecting mates; nurturing and protecting offspring; evaluating, co-operating, outwitting and communicating with members of our group and other groups etc. In this respect it is distinguished from many other systems which are action specific.

The adaptation had one other profoundly significant feature, which is unique amongst adaptations. It was able to bootstrap itself *independently* of the evolutionary process, to overcome limitations of memory on which its basic operation depended. It was able to gear itself, unconstrained by capacity limitations which embryology and physics imposes on all other adaptations. Thus, in the current era we are able to use reason to chain thoughts into infinitely long sequences, unencumbered by the restrictions of memory or the restrictions of a single brain. Aided by language (probably an adaptation) (eg Pinker 1995), and, critically, writing (certainly not one), thoughts can be committed to a more reliable and infinitely capacious long-term memory for subsequent process by ourselves or others, and transmitted without deterioration over great distances and over generations. But the ability to create long chain thought was not itself adaptive. Reason is the foundation of civilisation, but it is fortuitous non-adaptive artefacts that gave it potency. This remarkable consequential power remains simply a spandrel .

This model was challenged by a question posed to me. Is poetry possible without writing? Poetry in the terms of this model are certainly long connections of thought of a complex and subtle type structured by form as well as content. My answer is that it is possible (and legend, structured music, sophisticated law, and other accoutrements of culture as well), making use of our capacious long-term memory for which the adaptive advantages are clear, as a substitute for paper. But the essence of long-term memory is that it contains learnt thought and whilst retrieval is speedy, storage is generally not. Paperless poetry depends on a process of learning and refinement and, if of length and complexity, is likely to be the result of process of several minds over generations. Writing provides pace, accuracy and capacity beyond even our large long-term memory.

Some support for the supposition of simple capability can be found in the computer analogy. Simple machines with few logic elements can solve very complicated mathematical problems. Most people of modest education could devise a method for working out square roots by reasoning, aided by pencil and paper; but how many could quickly calculate the square root of 5 to even one decimal place? Even those who can find it with difficulty are taking deductive short-cuts abandoning reason for memory and recognition; we know the squares of 2 and 3. Hunter (1966 p341) describes the skills of a mathematician who, by contrast had prodigious skills of mental arithmetic. He was able to retain more in his memory than most of us, but largely depended on his knowledge of numerical properties, and was still very less competent than a cheap modem calculator.

Yet our capability for recognition is of quite a different order over that for reason. My computer has a character recognition program for the scanner; the program and the library of subroutines associated with it occupies 12MB. It does a satisfactory job but frequently makes errors; for example, not distinguishing dirt from text. Yet the variety of its task is trivial compared with the problem of recognising faces or places, or identifying objects, tasks which we appear to do with ease. Standing (1973) found that people were able to recognise 6600 out of 10,000 "Normal" pictures exposed for only 5 seconds each two days earlier. For "Vivid" pictures he considered "memory capacity is almost limitless"; abstract material was not retained so effectively. Against such unconscious power, our reason seems limited.

We should also avoid imputing to ourselves innate abilities we may have as a result of other facilities of the mind. Our ability to leam means that "long chain" reasoning, can be "schooled" and we can acquire sufficient familiarity to believe we have an extensive innate ability. In reality the unsupported mind can still only get itself around "short chain" problems. Compared with the ability to *learn* (an adaptation which we share in varying degrees with other organisms), *reason* is inefficient. It is superior in the solution of essentially original problems but it is a wasteful for addressing the many more that we have solved before.

Reason gives us an edge. But reason isolated from modem artefactual procedure may make only a limited contribution to good decisions, relative to other systems within us.

3.7 Decisions in the Ancestral Environment

Our ancestral environment cannot be known with certainty. We can say that we were Hunter-Gatherers for 99% of our existence as distinct species, without the benefit of agriculture, far less civilisation or organised economy. Indeed it is possible that hunting is an explanation of our speciation (Hill, 1982) and it is any case the determinant of (or intertwined with) many of the other adaptations that particularly characterise humans eg the use of tools, sexual biochemistry and mores, gender roles, group size and relations, communication, bipedalism, sharing, reciprocation, co-operation and intelligence (Hill (op cit); Washburn and Lancaster, 1968; Laughlin, 1968). Laughlin suggests, "Hunting is the master pattern of the human species. It is the organizing activity which integrated the morphological, physiological, genetic, and intellectual aspects of the individual organisms and of the population who compose our single species". However, Mithen (1996, p46) questions this, believing it is incompatible with, for example, facility with creative mathematics.

For a significant proportion of our evolutionary time we were people of the African savannah, though descended from apes who are and were mainly creatures of the rain forest designed for that environment. We may reasonably assume that we operated in relatively small hunting determined groups but with sufficient integration with other groups to facilitate exogamy (which Hill argues would have been necessary for economically viable sexual balance) and, possibly, to negotiate avoidance of gratuitous competition. Population would be sparse, though the range of individual groups would be very large compared with other primates. Groups would expand to fill the resources available in the good times, divide, and compete when conditions regressed. We can assume that the amazing versatility provided by our unique trait Reason would enable us to exploit either temporarily or permanently, different, marginal and interface environments (perhaps rain forest/savannah boundaries), an ability evidenced by the range of our species which is wider than any other animal and includes habitats for which many of our physical attributes constitute a severe handicap. (How would our hairless bodies manage to survive even in temperate zones, without our intelligence? How else could we survive on the savannah without a grazer's or browser's digestion, or evade being hunted, without living by our wits?).

Our environment would nevertheless have been stable. That is not to say that life was devoid of uncertainty or mobility; on the contrary at an individual level uncertainty and hazard would have been a dominating characteristic. But there would have been stochastic stability. Our ancestors would have faced the same types of problems as their mothers and grandfathers. Unusual events would have occurred but rarely outside living memory or the inherited vicarious experience of legend, available for the guidance of action. Life would be lived in the fast lane of action but the slow lane of types of action.

Within that environment we can assume that our intelligence would be applied (practically and adaptively) to the production and use of artefacts such as tools, clothing, and shelter. (The use of simple tools is observed amongst other primates so this was likely an early skill). It would also be applied in like terms to organisation and the politics of within group. Mithen (op cit) noted that brain size increases in new species of man correlated with and, he suggests, was causally linked to group size, and he accordingly puts major emphasis of the intelligence demands of larger group living. More caution is required here. Organisation requires elements of negotiation or instruction and the politics of society may give an adaptive advantage to the smarter human. But, many species live in large groups and "co-operate", and simple "unintelligent" rules can simulate quite complex group behaviours).

We can assume intelligence would be applied to various forms of taxonomic discrimination and other forms of understanding the natural history domain. Departing from a narrow species domain, food sources would cease to be "obvious", and abundant in any narrow range of types. Food would need to be explicitly selected from a far greater range of possibilities than usual in the animal kingdom. Intellectual classification secures reproductive advantage.

Tools brought the ability to hunt but our ancestors still suffered severe disadvantages. Behaviour of potential prey, and those for which they might themselves be prey, needed to be understood and such facility would be adaptive. To be versatile environment-wise, we could not acquire domain specific instincts (an alternative mechanism). These could not evolve with sufficient speed to ensure our species survival in the face of what, to the rest of the biological world dependant on narrow environment adaptation, would be radical change.

Other parts of the animal kingdom inter-relate with members of their own species without intelligence but given that intelligent "behavioural" appreciation of other species is adaptively created, the skill would be available for "reading" and out-manoeuvring our own. As there would be an adaptive advantage in doing so to secure more food, sex, desirable mates, allies, and the trappings of power to underwrite these indefinitely and to dispense nepotistic privilege, the trait would spread in the gene pool and be capable of relation to specific groups and individuals within groups. The ability to track, navigate, plan, conspire, outwit, make, belong, build, use, correlate, classify, assess, explain, inter-relate, charm, frighten, control, submit, befriend, share, distrust, lead, follow, listen, persuade and to do so *discriminately* are *all* traits for which there is adaptive advantage. The intra-species application of reason would give an advantage to individuals exercising it in this social way, though ultimately it has to demonstrate its viability exogenously in the "games" played by the species against the environment and other species within the environment.

Now let us look at the mechanisms at work in some of these facilities. Many of them are decisions with a single clear purpose which are not determined from first principles and are predominantly dependent on our ability to learn and consign the results of experience to long-term memory. Plant taxonomy, navigation, artefact manufacture, and the implications of reciprocation are simple examples, but the same principle applies to more complex tasks, say, a judgement of trustworthiness. In the first example, a classification may be tagged with an action-oriented conclusion implying a pre-tabulated decision (eg "tasty"→ eat). We may also assume that decisions of "how to do" whether in matters of craft, domestic and providing skills, or organisation, were also principally of demonstration, observation, and learning, as they are now. A young chimpanzee observes another breaking a nut between two stones and copies it, his success causes him to repeat the experiment. He learns. So do people. Much of such skill at this level might be considered to have no decision content beyond that of securing the end that the tool or method enables.

However, issues of design, selection, and approach, quickly enter the equation, and they require choice from options. What location for the shelter or trap, what people for the team, what piece of wood for the shaft of a spear, what forage plan? Guided by personal or vicarious experience a particular decision must be made cognisant of the circumstances of the moment. Then, as now, only so much could be taught or observed, accumulation of experience then allows for more comprehensive classification, more explicit action tagging, or more effective design.

We may note that many of the these decisions are multi-factored and these may interact in quite complex ways. Thus even the simple spear shaft has to be selected and crafted with attention to length, thickness, hardness, straightness, weight and springiness. But its function (a predominantly *single* function), is clear and experience would lead us through feedback and "hill-climb" search heuristics to an adequately optimum design decision, in control terms requiring no more perception than the ability to relate a difference in parameter to improvement in performance. So too with methodological or organisational issues; though here we might be tempted to use Lindblom's (1959) similar idea, Successive Limited Comparison.

The selection of a mate may be a more difficult multi-factor and multi purpose choice and, as a one-off or occasional decision, there is less opportunity for corrective feedback. However, cultural ideals (arguably an encapsulation of vicarious experience) biochemistry and our emotional systems make this complicated choice easier at least from the view point of individual parties. The structural complexity of this decision would seem to arise not from its multiple factors, or unclear objective structure but from its multipartite structure. Games of such complex structure are still unyielding to computer optimisation. Thankfully, (thanks to evolution) even here we have within us systems to avoid the indefinite stand-off at risk and, often, to convert an achievable compromise into a highly desired outcome. Ultimately this complex choice is simplified to one of winning, submitting to, or evading one person.

The other feature of the ancestral decision environment is that in information terms it would have been "noisy". Accordingly the practical and adaptive premium on precision would have been far less than the practical premium today. Adaptive sensitivity would be far less than the sensitivity that attaches to the highly geared practical decisions of modern commercial life. It would have been important to be competent, say, in judging who to trust, but the precise weighting of cues would be relatively unimportant as, however refined, mistakes of trusting and of not trusting would still be made. Adaptive decision making could be achieved with a broader brush.

3.8 Objectives and Optimality in the Pleistocene

For modern economic man, in his working habitat, decisions go hand in hand with Objectives. He lives in a world of objectives, targets, strategies, and missions and in a world in which best is most or least, and is precise. They are central to normative decision making. We know them and we often declare them. We want the most profit or the shortest time. We pro-act in the light of them. We cope with these modern competitive needs and because we are so familiar with them we may think that we are innately adept at handling them. Are we? In Chapter 2 I postulated that, although we may have a strong qualitative ideas of what we wished to achieve and an appreciation of the attributes determining the goodness or badness of decisions, we were Vague in the specification of more than one (or, at most, two or three) objectives. What insights emerge from EP considerations?

Let us first consider objectives and strategies. Our ancestors would have had the most simple overall objective; to survive, and insofar that this objective allows additional flexibility, to do so as pleasurably as possible. But such an overall objective, even though simple, would have little meaning, because objectives imply pro-active purpose and provide criteria for the selection between possibilities. However, at least at the strategic level man would have been entirely reactive, the environment was stable. Life was to be lived in the same way as their Great, Great, ...Grandparents did and their Great, Great, ... Grandchildren would. There was no strategy. Any unforced changes in the habit of life would be by very slow gradualism.

The same would also have generally applied at a personal level. There were some personal strategic choices, the means to secure our preferred mate, whether to befriend X, whether to bid for leadership within the group or whether advantage can be secured by submission to another. The task and options would be simple. Their solution would have been assisted by the application of reason (although again our emotions would be at work) but the process would not have required concepts of objectives or strategy anymore than these familiar problems require them in the animal world.

Our ancestors would of course have had roles; and, within these roles, tasks. These would have been basic; hunter/ provider, homemaker/ mother and, possibly, group leader and family head. At least as far as the first two of these were concerned our effectiveness in these roles would have been reinforced by the evolutionary selection of reinforcing instincts, and we can perhaps discern these in our own natures. We can also see that there might have been adaptations to suit some of us for leadership

There might, in due course, have been other specialisation of task within basic roles (eg tracker, shaman, basket-maker, water carrier). Such specialist roles would not be adaptive (there are too many and the advantage of particular roles may have emerged too recently in the evolutionary time-scale). Some may have been determined by tradition or inherited, but others simply determined by gender, anatomy, assignment or self-selection, as in any community there would be a natural but exploitable variation in the skill and physical profiles of members. Facility to flexibly adopt or assign complex roles would, however, depend on adaptations. In addition to the basic human trait of reason these would include the facility to perceive those skill differences in oneself and in others and to make judgements of relative advantage. A group with these skills in these areas would be more practically and genetically successful than a group without them. However, dexterity or intuitive familiarity with "objective setting", per se, would not be required as the profile of roles necessary would be stable over generations.

However at the tactical level the situation changes. There are issues of choice to be made on a fast changing time scale. If we consider the hunt for example, there are questions of where to go, what to try to catch, and how to do it. Goals and plans and the ability to cope easily with them become important and competence therewith may be adaptive as genetic survival would no doubt be enhanced (but remember this is gilt on the gingerbread, other carnivorous species cope without these human skills).

It is nevertheless instructive to contrast such goals and plans with modern equivalents. Ancestral objectives are likely to have been implicit, qualitative, classificatory, of short duration, imprecise, liable to pre-emption or replacement, with success measured on a binary or a simple classificatory scale, and one-dimensional. Thus the objective might be to "catch a gazelle", "gather berries, "ingratiate myself with the son of the Chief"; it might be replaced by another if it rained and would have no validity beyond the day or trek or the natural time to complete a task. Generally one would judge success by crude measures, "nothing", "one elk", "enough", perhaps principally judged by exceeding or falling short of unarticulated prior expectation. There would not be multiple objectives such as "get elk and boar": both might be possibilities for the mission but in a statement "get elk or boar" there is likely to have been a well understood hierarchy of desirability. Quantitative considerations would be minimal. "Enough for the family or group" might be a an implicit goal with "too much" being determined by practical constraints such as what one can carry, eat, and protect. Often there would be a natural binary test of goal achievement eg win-lose, sex-no sex, miss-hit, life-death. Matters involving degree would be governed by notions of satisfying and satiation similar in kind to the modern criterion of satisficing. They might be "converted" into binary conclusions by the emotional adaptations of disappointment and elation when we failed at the hunt, or our sycophancy is rewarded with a token of recognition. We can see our natural inclination to "binarise" in our modern lives; soccer goals to win-lose, exam marks to pass-failure, and our bonus into whether it was more or less than last year's.

There would nevertheless be a large number of value assessments to be made in determining goals, switching or modifying goals and executing goals. Is it best to try for boar or elk; is Fred reliable; does one stop to pick up walnuts blown-off in the storm? Skill in such choices would be adaptive and that man would have acquired intuitive skill in such selections, but we might again assume imprecision would be tolerable. Moreover, as such choices would also often have been binary, sometimes with a few options, but rarely at one time with many, we should not assume innate facility with multiple option evaluation.

We can also look at objectives through our ancestors more basic needs. They sought food, sex, warmth, power, fun, and companionship. We are multiple needs organisms in our very natures are we not also multiple objective organisms? A short examination shows that it is adaptive for our nature to be the contrary. It is adaptively important for us to be interested in sex but not in flight or battle when greater dangers to our procreativity exist. We may seek power but it is not adaptive when we are hungry or cold. As Maslow (eg 1970) pointed out, we have hierarchies of needs, multiple needs. "Man is a wanting animal and rarely reaches a state of complete satisfaction except for a short time. As one desire is satisfied, still another comes into the foreground, etc. It is a characteristic of the human being throughout his whole life that he is practically always desiring something" (p24). Our wants "seem to arrange themselves in some sort of hierarchy of prepotency." (p25). We have *serial* objectives and the selection of the objective of the moment would be primarily or exclusively a matter of adaptive biochemistry rather than reason; with reason contributing more to achievement. Even today we may find our more modem problems of decision priority affected by primeval objective choosing mechanics, for example, by sexual game plays in office situations.

The concept of the serial single goal obviously requires a concept of priority and this might be seen to parallel the modern need for relative values and weights within multiple concurrent objective problems. However, there is no reason to suppose that a material adaptive advantage would have accrued through intellectualisation. The long-run balancing point for sex versus food, for example, would be resolved through evolutionary mechanisms largely independent of the mind and at the practical level there would be insufficient stability in the sex versus food relative pay off for a "weighted objective" to simplify future evaluation. Indeed, any individual exhibiting such a trait would suffer reproductive *disadvantage*.

In the modem world our objectives lead us towards good action, indeed we seek the best or optimal action. It is therefore also instructive to examine the notion of optimisation within the context of the environment of evolutionary adaptation (prima facie, still a useful thing to thing to achieve). Were our ancestors to have spent several hundreds of thousand of years evolving in oil refineries where procreative rights were linked to successful control of the process, what would have happened? We can be certain that the operation of the oil refinery would have been optimised, but would this mean that we would have become good optimisers. If during evolutionary time, the crude oil offered to the refinery varied over days in price, quantities, and characteristics, and the required products varied similarly, they could indeed have become brilliant intuitive LP optimisers inverting large matrices in their heads. If altematively the oil demand and product prices remained steady but crude types and prices varied, we would have become expert dealers, knowing the value of every crude on offer and literally smelling whether it could be economically blended with other types. If supply and demand and all prices remained steady we would become zombies whose reason had become vestigial, but we would be capable of reflexively responding to minute movements in temperature gauges. In all cases the refineries would be optimised. But evolution would have taken its time and created optimising adaptations suited to the environment. If the environment is strategically stable we do not need individually to be able to optimise in order to operate in an evolutionarily optimal manner.

Optimal foraging (Harris, 1993; Smith, 1983) is an example. In principle the selection of diet to maximise energy intake relative to foraging time, subject to constraints on other nutritional needs, is a knapsack problem of profound adaptive significance. But, it is a problem solved on an evolutionary time-scale by the development of tastes (physical as well as mental) favouring one food rather than another and on a shorter one by societal preference or taboo. This would be a more efficient solution than providing an intellectual mechanism unless the ancestral supermarket habitually offered completely novel options. Moreover, as species without the capacity to reason also show optimal foraging behaviour (Krebs, 1973), unicellular predators exhibit behaviours that constitute effective search procedures (Charnov, 1976), and foraging models with simple rules can be developed which deal with the survival needs of the modelled "organism" Simon (1957), it is apparent that the fact of optimisation does not require intuitive understanding or intellectual facility with the concept. I conclude that the concept of optimality and the capacity to optimise in original circumstances is not adaptive and not innate. Simon (1965, xxiv) observed that "Administrative theory is peculiarly the theory of intended and bounded rationality of the behaviour of human beings who satisfice because they have not the wits to maximize". Administrative Man is much like his primeval ancestors; he does not have the wits to maximise because his forefathers never needed to.

I suggest that the concept of long-term objective as a solid criterion for focusing and evaluating relative achievement is not itself directly hard-wired within our mental mechanisms. We might plausibly argue that the step from short-term goal to long-term objective is not an issue of quality but of degree: a mind tuned to the former should not find the latter too alien. However, we probably cannot say that we are intuitively adept at defining objectives which are not implicit in the roles we perform. Moreover, although we may readily perceive the interacting impact of many complex factors to a single end, we seem unlikely to be endowed with inbuilt mechanisms easily to perceive problems as *simultaneously* requiring the achievement of multiple objectives or to balance multiple objectives.

Looking introspectively and observing subjectively, we do seem to be adept at lexicographic objective formulations and the concept of single objectives circumscribed by "subject to ..." qualifications. This seems consistent with Simon's notion that we satisfice and Lindblom's Successive Limited Comparison. We appear naturally suited to moving dextrously between serial short-term objectives whilst moving the overall game plan by small adjustments to established norms. We might intellectually recognise that some or many conflicting objectives might be simultaneously dealt with by striking a relative balance, but we remain vague about how to do this. Furthermore, we may not sustain a consistent balance between them. There were few evolutionary Brownie points for doing so. For the same reason, it is unlikely that we have unschooled mechanisms which graduate the achievement of long-term objectives (in distinction from choosing the important objective of the moment). We must question our facility with the idea of achieving objectives better or more economically, in contrast to merely achieving them.

This appears to be consistent with the assumption I make in the approach developed in this thesis: that people do not have a innate affinity with complex objectives defined in any precise way to address economic problems, despite having good appreciation of their broad intentions and the factors involved. They are vague. Objectives that can be directly mapped to decision desirability rarely exist and cannot readily be defined. They need to be constructed.

3.9 Ancestral Logic

Modern analysis of decisions depends on quantitative evaluation by decision maker or analyst but also quantitatively expressed judgement of values, risks and, in some cases, of factors. It thus depends on numeracy not only in the generation of conclusions but in the provision of information describing the mind of the decision maker. I will explicitly discuss both issues of number and risk shortly.

Both concept of calculation and the ideas underlying statistics share common ground with the ideas underlying classical logic- the manner in which conclusions may be drawn from given propositions. Classical logic also lies at the root of concepts of normative rationality. If there are any grounds for believing that concepts of testing thought which we call Logical are not innate, we are entitled to question our intuitive comprehension of its close cousins.

I therefore explore in this section whether all ideas which we call logical would have adaptive value to our ancestors. In particular I examine the Wason Selection Task, a simple logical problem examined by empirical psychologists.

The ability of a species to infer a logical proposition of the type $P \rightarrow Q$, $(P \text{ implies } Q)$ or $(\text{if } P \text{ then } Q)$, should afford an adaptive advantage. It should allow it to select better food, better alliances, avoid danger, and hunt better than by employing an unreasoning or unconscious facility alone. Indeed, such a relationship is at the root of an ability to reason action-based conclusion from facts, to analyse decisions, which I have suggested is the embracing adaptive problem to which Reason contributes. Reason cannot be adaptive without the ability to discern $P \rightarrow Q$ also being adaptive. It is the key building block of connected thought. We might say that "Reason" $\rightarrow$ "Facility for $P \rightarrow Q$".

Why is it then that we appear to be intuitively poor at the Wason Selection Task which explores this implication relationship? The task designed by PC Wason, together with subsequent research, assessing the effect of content using the same problem, is described in Griggs (1983). In the test, subjects are presented with cards representing 4 logical conditions $P, \widetilde{P}(= \text{not } P), Q, \widetilde{Q}(= \text{not } Q)$ on their exposed sides. On the reverse of each of P and $\widetilde{P}$ are corresponding conditions which may be either Q or $\widetilde{Q}$ consistent with a logical rule. Similarly on the reverse of each of the exposed Q or $\widetilde{Q}$ there is P or $\widetilde{P}$ consistent with the same rule. The subject is then asked to turn over two cards which can prove or falsify the proposition $P \rightarrow Q$. Thus he might be shown cards with E, K, 4 and 7 on one side and be asked to prove the rule that if there is an E on one side there is 4 on the other. The correct answer, to turn over P (or E) and $\widetilde{Q}$ (or 7), is rarely chosen by more than 10% of subjects in abstract presentations. Even well-educated subjects perform poorly. Results are improved with content specific presentations but, except in one case mentioned by Griggs (op cit), are still quite poor. Why has not our intuition been tuned by evolution to lead us to the correct answer with greater reliability?

Let us examine a Wason look alike that could have been faced by our ancestors. Let the four conditions be "Stripes", "No Stripes", "Dangerous", "Benign". At first sight this seems a stylised version of a useful problem for our ancestors to be able to solve, (whilst recognising this and other danger discernment problems are facilitated by the selection of traits of fear, caution, learning etc.).

Nevertheless, the intellectual discrimination of second order effects in order to appreciate nuances of danger, or to deal with environmentally less familiar information, was a practical problem faced by our ancestors. As their success in solving it would have influenced reproduction, it was also an adaptive problem. An ancestral logician might therefore usefully ask whether "No Stripes" $\rightarrow$ "Benign". He can investigate "No Stripes" well enough but when he asks his assistant to check "Dangerous" he is faced with a problem: No-one has placed "Dangerous" labels on the animals. The assistant may legitimately ask how he can tell and may also be excused for thinking why does he need to know about "Stripes" and "No Stripes" as it is "Dangerous" or "Benign" that is actually the issue. Conclusions concerning covert qualities need to be inferred from overt cues. It is not an answer to see whether "Sharp Teeth" has "No Stripes" as that calls for another inference. Besides, even the logician's four-year old daughter knows that "Dangerous" is the one with "Stripes" demonstrating an ability to *receive* the implication $P \rightarrow Q$. This ability to receive such information would be adaptively more important than to *perceive* it from first principles. She has every prospect of passing her genes to the next generation but gratuitous curiosity could be fatally damaging to her genetic bequest!

Nevertheless, for some to receive others must perceive and Neolithic Logician has a role to play. But he has a different problem from and epistemology to Classical Logician. The latter puts all facts within propositions on an equal and symmetric footing; there is thus no quality distinction between "Dangerous" and "Stripes" within a classical proposition such as, "All animals with stripes are dangerous". Moreover all classical entities are in principle observable and independent of the propositional structure. For example, in "Some small cars have four doors"; we can see both the cars and the doors. Neolithic Logician has a different task. He wishes to use reason and logic to add value to information by processing a less valuable organisation of information into a more valuable one, to draw a conclusion from a directly observable fact or cue regarding unseen qualities. This requires a *hierarchical* quality of propositional facts, and an asymmetric relationship between "Stripes" and "Dangerous". He becomes interested in $P \to Q$ *only* when Q is part of a more valuable orientation of facts than is P, moreover Q may frequently (perhaps, nearly always) be a covert fact or classification dependent on cues of similar type to P. Thus P's in the ancestral environment would generally be *observable but directly unimportant* qualities such as "Stripes", "Avoids Eye Contact", "Athletic", "Upwind" and Q's such adaptively important *conclusions* concerning *unobservable but relevant* qualities such as "Dangerous", "Untrustworthy", "Good father", "Poisonous", "Easy to catch"; properties which might be retrospectively tested but cannot be observed in advance except by using other cues or proxies of type P. The essence of the ancestral logical problem, the only problem, is to infer exploitable *hidden* dependent facts from not directly useful observable ones. Our ancestors learned what was implied by P by observing over very many occurrences what hidden attributes were subsequently revealed.

Nor would our Neolithic Logician have seen eye to eye with Classical Logician on another issue. He would find the range of available logical options (ALL, SOME, NO) an extremely limiting classification. Frequently, in the classical view, the best that could be concluded would be of the form "Some animals with stripes are dangerous", "Some animals without stripes are dangerous", "Some animals with stripes are benign" etc. He needs categorisations which can be better related to action conclusions. Thus SOME would be more usefully split into MOST and FEW. As the action consequence of MOST would be more similar to that for ALL, and FEW to that for NO, the classical structure accords little practical benefit.

Another issue is the parsimony of the Wason structure. It is unexceptionable that the logically correct solution to the Wason Task provides the maximum conclusive power for the minimum of information. But such economy has only puzzle merit; it is neither adaptive nor practical to do the equivalent of turning only two cards in the real life situation. Indeed, the natural selection process is the antithesis of frugality. Evolution seeks to solve adaptive problems but its criterion is not information efficiency. Our ancestors would have solved the practical problem by the equivalent of turning over all the cards, with a good few thousand others with varying labels such as "Spots", "Difficult to Catch" etc. on many hundreds of occasions, together with absorbing the reports of the experiences of others. This would have been integrated into a sophisticated classification with implication mappings of a far more complex type than $P \rightarrow Q$. Skill in doing this would have been selected. This leads to the intriguing possibility of intuitive skill in regression analysis. But there seems to be no mechanism for the selection of an intuitive classical logic ability.

Wason subjects often turn the P card as their first choice (we can say that this is their intuitive choice) this is consistent with this model. If you want to know what P implies look. The Q card is apparently a frequent second choice. This too is an action that can be understood in terms of adaptive decision needs before the dawn of history. I will remember the characteristics of an animal who attacks me and the body language of a man who lies to me.

The importance of this issue is that we indeed "ought" to assess our modern practical problems through a rationality based on classical logic. We are tempted to believe that the same mind that comes to this view will co-operate in its execution. But our actual cognition has been tutored in a different training ground.

3.10 EP, Risk and Uncertainty

Although closely bound up with Number (which I discuss in the next Section), I consider concepts of risk and uncertainty first as these appear to depend on more fundamental considerations of the operation of the conscious mind.

To a large extent our ancestors' response to risk would have been programmed through the fear and apprehension mechanisms. Some modern psychological maladies might thus owe their selective origins to avoidance of dangerous situations (eg vertigo, agoraphobia etc.) Nevertheless, there could have been a selective advantage if they were to have acquired an intuitive appreciation of the magnitude of risk. To what extent could this have come about and what characteristics could we expect such a trait to have.

Let us consider what would be the attribute of a person without appreciation of uncertainty. Such a person would respond to events with some expectation of the future but blind to the possibility of more than one outcome. This deterministic outlook would require an in-built forecasting system and we could expect the person to have foresight selectively programmed to be optimally pessimistic or optimistic, a balancing act which would be performed by evolution to provide the mix most successful at reproducing itself. However, life, although possessing the right balance of optimism/pessimism, would be continuously full of surprises. The perpetual capacity to be always completely surprised would require that such a person would not have the capacity to learn that things do not turn out as expected; and we cannot consciously learn at all without the ability to assess a deviation from expectation, by which we can store some modified response for the future. Our subject would remain surprised for just an instant because he is unlikely to recognise that there was more than one possible thread from the past if there is none into the future. Our deterministic man, not recognising the possibility of alternative futures, would take actions independently of the possibilities of the future and in doing so he would have miss reproductively important opportunities to which he could have applied his trait of reason. He would not always fail but he would fail more often than the person with the trait, which having an adaptive advantage would eventually dominate the gene pool. A *sense of alternative* and, in consequence, the concept of an uncertain future, and decision making in response to uncertainty not only has to be adaptive but has a more fundamental status. It is a concomitant of consciousness, reasoning, and thoughtful learning.

We can argue similarly that a capacity for appreciating *comparative* possibilities of future scenarios would be built in. Our imaginary intelligent ancestors could hardly manage life where the full panoply of futures was accessible but they were denied appreciation of some form of assessment of the relative likelihoods of those futures, that is to say, they lived in conditions of Strict Uncertainty. They would be little better placed to survive than their deterministic cousins. Naturally, Strict Uncertainty Man would develop an intuitive grasp of the type of modern heuristics suggested dealing with these problems, such as Minimax, but some recognition of likelihood would give the person who had it a decision making advantage on adaptive problems. But we must be cautious in attributing facility which is more than sufficient for the task. I suggest it is reasonable to attribute him with the following intuitive adaptations:

- the ability to recognise alternative futures

- the ability to judge or assign broad degree of possibility to futures or events (eg impossible, quite possible, likely)

- the ability to assign broad comparative likelihood of one future, relative to another (eg more likely than, most likely, similarly likely)

- the ability to assign that an event is more, less, or similarly likely to occur than not to occur (eg it is more likely to rain tomorrow than not to rain).

The fourth of these is simply a special case of the third but it is worth special mention as it provides a mechanism for graduation of important events which might be described as moderately frequent. It also provides a basis for an intuitive appreciation of a single point in a probability scale, 0.5.

Associated with the idea of recognition of alternative futures is that of action-influenced alternative futures. If one accepts that the adaptive function of reason is decision-making, or the selection of choices, it is a *sine qua non* that one must be able to anticipate alternative consequences in order to effect a choice. Thus, again, reason could not be selected unless the notion of an action dependent future was not also selected. I suggest we can therefore also ascribe to our ancestors and ourselves further additional intuitive competencies:

- the ability to recognise that a course of action might have a different relative prospect of success from an alternative to meet the intended end (eg that one is more dangerous or likely to be successful than another)

- the ability to assign a degree of possibility to particular outcomes of action choices (eg, impossible, quite possible, likely)

- the ability to assign comparative likelihoods to particular outcomes of action choices (eg more likely than, most likely, similarly likely) and the consequent ability to assign *ordinal* likelihood of outcomes

This is not at all to deny the possibility of leaps which short-cut explicit prior visualisation of the future in order to effect a decision. Experience or hard-wired processing of present information may allow some decisions to be made without the intermediary of explicit anticipation of consequences, but the ability to short-cut requires that experience be gained through some retrospective review process dependent on similar mechanisms. Thus a person may, for example, uncritically and unconsciously assume that the best prospects of success at fishing to be after it has rained in the evening.

We are now faced with the issue of whether cardinal risk, probability, is an adaptation. I briefly mention that there must be some doubt that Chance, as we understand it, would be conceptualised. The historical and anthropological evidence suggests that people have a tendency to invoke an unseen agent, either superstitious or spiritual, to explain what might otherwise considered to be random events. They assume that they in turn can influence these events by ritual, talisman, or communication. It seems unlikely that this notion of personal influence can go hand in hand with an intuitive concept of chance. The *adaptive* value of superstition and spirituality is outside the scope of this thesis. However, in any case, I attempt in the succeeding paragraphs to set aside unseen intervention and to review the possibility of intuitive probability without this factor. Naturally, some issues of numeracy (which I have still to consider) impinge here.

French (1986, pp 210-254) discusses the three philosophic outlooks of probability from a decision analysis viewpoint. These are the Classical or Laplacian view of probability, Frequentist Probability, and Subjective or Personal Probability, of which Bayesian statistics is one operationalisation (though the primary issue is that Subjective Probability relates to viewtaking about a future single event assembled from general experience rather than repeated incidence of similar events). The use of the terms "frequency", "chance", "odds" and "probability" are legitimate within all these frameworks. I use the term Frequentist, as French does, to describe a philosophy of probability which derives from observed incidence, not as Cosmides and Tooby (1996) who use the term when describing the encoding of frequency with extra information by including both event instances and opportunities, as in 3 out of 30.

I start by looking at the nature of uncertainty within the environment of evolutionary adaptation as a clue to which philosophic frame our ancestors might have experienced. We can rule out a classical perspective. It is improbable that our ancestors had games of chance, which used artefacts generating events of exactly equal likelihood (as with coins or cards), until comparatively recent millennia, and if they had them they would not have formed the basis of an adaptation.

However they would have dealt with some environmental data to which a frequentist view could be attached. For example, in principle they could have kept data on the frequency of rainy days (though a question is posed here which I will return to). Less describable in frequentist terms would be, for example, the group's success at hunting. Indescribable would be the risk of certain dangerous activities for which there would be an awareness of risk but for which actual experience of the risk was minimal or non-existent. A Bayesian situation faced could be one in which there was well established experience of a phenomenon in one area (eg hunting success) but no experience of it in another; the group would need to progressively modify a prior view of prospects as experience was gained.

However, whilst frequentist information was available in the environment, could a frequentist approach have been applied to it. The answer here is likely to be no. A frequentist approach requires the maintenance of statistics. The deferred issue from earlier is how? A detailed memory of such events can be ruled out on capacity grounds and writing, or even the marking of incidents on tree trunks, are artefactual applications of adaptations not adaptations themselves. There are further objections. Whilst frequentist data was available, it is difficult to imagine that frequentist data relating to the consequences of decisions could have been. Even in the modern world it is only under special conditions that we can acquire frequentist data relating to the consequences of decisions (in a laboratory experiment perhaps). Modern decisions are unrepeated events where the consequences of the unimplemented alternative are not known or they are repeated but in different environmental circumstances. So also in the ancestral environment. Moreover, frequentist information would have been of a passive type. One might know the frequency of rainy days, but so what? How could this govern action? To know that there is a higher chance of rain *tomorrow* could, but this is a non-frequentist question. Also to know an edible fungus is available at the time of year when the frequency of rainy days is increasing is useful but clumsy. It is easier to look for them during damp days in Autumn. I suggest we can rule out an intuitive frequentist philosophic concept on practical grounds alone without considering adaptation.

We are I suggest left with the concept of quantified subjective or personal probability, albeit that it is likely to operate by the retention and processing of sample size as well as relative incidence data, rather than by retaining only the ratio. (Cosmides and Tooby (op cit) report greater ease of understanding with problem framing expressed in this form). What are the prospects that we have been honed by evolution to accurately (or sufficiently accurately) assess and process risk magnitude of a single event possibility in a way that can be related to modern concepts of measured probability, at least crudely, based on Kolmogorov's laws (French, 1986, p213)? In order to establish this we must at least establish that such a skill would solve an adaptive problem and do so better than skill in the assessment of comparative likelihood married with aptitude in broad qualitative assessment of the possibility of particular futures and outcomes, for which I have argued a case exists. The first issue is could an unbiased estimator exist?

We can readily argue that an unbiased estimator might serve the personal *practical* interests of a organism. A precise estimate might enable it to better calculate its self interest. Let us allow that an ancestor is able to assess the following matrix:

Table 1 Probability of catching animal in hunt

	If Rain	If no Rain
Option A	0.7	0.6
Option B	0.8	0.5

He might conclude that if the chance of rain is 60% it is better to take option B, and if there is a 40% chance, option A. This is one of the simplest pure probability (as distinct from utility) problems that we could conceive which stylises an ancestral choice. It is also a well differentiated problem in the magnitudes of the data used; yet there is computationally just a 2% swing in each case. Would this be sufficient to secure a material adaptive advantage over a heuristic that concludes that when it is very likely to rain do B, if it is very likely to be dry do A? If you can't tell, choose the option that takes you by the nut tree, which is beneficial anyway. Even in such a manageable case one can see that the fine structure of probability may be less important than ancillary considerations of the unbounded problem. Knowledge of the fine structure does not therefore lead one to an inevitable conclusion about the best solution and even were it to for practical problems, in evolutionary terms the adaptive advantage could well be swamped by system noise.

However, there is a more critical objection. Objectivity of the organism about its own best interest is not necessarily adaptive. Clearly there is some coincidence. If we did not have a survival instinct, we would soon place ourselves in sufficient danger to severely compromise the ability of our genes to replicate the trait. But what we want does not always serves the best interest of reproduction of our genes. There are circumstances where the correct assessment of risk was inimical to the interests of the procreation of our ancestors' genes. We can argue that we and they would be anxious to avoid events which risk death. Accordingly, an objective personal view could lead us to take less risk, say, in the pursuit of food, and this might indeed maximise our longevity at the price of temporary hunger. But the life prospects of our infant children (transmitters of our genes), precarious at the best of times, could be additionally and far more severely hazarded by our failure. Moreover, our mates, who propagate our genes, will have a completely different agenda related to their own propagation and could be largely indifferent to the risks her mate takes. He seeks to avoid his risk, she seeks to increase his. This would generate the optimal admixture of bravado and caution for the propagation of the species but it is unrelated to facility for unbiased estimates of cardinal probability. Indeed, it is most probably adaptive for a provider to adopt strategies injurious to his personal prospects of survival. We seem to be programmed for this problem with instincts that deal with it. But our programmed desperation to save the life of our starving child will not be realistically assisted by the balancing the finer issues of probability. Indeed, we know our emotional systems kick reason out of the window in such crises.

Bayes Law, is often suggested as a basis for the modification of subjective probability but its one line formula belies a demanding computational problem. It requires a *distribution* of estimated prior probabilities of the observed event. It is a many parameter model requiring mental Integration, and on capacity grounds is suspect. In any case, it can only be applied to relatively frequent events for which

an off-the-shelf instinctive, learned, or cultural response would be available. More relevant would be the assessment of events that we might classify as infrequently observed but not yet familiar. However, it is likely that an adaptive behaviour for such circumstances would be to apply a simple heuristic which assesses risk conservatively. Assume danger until safety is demonstrated. With a robust simple heuristic, I can see no adaptive benefit in cardinal probability.

Before leaving this question it is worth commenting on the impact of technical requirements of a scale of probability. Modern man with some numeracy schooling and knowledge of the properties of probability (eg that probabilities of disjoint events must sum to unity) may have little trouble in generating a scale of likelihood *resembling* probability. The innate skills of assessing "degree of possibility" of an event and the "comparative likelihood" of one event relative to another event or of the event relative to its non-occurrence is sufficient to generate a cardinal-like scale. Indeed this, though biased, might serve adequately for many modern decision making purposes. The additional requirement for a proper scale of probability is that *intervals* of likelihood must also be explicitly or implicitly expressed (eg of disjoint events; A and B are equally likely, C has the same likelihood as either A or B occurring). As events, in contrast to roulette wheels, don't label themselves neatly, there is no basis for building a mental standard. Accordingly bias and inconsistency would seem largely unavoidable.

This conclusion would seem to be consonant with many empirically-based findings and views of others. Keen (1977) suggests, "Man is not a good statistician. He has a poor sense of variance and in clinical assessment or actuarial judgments, his performance is virtually always inferior to any simple linear regression model-assuming of course there is enough historical data available to build the model. He relies on heuristics which are highly economical and usually effective but which lead to systematic and predictable errors, such as insensitivity to sample size and prior probabilities and to the regression fallacy. He makes frequent errors of logic, especially in dealing with negative examples, but is generally able to rescue himself from his mistakes because of the self-correcting feedback of language. In even simple gambles, he tends to be biased by the payoff, overestimating probabilities if the set can result only in a break-even or win." Fox and Tversky consider that there is ample evidence that people's subjective estimates are inconsistent with the laws of chance and that estimates of the probability of the union of disjoint events is less than the sum of their estimated individual probabilities. Moreover, we should not be surprised given the EP scenario developed here to find the difficulties and biases of probability estimation described in the empirical literature (eg Tversky and

Kahneman, 1974; Bazerman, 1986, pp 14-41; Tversky and Kahneman, 1971; Estes, 1976; Slovic, Fischoff and Lichenstein, 1982). Hasher and Zacks (1984), Howell (1973) and Jones and Jonides (1992) on the other hand, report creditable accuracy in encoding frequency in formal word and non-verbal recall tests, though one should note that excellent correlation of reported frequency with actual can still be obtained even if there is bias in estimates as seems to be the case in some of the graphed results. Nor should we necessarily be surprised that the expression or framing of probabilistic problems which are computationally equivalent may sometimes influence the conclusions reached (eg Kahneman and Tversky, 1984). However, Huber and Huber (1987), in addition to reporting the work of others suggesting that both adults and child make preferential use of comparative probability, tested and confirmed the application of 6 formal principles of comparative probability that they suggest, and found that even young children apply this concept remarkably well. This also seems consonant with the notions presented here. Simon (eg 1965, 1979) also challenged the need for assumptions of heroic calculation of the type used in SEU.

It is for these reasons that I only assume that comparative likelihood (ordinal probability) and broad generalisations of degree of likelihood are intuitive amongst probabilistic concepts and that ideas such as probabilistic arithmetic and SEU are unnatural although competence in objective cardinal probability for relatively frequent experienced events can be schooled. This is the basis for my suggestions in Chapter 2 concerning the operational definition of rationality. We must accommodate as rational, views which are merely consistent with cognition of comparative likelihood whilst allowing conclusions based on a more liberal view-provided that the latter is not gainsaid by the first.

3.11 Ancestral number, quantity and calculation

If, notwithstanding, we temporarily allow that we have some cognition of probability what of other properties for which we take measurement for granted on which we depend in formal decision analysis. To what extent is measurement and calculation innate?

It is manifest that most animal species survive without the ability to compute or to count (beyond some sort of ability for some to determine, say, that individuals of their litter are missing). Number and Quantity, however, permeate the life of modern man with measurement of time, money, length, and weight being preoccupations over very many centuries. Number also underlies much of decision analysis, not only that which is normative (for which we can allow any effective aid) but descriptive decision modelling as well. This would be unexceptionable if it is a normative commendation or if it adequately encapsulates the consequences of another process. However, if it presumes a precise mechanism in which the mind weights a value of a possible decision outcome by a measure of likelihood (however we define it), integrates this measure over all outcome possibilities and compares it over all decisions, we must consider whether we could possess such sophisticated mental facility. Could numeracy be hard-wired within us? Do we compute?

Let us examine the usefulness of in-built numeracy in terms of evolutionary function. Let us consider measurement first. Judgement and the ability to learn from experience would certainly have been necessary in Palaeolithic times with respect to functions which we now seek to measure, for example, in the manufacture of artefacts or the construction of dwellings. Individuals or groups having the engineering or manufacturing judgement sufficient to perform the tasks, could be expected to prosper in relative terms. However, before the era of machine tools, patterns, and the high economic gearing that nowadays accumulates single activity inefficiencies into major living disadvantage, precise dimensional or resource measurement, and that aid to execution precision, calculation, would not have been critical to survival. Design would have been based on experience or learning and such measurement as was necessary might have been performed by direct relation to person or thing for which or from which it was constructed- the cut skin to the body, the house dimensions to the tree. Indeed, we might imagine such metrics as were used would be measured imprecisely and tautologically in terms of the conclusions that one would wish to draw from them or the purpose to which it would be put; "A day's march", A hole big enough for a bear"; or perhaps for identification with only sufficient accuracy to avoid ambiguity; "the rock 10 paces high".

Accordingly it seems unlikely that we would be predisposed by the environment to measurement in the sense or precision that we now understand it, that is as a metric scale concept. Nevertheless we can suppose that certain aspects of number would at least be closely intertwined with an adaptive function, that is man's important and unique facility for pro-active organisation. Our ability to organise, to inter-relate for sophisticated mutually advantageous purposes, must be one of the of the major adaptations which has caused our species to survive. Without it our vulnerability and limited physical abilities would soon confine us to oblivion in the difficult environments which we chose to adopt. Organisation requires an ability to plan, it requires a world which can be partly controlled by proaction, it requires communication and language, it requires the assignment of roles, and it requires decision. Decision itself implies alternative actions, assessed results, and that some outcomes are superior in the light of resources used. But organisation also necessitates at least a crude notion of Economy: that resource should be assigned where it is most valuable.

Hand in hand with notions of Economy go concepts, which we may view as mathematically related. We might include in this relational ideas such as "More than", "Better than", "Younger than", "Enough", "Large", "Best" (though the last would not connote Optimality merely primacy amongst discrete choices)". Organisation requires dexterity with the notion of Classes ("Men", "Women", "Hunting Party", "Family", "My"), and hence Sets. One might expect an intuitive grasp of set unions, "Men and Women", and intersections "Old Men". In certain content specific areas, say, defining relationships within the groups, relational understanding could be complex.

Within this framework there would also be an absolute need for descriptive and instructional concept of number, "5 men are needed in this hunting party". One might expect that in such a situation that facility with counting, as a system of *classification*, would become natural. Based on familiarity alone we could expect *descriptive* arithmetic "We had 5 chickens, we ate 2, we now have 3", and even *deductive* arithmetic based on experience "We picked 6 bags which we agree to share, that will be 2 for each of the three of us". This is not to say that this level of concept of number would be of itself adaptive, but unless our ancestors were at least able to relate to the idea by learning and accept the immutability of the relations, involved, including the notion of conservation of count, they could not have become effectively organised for which the adaptive value is very clear.

However the need would in be confined to numbers in the pragmatically countable range to relatively small integers and simple fractions. Arithmetic would seem likely to be similarly confined with large holes in the multiplication table. Levels of abstraction would be low. Large numbers of units either being compounded into generalisations like "Many" or subject to the adoption of another unit related to the practicalities involved for which there may be no explicit or constant equivalent. (Hallpike (1979, p101) notes that Cape Coast fish traders in relatively modern times memorised tables to deal with variations from standard purchasing quantities which were handed down from mother to daughter rather than learning simple calculations.

We can, I suggest, say that Number in this organisational function is largely Number as language. It is adjectival. The quality of sameness between, say, 5 fingers and 5 eggs would be recognised (though we might be more suspicious that similarity will be recognised between objects and intangible phenomena such as 5 paces and 5 days). We can reasonably accept that a set of 3 eggs and a set of 2 eggs would have been perceived, with adaptive advantage, as being identical to a set of 5 eggs. As this assignment of precise equivalence is not a feature of combinations of non-numerical adjectives (as in man, selfish man, very selfish man; or bobo, small bobo, small-small bobo) this marks the beginnings of distinction from other language. However, the essence of numeracy as calculation, is reasoning in abstract with number used as symbols. The adjectives are detached from the nouns, they are manipulated and the nouns are added back. It is doubtful that there would be sufficient domains of similarity in their numerical structure for content independent reasoning to accord adaptive advantage. It is also difficult to see many problems that would allow the non-schooled functional acquisition of other components of elementary calculation, eg sets of sets, recursion, commutative and associative equivalence etc (though no doubt some would be aware of curious properties which they could pragmatically exploit without adaptive impact).

There is indeed anthropological support for very circumscribed untutored quantitative cognition, despite considerable classificatory dexterity. There are languages where the largest explicit number is 5 (Harris 1993) and indeed the Tauade have only 2 (Hallpike 1979 p61). Hallpike noted that in the latter case lack of verbal numbers interfered with the practical issue of ceremonial distributions of pork, but in any case this does suggest that in primitive societies and the similar societies of our ancestors the need for number and calculation would not have been pressing. It is moreover difficult to perceive natural numeracy without a natural understanding of the concept of Conservation of Quantity (ie that length, weight, area or volume are unaffected by the shapes into which they are transformed), yet studies amongst unschooled adults in New Guinea and Australia suggest that this is not intuitive and a high proportion do not conserve quantity (Hallpike p60), though Hallpike also refers to other work suggesting that conservation can be rapidly learnt when members of traditional societies are given instruction. Later Hallpike (p257) comments "We seldom find, as far as I know, that primitives use terms for dimensions such as "weight", "length", "distance", and so on, as opposed to heavy/light, long/short, near/far, etc, ..." He also remarks that primitive modes of thought tend to view concepts like lightness and heaviness as opposing absolute properties rather than as points on a scale and that there is unclear distinction between such properties as weight and size, for example. He also (p352) mentions the 1940 research of Evans-Pritchard on the Nuer who although having a system of lunar units with which they could describe the occurrence of an event "it is with great difficulty that they reckon the *relationship* between events in abstract numerical symbols". (My emphasis). (However, I have difficulty with one implication of Hallpike's observations concerning Conservation. It may be that volume as a general measure would not be inter-related between one subject and another, for example a volume of liquid to the volume of a box or a house. However, it is difficult to conceive that as a specialist liquid measure (notably of water) conservation would not be understood. Its conservation in being moved from container to container of different shape would be too familiar).

We can, I believe, attribute intuitive ordinal and relational skill to our ancestors; reason would seem difficult without it. We can also assume content explicit classificatory dexterity as these accord clear adaptive advantages. It seems doubtful that one can go further. However, If an inbuilt general numerical facility (involving innate cardinal appreciation and calculational dexterity) seems unlikely, could we have domain specific capabilities in the decision evaluation area, perhaps to weight value (a concept I will shortly come to) by likelihood, or otherwise, in complex ways? I think not. I have argued that objective cardinal probability assessment is in any case unlikely to be intuitive and I will discuss issues of Value which are similarly inhibiting.

Risk problems, I have already suggested, would have been adequately dealt with (from an adaptive perspective) by simple heuristics and non-deliberative mechanisms. Similar arguments could be advanced for other complex computations of value. Simple non-calculating search algorithms can be shown to be effective mechanisms for finding nutrition (eg Simon, 1957). Simon concluded, "The principal positive implication of the model is that we should be skeptical in postulating for humans, or other organisms, elaborate mechanisms for choosing among diverse needs". Even for modern day judgement and decision problems strong arguments have been presented that sophisticated arithmetic actually does not matter and straightforward heuristics can cope adequately in multiple factor problems (eg Wainer, 1976; Dawes, 1979; Edwards and Barron, 1994; Barron and Barrett, 1996; Gigerenzer and Todd, 1999). I only disagree to the extent that the long chain reason facility provided by decision theory and the computer, today provides an opportunity to do better and the gearing of modern economics makes small gains normatively important in commercial and administrative organisations.

Even so is there still a possibility we possess a sophisticated unconscious complex calculating system in the same way that we possess vision- after all we could not conceive the idea of vision if we did not possess it. We could allow the possibility for the primeval problems we still face today, but one which we can also apply to modern problems presents difficulty. Such problems must have a conscious interface to be used for conscious problems. This seems implausible in the same way for example that we cannot hi-jack vision to image temperature profiles. It is likely we use relatively simple rules. Evolution fine-tunes the simple rules but would not generally extend their structural complexity excessively and will not do so at all if the computational payload exceeds the benefits. However, I will qualify this view to some extent shortly when I discuss what I will call holistic integration.

As with probability I conclude for working purposes that comparative and ordinal concepts are innate, together with count in the practical range. Adjectives of broad magnitude are innate, and, perhaps, small integer arithmetic. Concepts of cardinal measurement, scale and quantity other than count, and calculation are concepts which need to be learnt. Decision concepts which may depend on them, need to be carefully qualified.

3.12 Concepts of value and compensation, and the availability of standards.

Concepts of a value and preference, what causes us to choose, would appear to be quite complex operations of the mind depending willy-nilly not just on the intellectually reasoned contributions which in a basic neurophysiological model of the brain such as in Sagan (1977) might be controlled by the neo-cortex, but also on our emotions, and on instincts for aggression and territoriality, controlled by the more distantly evolved features of the Limbic system and the reptilian R-complex. Damasio (1994) further suggests that the emotions and "somatic markers" are essentially entrenched in actual human as decision making mechanisms themselves (not just as value influences) and are necessary to constrain the otherwise open-ended nature of all decisions problems. In any view, value influences on the decisions we take are multi-functional activities of the whole mind.

In our single money-scale modern economic view of value we have entrenched value into our civilisations for many hundreds of years as a hard, conscious and measurable concept. They are familiar and incredibly useful to the solution of problems of the organised world, but are they innate? We again ask the extent to how reasoned and conscious ideas of value, particularly as a metric, would have been necessary amongst our ancestors to secure adaptive advantage in their environment, the necessary and sufficient pre-condition.

The concept of value is helpful as a descriptive mechanic as soon as one allows at least binary choice. Faced with two possibilities I must choose one. "Which one? The one I prefer. Which is that? The most valuable". Of course such an argument would, unmodified, also apply to animals but becomes tautological. An animal chooses what it chooses but value is *perceived* by humans independently of the action. Value is the *conscious* property which causes us to prefer one outcome from another prior to the commitment of action designed to achieve the preferred outcome. Once we allow conscious choice, value comes in by the same door. Choice can only be adaptive if a sense of value is associated with it. Accordingly we must suppose that at least the ability to make comparative value selections between binary choices is innate, notwithstanding any wider capability.

How much further do we expect our unschooled ancestors to have been able to go. Clearly if he can make a value distinction between 1 Elk and 1 Buffalo he can distinguish between 1 Elk and 2 Elk, and between 2 Elk and 3 Elk. A person unable to do so or who believed that 2 Elks were less valuable than 1 would be less likely to pass on their genes. Do we have then the basis of an intuitive cardinal scale of value? I suggest this ultimately becomes the basis for money, but only once a need for money, to store value on a standard measure however temporarily, is established as a practical need. But this is a necessity of only the last few millennia. 4 Elk may be more valuable than 3 Elk but do we actually *need* 5 Elk. This would be true of spears, sons and all else within the countable range. On these grounds an inbuilt concept of an extendable cardinal scale of value involving intervals of equal worth seems unlikely, despite facility with comparative value.

Trade also evidences some type of concept of value and this could be deeply entrenched. Neolithic flint tools have been found many hundreds of miles from their place of manufacture. Chimpanzees display "reciprocal altruism" (Goodall, 1986, p380) and have been observed giving away "low value" food items to avoid being pestered by begging members of the group: surely rudiments which can build towards a concept of value? However, as Harris (1993, p255) observes, "Modern-day price markets and buying and selling are not universal traits. The idea that money can buy anything (or almost anything) has been alien to most of the human beings who have ever lived. Two other modes of exchange- reciprocity and redistribution - once played a more economic role than price markets." Some primitive trading systems which bear some resemblance to barter, offer packages of mixed goods to the other "party", who in turn offer a tranche of other goods. The whole exchange takes place or none at all. This is a trade system largely dependent on receiving goods of "value" for surpluses of small value from a party who sees the process in exactly reciprocal way, *cardinal* value is not in the minds of either party. It involves only binary comparative assessment of the type I argue is adaptive- "Do I value what I am offered more than what I have?." Trade does not imply a *scale* of value.

If we are nevertheless tempted to argue that the existence of money is a modern manifestation of an innate internalised scale sitting within the mind, one should be aware that single scale money is not a universal given in all money-using cultures.

Specific forms of money may be used for specific purposes and not be subject to an "Exchange Rate" between them. Without such reduction we still have problems. Moreover, we know that were an internal scale imagined, that it would be no more effective operationally than that scale raised to any arbitrary power which would be a strategically equivalent scale leading to the same operational choices. As discussed in Chapter 2, without the existence of two exchanges of designated equal value a value interval cannot be fixed. What phenomenon would make a particular mental scale better than another?".

I suggest only the existence of a Standard. Scales depend on standards, things we can look at (literally until comparatively recently for the Imperial system). From one yard, or gallon we can by successive binary comparison find an equal measure from which we can then define two units and we can multiply or subdivide our scales indefinitely using the laws of physics or geometry. The concept of money, once similarly dependent on the use of quantities of precious metals, provides modern man with a conceptually identical scale for value. Whilst older "scales", say based on the length of a man's pace, had a descriptive value and helped to organise an understanding of the world they were not available as precise concepts without precisely engineered artefacts. We have only limited ability to retain in the mind things for which artefacts do exist. It is far more difficult to perceive that one could retain in the mind as an invariant something that does not exist in physical substance, such as a scale of cardinal value. The notion of probability is alternatively used to standardise measures of value through expected Utility but use of this mechanism itself depends on an internal concept of well defined cardinal probability which I have already questioned. In the modern era we may be able to benchmark our internal value through external standard of common use- money value, but it is not inherent.

Can we then go beyond comparative testing of value of binary choices? The situations tested by our ancestors would often be binary alternatives but they would certainly involve measures of degree. Thus choices such as 1 Thin Pig or 1 Fat Elk or between 1 Fat Elk and 2 Thin Elks and, importantly, notwithstanding a lack of conception of cardinal measurement, choices somewhere between Fat and Thin would be frequent in the environment. A trade-off between one quality and another, a mechanism for comprehending *compensation*, would be called-for. We can imagine there being sufficient two variable trade-off choices generally to accord adaptive advantage to those able to discriminate; whether food for effort, illicit sex for risk, current favour for future return.

If we go beyond 2 variable qualities, the situation becomes more difficult. Larichev (1992) considers that we cannot reliably express preferences in situation involving the trade-offs of more than 2 variables, other variables being held constant. Looking introspectively I feel unsure when examining more complex situations. I illustrate the problem in what I call the "child's sweet dilemma". If a child is offered the opportunity to trade 1 Humbug for 1 Jelly Baby he or she is likely to exhibit a quick preference. If the problem becomes 2 Humbugs for 3 Jelly Babies I suggest that the problem is not inherently more difficult and only requires extra consideration if we hit a combination around his or her indifference level. However, if the offer is 2 Jelly Babies and 1 Chocolate Bar for 3 Humbugs and 2 Bubble Gum the decision could present a dilemma and lead to subsequent tearful regret with greater frequency than the simple 2 variable choice.

3.12.1 Innate Holism

Yet despite our difficulty with explicit value trade-offs beyond two factors, our poor intuitive grasp of multiple objectives or long-term strategy, and our inability to deal with cardinal quantity or to compute, our ancestors would very commonly face adaptive problems that had the characteristic of being many factored and frequently urgent. Palaeolithic decision making analysis based *only* on a potentially time-consuming factor by factor trade-off does not seem a convincing approach for many of the decisions faced, given the combinatorial computation issues that would quickly emerge. In large part, at least, our ancestors would have secured an adaptive advantage in assessing the relative merits of alternative courses by quick, unarticulated, subjective leaps to a comprehensive evaluation in a process which we might call holistic integration.

Such a process still involves reflective comparison although it may be intuitively based. We can observe in our modern lives that we confidently take many important and complex decisions without excessive analysis or soul-searching; the mate or friends we choose, the career or job we select, the pastimes we participate in, the clothes we wear, are problems of great and unprogrammable complexity, even if not all are of consequence. Complex decision making is something that we believe is inherently simple. It is usually the OR Analyst who believes that a problem cannot be efficiently solved by subjective methods; not the decision making manager. The latter often confidently believes in his abilities to do so effectively and needs convincing that formal analysis can provide instructive insight. This is not surprising as his ancestors have been decision making for hundreds and thousands of years and doing it sufficiently successfully for their genes to be handed on to him. (One might add parenthetically that decision confidence is the evolutionary antidote for the counter-adaptive trait of prevarication. But the same confidence in the same intuitive decisions in our modern world will frequently be misplaced).

The nature of mechanism to achieve this is even more speculative. However, we can rule out one possibility and favour others. We are certainly pre-programmed to evaluate certain situations in certain ways. For example, avoidance of venomous snakes. There is evidence that humans are predisposed to acquire a fear of snakes (Hinde, 1995). In an environment of venomous snakes, a Snake→ Fear→ Caution production would have reproductive advantages and this may be extended to embrace "Risk of Snake". Could we not have extended facility of this type? The difficulty is that it encodes a very limited, domain specific, decision valuation. Whilst it is possible that over millennia that a few dozen environment-general but specific situations might be encoded, the mechanism cannot encode the circumstances of particular individuals. Moreover, even were it to be possible to do so, the quantity of information that would need to be hard-wired in inherited structures would quickly become impossibly large. A soft-programmed structure, based on the environment that the individual lives in (that tree, that ford, that potential ally) and that can cope with normal changes in it, may remain formidable but is potentially orders of magnitude less capacity-demanding.

One can envisage a holistic system based on recognition and learning. In effect an experience based system, though embracing not just information in the hands of the individual but vicarious experience, cultural expectation, and the injunctions of the powerful. However, unmet circumstances, and the variation from the norm would also need to be accommodated. Saarilouma (1995) debates the process by which chess players "apperceive" chess problems and experts exhibit superior skill.

A major factor he identifies (p62) is that experts have "seen thousands of positions and this has created in their long-term memory a large store of chess specific information" and "If skilled subjects select their cues properly" they can access this though they do not have a capacity to "generate long move sequences" (p91). Experience maps the area in broad terms and a mechanism for relatively limited deviation enables navigation into uncharted territory. So possibly with a more general holistic appreciation.

A holistic integration model is not necessarily incompatible with a binary compensation model, although at first sight they appear to be extreme alternatives. Indeed, the latter may be necessary to the operation of the former. How would an ancestral or contemporary decision maker judge factors deviating from his or her experience? How does he or she audit the success of the multiple variable outcome of the quickly made decision before consigning the experience to long-term memory? Hindsight processing can, after all, take place at leisure. A trade-off of factors is likely for both, and binary trade-offs may be sufficient for the task without capacity for innate multi-way competence. Indeed, this may be computationally efficient. After all the Simplex Method of Linear Programming effectively reduces to a succession of binary trade-offs.

As a footnote I should note that what I have termed holistic integration is similar to what Khatri and Ng (2000) describe as *intuitive synthesis.* Their work illustrates the effectiveness of intuition, which they describe as "... a complex phenomenon that draws from a store of knowledge in our subconscious and is rooted in past experience". Drawing on Prietula and Simon (1989) they suggest that "It is a sophisticated form of reasoning based on 'chunking' that an expert hones over years of job-specific experience". In the work here I have envisaged an unconscious leap to decision as being effectively a processing of *value.* However, an interpretation of the Khatri and Ng view is that the intuitive decision is made within a larger domain of *facts.*

Whilst its mechanism should remain open, for the purpose of this thesis I recognise the possibility of a special, and exploitable human skill, in overall assessment of the complicated alternative options in domains where the subject has considerable experience. However, I do not assume that competence would be of the same precision as binary trade-offs.

3.12.2 Stability of the value fame

An adaptive advantage accrues from being "good at" comparative value estimation and a facility for trade off. However, does an advantage accrue from stability in the values that we retain within our mind? The suggestion that we are better at handling multiple objectives serially rather than simultaneously already casts doubt on the matter as this is equivalent to altering the value of the objective or objectives we choose (or our biochemistry chooses) to bring to the fore at any one time. Seriality seems to depend on value lability.

We may see why stability of value seems useful today. In a complex modern world, slow-changing policy is advantageous as our environment is principally endogenously determined (in a species sense) by a myriad of human policies which are in turn determined by community values. Stability of policy (eg in our schools or railways) follows stability of value. This causes us generally to favour slow movement in value in our institutions and economic systems, despite the process of politics causing intensive and continuous review of those values. But this does not make us innately capable of maintaining stable values as individual people.

In the era of evolutionary adaptation, the environment was principally exogenously determined. Accordingly, stability of values, the stability of binary compensation relationships, would have a less clear link to stability of policy. Indeed, learned behaviour would seem more important than assessment from first principles from established values, in these inherently slow changing circumstances. Having evaluated and implemented a particular policy in a particular situation, the learned outcome becomes an important factor in the determination of future policy. Moreover, It is arguable that better learning goes hand in hand not with stable values but with labile values. We can anyway confidently assert the converse, fixed values imply no learning.

Moreover, fixed and precisely determined values is a concomitant of finely discriminated decisions. If achievable, this may have merit today but not necessarily in the ancestral environment. Selection is unable to test for decision optimality in an environment where no two situations are identical, so precision itself is unimportant over quite broad limits. We might go further, the fast changing nature of (individual) ancestors lives required that value frames not only need not be stable but should not be. Fixed values also imply unrelenting obstinacy. The man who always distrusts the man who will not look him in the eye may overlook that this young potential ally may do the same thing for different reasons, or who always values a fat elk as more than a thin one may be disadvantaged far from home. Value in the ancestral environment would be frequently determined afresh from almost entirely internal considerations.

This is not to deny that there would have been slow changing constraints on the formation of value: The family must be fed, honey is always a treat, and a physically fit husband is a better bet than an old frail one. Moreover experience would ameliorate haphazard valuation, "I can see from my sister's problems that a fine physique does not compensate for laziness in a provider." That a good provider is valuable may be invariant in broad terms, but its comparative value cannot be. Quite apart from the additional difficulties caused by effecting a physical mechanism for a mental Standard, there is no functional reason to expect that we should have an innate value and preference stability.

Value issues and their relationship with decision analysis were discussed in Chapter 2 and this included a discussion of labile preference and value, and the evidence for it. The empirical evidence for the lability of *expressions* of preference and for sensitivity to problem framing would seem well founded. I attempt to make a stronger point here suggesting that it is preference itself not just expression that is labile. This seems consistent with the findings related to Judgement and the ability of the human mind to discriminate sensory phenomena which I also discussed in Chapter 2.

In summary whilst we can properly assume an innate recognition of comparative value and some facility to compensate the value of one attribute for another, we have no reason to suppose an ability to attribute cardinal value nor to have sustained stable values or preferences. We should be cautious about depending on these ideas in normative decision analysis.

3.13 Summary of cognitive assumptions in decision making

Based on the discussion above, supplemented by observations made in Chapter 2, I suggest the working assumptions of cognitive competence for the purposes of this thesis, which are summarised in the following pages. I believe these are consistent with the main empirical research and the other subjective triangulators I referred to; introspective assessment of my own capabilities and pragmatically observed behaviour of managers and people making decisions in everyday life.

An overall encapsulation is that humans are competent comparators but are less capable in operations requiring assessment of degree, and we have not been moulded by the evolutionary environment to be adept with the concept of simultaneous multiple objective or *conscious* trade-off of multiple attributes. However, we may have skill at undeliberative holistic assessment.

In the following tables, the third column represents a view on the ease of the concept described in column 2, and the fourth the basis of that view in terms of adaptive function, supplemented by an indication of empirical support, if any, usually attributed more fully in the text.

Ngl= Negligible, SAP= Solves adaptive problem, UAE= Unnecessary in adaptive environment, FAE= Familiar in adaptive environment (though adaptive value may be unclear), FMP= Familiar modern problem or approach, CAC= Can acquire competence, PCA= Possibly counter-adaptive K&T = eg Kahneman and Tversky amongst others,

DXY= derivative or consequence of item Y in list X.

Objectives and Goals (O)

1	Appreciation of relevant factors	Intuitive	SAP
2	Formulating single "pass-fail" goals	Intuitive	SAP
3	Formulating single "more is better" objectives	Adept	DV1
4	Formulating multiple simultaneous "pass-fail" goals	Adept	SAP Simon
5	Formulating multiple simultaneous "more is better" objectives	Unnatural	UAE, FMP
6	Prioritising multiple "pass-fail" goals	Intuitive	SAP
7	Condensing multiple objectives using cardinal arithmetic weights	Ngl facility	UAE
8	Setting satisficing standards	CAC	DO4
9	Setting lexicographic thresholds	CAC	DO8
10	Ranking weights of multiple "more is better" objectives	CAC	DO6 Larichev
11	"Successive Limited Comparison"	Intuitive	SAP, FMP Lindblom
12	Precise specification of objectives	Unnatural	UAE Lindblom
13	Stability of Objectives	Unnatural	UAE
14	Serial switching of objectives	Intuitive	SAP

Number, Quantity, and Measurement (N)

1	Counting in practical range	Intuitive	SAP
2	Concept of comparative magnitude (eg bigger, biggest)	Intuitive	SAP
3	Concept of graduated classification (eg small, large, very large)	Intuitive	SAP
4	Quantity, cardinal magnitude, scales.	Unnatural	UAE, FMP
5	Relationships and association	Intuitive	SAP
6	Objective causality	Unnatural	UAE
7	Basic arithmetic with small integers using learnt relationships	Adept	SAP
8	Geometry	Unnatural	UAE
9	General purpose calculation methods. Concept of square and cube. Mental arithmetic	Ngl facility	UAE
10	General concept of conservation of quantity for cardinal metrics (eg area)	Unnatural	UAE Hallpike
11	Specific conservation of quantity for countable items and volume of liquids	Adept	FAE Notwith-standing Hallpike
12	Capacity to "chain" calculations	Limited to short-term memory	UAE
13	Unconscious complex computation	Unlikely	UAE Saari-louma

Probability and Uncertainty (P)

1	Recognition of unpredictable futures	Intuitive	FAE
2	Recognition of action influencing uncertain outcome	Intuitive	SAP
3	Recognition of distinction between chance and unseen agent	Frequently not distinguished	UAE Hallpike
4	Concept of probability as an enduring property of systems and events	Unnatural	UAE
5	Concept of comparative likelihood (eg more likely, most likely)	Intuitive	SAP Huber &Huber
6	Concept of cardinal probability	Unnatural	UAE
7	Realistic estimation of objective probability	CAC, possibly	UAE DQ1
8	Realistic estimation of subjective probability	Ngl facility	UAE PCA K&T
9	Probabilistic arithmetic; Kolmogorov's Laws	Unnatural	UAE
10	Intuitive Assessment of SEU	Unnatural	D 8 Simon

Perception of Value and Trade-Offs (V)

1	More/less is better	Intuitive	SAP
2	Value scales; value having continuous relationship with magnitudes of attributes of choices; value as an additive concept	Unnatural	UAE
3	Preference between choices varying in 2 attributes	Adept	SAP Larichev
4	Break-even points for 2 attribute trade-offs	Unnatural	UAE
5	Preference between choices varying in few (3+) attributes	Difficult	See text Larichev
6	Deciding between choices varying in many attributes. "Holistic Integration".	Intuitive Efficiency unclear	SAP Larichev
7	Ranking value weights of multiple attributes	CAC	Similar to O10
8	Stability of value (Related to D4)	Unnatural	DD 2,3

Discrimination, Preference Stability and Precision, Configurality (D)

1	Discrimination of small differences in unidimensional magnitudes.	Adept	SAP
2	Reliability of classification of absolute judgements of unidimensional magnitudes. (Transmission or channel capacity of subject)	Limited	UAE Miller
3	Reliability of classification of absolute judgements of multidimensional magnitudes	Better overall than D2. Each attribute worse.	UAE Miller
4	Constancy/ endurance of preference	Unnatural	UAE DD 2,3 Chap 2
5	Consistency of judgement/evaluation with similar information	Limited	UAE Chap 2
6	Ability to process complex interactive information	Limited	UEA
7	Ability to be configural in unextreme (eg Minkowski) forms	Limited	UEA
8	Ability for extreme disjunctive or conjunctive information assessment	Adept	FEA DO8, 9

Chapter 4 The investment portfolio decision

4.1 Introduction

At this point I digress from discussion of general decision analysis considerations, to introduce the problem to which the approach developed within this thesis will be applied. I also summarise and comment on features of Modern Portfolio Theory which is the basis of existing "Quant" decision analysis in this area, where it is applied. There is a parallel thread in this thesis and I will return in Chapter 5 to further consider methodology without special reference to this application area. A reader who wishes it could therefore alternatively address this chapter prior to Chapter 8 which places the developed approach within the context of this problem.

In this Chapter I start with a consideration of the nature of objectives and approaches to the investment portfolio decision problem, considering the extent to which private and professional needs and capabilities correspond. I introduce my personal tastes as a decision maker. I mention that amongst established professional analysis there are two major strands of approach, Quantitative and Qualitative,

I then discuss Modern Portfolio Theory (MPT) which provides the theoretical glue underlying more formal quantitative professional analysis. I introduce the concept of systematic risk, or Beta, and discuss aspects of the Capital Asset Pricing Model. I will later borrow from these concepts. I argue that MPT has limitations as an exclusive normative methodology being very data intensive and, in essence, only a two-dimension model for which questions are begged. Its use by private investors is effectively precluded by cost. I conclude by suggesting that the traditional approaches can be considered as polarised paradigms. Each of these viewpoints do not need to exclude the application of the other, but the model based optimisation approaches, which occupy middle ground by characterising portfolio formation as a many dimensional problem are rare. Hallerbach, however, pursued the issue in his PhD thesis (1994) and the general arguments for such a framework were developed by Spronk and Hallerbach (1997) whose suggestion is referred to. They have subsequently continued to develop multiple dimensional analysis methods for the financial area.

4.2 The Core Problem background

One of the more complex private decision problems that can be undertaken by an individual, who chooses not to use advisers, is the selection of investments. The writer is one of many private investors who do this for themselves or within investment clubs. The Core Problem addressed in this thesis is the development of a methodology to facilitate selection of a portfolio of UK Equities for him and his wife.

It is appropriate to briefly examine the investment methods that can be adopted by private investors and the fundamentals of the model-based approaches that are available to professional managers. At root the issue is simple; "How does one choose a selection of "good" shares, which as a joint package have desirable characteristics, and decide when to buy and sell them in a way which maximises future value of purchased assets?". Although presentable as separable concepts, the two elements within this question are interdependent. Thus, one feature of a good share is that its value is expected to appreciate or that it has a low risk of declining.

Nevertheless it is probably reasonable to distinguish two polarisations in a spectrum of approaches to investment:

(a) Those whose primary interest is in calling the timing- judging the turning points in the market or an individual share price so that, in a perfect world, they are invested in the market on the climbs and liquid in the declines.

(b) Those whose primary interest is in finding shares which are good value for money over the long-term, judged from fundamentals.

The first group are likely to be interested in trends, charts, economic developments, and other situational reasons why the market or sector is over or under-valued. They may also depend on intuition to judge whether the market may be topping or bottoming out. Potential movements tend to be judged over a time scale that is short compared with the income generation rate of the enterprises which underlie the investments. (At the time of editing of this paragraph two weeks after the September 11 World Trade Centre disaster, the stock market is reeling and there is talk of recession. But will the market continue to tumble as consumer confidence is sapped, or is the present lack of purchasing merely the result of the emotional and temporary shock and unexpressed view that at such times self-indulgence is unseemly, which might change soon).

However, it may be argued there is little point in attempting to forecast the market in this way. The Efficient Market Hypothesis could be invoked to suggest that all available public information would be discounted. I am disinclined to accept this as an absolute statement. There seems sufficient (albeit non-scientific) evidence that financial market actors' attitudes feed on each other enough for the independence requirement to be questionable; which the canny observer could exploit. Witness the boom in dot-com stocks that could not be justified even in foresight by any realistic earnings model. Nevertheless, I have neither the time that would be necessary to read the changing influences, nor, I suspect, the insight to beat the market at price forecasting.

The other approach is to judge investments in the same way as a business might look at a capital expenditure, new product, or acquisition project; and to do so on a longer time-scale. Will what we get out of it justify what we commit? Is there something better? Does the expected return justify the expenditure in the light of the risks? Is it good value? This is what some describe as the Value approach to financial decision-making. It depends, though not exclusively, on consideration of Fundamentals, that is by examining the characteristics of the business that underlie the shares. The investor is likely to look at the hard facts of the business's financial performance, both historic and future forecasts. However, he or she may also look at softer considerations. Is the sector growing? Is their declared marketing strategy convincing? Is the management staid or do they recklessly seek change without regard to its true benefit to the business? What do other people think the prospects are, and so on? A private investor might depend for his data on the financial pages of newspapers, or magazines such as Investors' Chronicle, Stockbrokers' reports and recommendations, companies' Reports and Accounts, digests of financial statistics, on line sources of share prices and other share data etc. I subscribe, for example, to Company REFS (a regular CD providing common basic financial data, news and reports summaries, details of key shareholdings, Directors, and a summary of key stockbrokers' forecasts), and an on-line source of share-price and key events data.

Predominantly, private investors are likely adopt an essentially qualitative approach to assessing the overall value of a share (though may use their calculators to calculate key statistics such as Price-Earnings ratio, Gearing, or Dividend Cover etc if these are not already calculated for them in the papers they read). Certainly, the style of press information supports this approach. They assess shares (anticipating the description of the Core Technique, we can say they assess the Attributes of the shares) against their Objectives whether firm or confused. Some investors may make use of filtering methods or test options against an index of desirability. REFS, for example, provides both a means of short-listing shares against attribute thresholds and also calculates "PEG", an index intended to identify good value shares by highlighting those with good growth relative to their Price-Earnings ratio.

The principles adopted in portfolio formation by private investors are unlikely to be sophisticated though they may pay attention to exhortations in the money-advice sections of newspapers, for example, to split their portfolios into different investment types on the basis of their attitude to risk and their financial situations. They may also follow more detailed advice in books directed towards them eg Graham (1973) (which commends attention to choosing securities suited to the time and interest available to manage them). Not putting all one's eggs in one basket is a powerful risk-mitigation concept that prudent people may adopt instinctively. However, other considerations may prevail. An investment club of which I was a member seemed to want to find room for an attractive share without applying a relative test. More shares implied a more interesting portfolio and it was the dealing costs of small share parcels that seemed to keep the portfolio to around a dozen.

Some professional fund managers' approaches to share selection and portfolio formation may not be very different in principle. It seems that some houses are happy to depend on the flair of their individual managers, and perhaps their approach is akin to the qualitative private investor approach. However, their objectives, which describe fund specialisations, are likely to be quite explicit and they will be more explicitly aware that they are diversifying to reduce risk. The qualitative and quantitative data available to them will be far more extensive and through their own research staff, purchased research services, brokers' reports, their reading of the financial press, discussions with colleagues etc they have the opportunity to understand sectors in great depth. Their back-up analysis is likely to be considerably more thorough, but at root it remains a judgement or intuitive process.

Other "Quant" Houses may be more attentive to objective and quantitative assessment of value. One major fund management company I had discussions with, evaluated shares using two approaches. They paid great attention to the assessment of future cash generation, and its prospective variability, of the companies underlying candidate shares. From this they calculated a measure of fundamental value using Discounted Cash Flow principles. From a model of how the market seemed to translate share characteristics into share values, they also assessed possible movement in the market valuation of the shares. The bases for valuations were generated through a group process and accordingly the same corporate assumptions were made for all candidate shares. The relative attention paid to each of these figures depended on the objectives of a particular fund. Individual fund managers, who had discretion in portfolio formation could simulate the properties of alternative Portfolios' attributes across many factors, using a database based on the common data developed and an in-house analysis program available to them. However, they did not use computational optimisation.

Some professional houses, at least in part, develop decisions based on mathematically optimal portfolios using packages developed from Modern Portfolio Theory (MPT). MPT is a model based statistical approach owing its origins to work by Markowitz (1952) (expanded in Markowitz (1959)), It is described in full by Elton and Gruber (1987).

4.3 The elements of Modern Portfolio Theory

MPT shares with Multiple Criteria Decision Analysis and Data Envelopment Analysis the concept of efficient sets, choices and frontiers. It focuses on Return and Risk which it translates into the expected value of return and its variance. It seeks to define relationships which define for any combination of investments constituting a potential portfolio, its Expected Return and estimated Variance of that return.

The information used to develop the evaluation are the estimated return means, and variances for each of the individual investments which are candidates for inclusion in the selected portfolio, and, *in principle*, the covariance of return with each candidate investment with each other investment. Following Elton and Gruber (op cit) the expected return of a portfolio is:

$$\begin{array}{l} \overline{R_p} = \sum_{i=1 \text{ to } n} X_i \overline{R_i} \\ \text{Where } \overline{R_p} = \text{expected return of the portfolio} \\ \quad X_i = \text{proportion of investment } i \in (1,\ldots,n) \text{ in the portfolio} \\ \quad \overline{R_i} = \text{expected return of the investment } i \end{array} \tag{4.1}$$

The variance of the portfolio return is given by

$$\begin{array}{l} \sigma_P^2 = \sum_{i=1 \text{ to } n} X_i^2 \sigma_i^2 + \sum_{i=1 \text{ to } n} \sum_{j=1 \text{ to } n, i \neq j} X_i X_j \sigma_{ij} \\ \text{Where } \sigma_P^2 = \text{variance of the portfolio return} \\ \quad \sigma_i^2 = \text{variance of the return of investment } i \\ \quad \sigma_{ij} = \text{covariance of the returns from investments } i \text{ and } j \end{array} \tag{4.2}$$

Markowitz developed this fundamental formulation in the relatively early days of mathematical programming and before the availability of easy, cheap and powerful computation. He accordingly developed special solution methods to trace the efficient frontier or path, tracking maximum return for increasing levels of portfolio variance, or conversely, minimum variance for increasing levels of return. Nowadays, the problem *structure* presents no great computational problems and is easily manageable using standard non-linear programming software. However there are two features of the above equations which have significant impact, one simplifying and one complicating.

The first of these is that as the number of investments in a portfolio gets bigger (ie as typical X_i becomes smaller), the weighted sum of variances term tends to zero but the value of the covariance term tends to $\bar{\sigma}_{ij}$, the weighted average of all the σ_{ij}. Thus the covariance terms come to dominate. At least with the number of investments likely to be included in professional portfolios, individual share variance is unimportant. This element is frequently referred to as *diversifiable* risk. The double summation of covariances is the *systematic* risk on which MPT tends to concentrate.

Ignoring for the moment the approach to calculating the covariances, it is manifest that a severe data problem exists. A portfolio analysis involving selection from 300 investments requires 45,000 parameters and there might of the order of 30,000 items of data concerning investment return required to calculate them. Professional portfolio selections might be from a pool of potential investments at least ten times bigger than this. This was unmanageable in the fifties and would be formidable now, even were it to be appropriate. Much of MPT concerns itself with work-arounds.

The most basic of such simplifications is the single-index model:

$$R_i = \alpha_i + \beta_i R_m + e_i$$

Where e_i = a random variable

such that $E(e_i) = 0$

$E(e_i e_j) = 0$ (4.3)

α_i = return of investment i independent of the market

R_m = return of the market

β_i = proportion of the market return reflected in the return of investment i

One can establish (eg Elton and Gruber, op cit, p132) that

$$\sigma_{ij} = \beta_i \beta_j \sigma_m^2 \tag{4.4}$$

Where σ_m^2 = variance of the market return

β_i is sometimes referred to as the systematic risk, the Beta risk or, simply, the Beta of the investment. It depends on correlation with the market return and is equal to the correlation coefficient of the investment with the market, multiplied by the ratio of the standard deviation of the investment return and that of the market. Statistics of Beta measured on an uncomplicated basis are available, for example within REFS, and data providers supply suitable statistics to investment houses. This approach can be extended to multi-index models such as industry or market sector indices or macro economic indicators. One might also condense the information within the

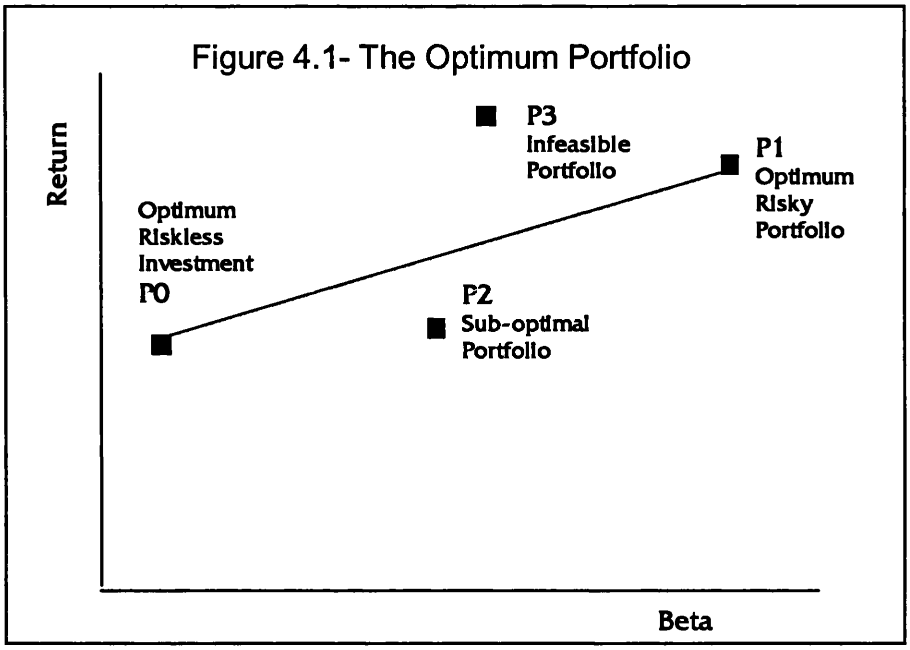
Covariance matrix by seeking its principal eigenvectors and thence the principal components, a method which produces orthogonal or independent components.

However, returning our main attention to the single index model, it should be noted that Beta has a valuable property. The Beta risk for a portfolio is simply the weighted average of the Beta's of the individual investments which comprise it. That is:

$$\boxed{\beta_P = \sum_{\text{all } i} \lambda_i \beta_i, \quad \sum_{\text{all } i} \lambda_i = 1} \tag{4.5}$$

This implies that that $\lambda\beta$ is a conservable quantity within the market (at least for the time that the Beta parameters for individual investments are unchanging) in much the same way as volume, energy or gold. A decision maker can thus attach linear value to it. It is also the property which gives rise to a key element of the Capital Asset Pricing Model (CAPM).

The first "leg" of the CAPM is that if an optimum well diversified risky portfolio P_1 with a defined return, R_1, and Beta, β_1, exists; and there is also an optimum "no risk" investment, P_0 ,which gives the maximum return with zero Beta; then the return R_2 for any other optimum portfolio, P_2 , with different Beta, β_2, is identical to the return given by that linear combination of P_1 and P_0 which has a Beta of β_2. This to say, the returns of all optimum portfolios when plotted relative to their Beta will lie on the straight line joining P_1 and P_0 (see figure 4.1).



This can be easily seen. If a portfolio exists with a return that lies below that line it will be dominated by the portfolio of corresponding Beta which can be formed from the combination of P_1 and P_0 which has the same Beta. If one lies above the line then P_1 cannot be optimal as a superior portfolio could be formed by generating a portfolio with β_1 by buying portfolio P_2 and selling P_0 ("short" if necessary). This is true for any individual, however he or she estimates the individual investment returns and Betas.

The CAPM then goes further. Working on assumptions akin to those of the perfect competition assumptions of economics (and other simplifications of conditions, enumerated in Elton and Gruber (op cit) which I will overlook here), it asserts that all investments within a market will be positioned on the same line. This can most easily be explained in terms of there existing a market for investments of a particular Beta. If we assume that there are known returns associated with those investments, an actor in the market will wish to replace all investments with below average returns, with investments of higher return. This creates a demand for the latter, which will boost their prices, and hence lower their returns, at the expense of the former whose price will drop. This opportunity will persist until all investments are restored to the line, when there will be no incentive to buy one investment and sell another.

In one sense this is unexceptionable, almost tautological. The capital market line can be said to represent the market view (in terms of being an average of all the players), and, for a given expected Beta, the corresponding point on the line can be said to represent the market expectation of the return of an investment of that Beta. However, one can neither say that the market expectation of return is "true" (ie the market actually correctly assesses all information, appropriately discounts it, and correctly weights all the factors that contribute to value), nor that "return" is the common unidimensional objective known to all players and to which they operate. Investments are differentiated and although traded as commodities they are more akin to brands such as perfume or motor cars, than to single utility goods like iron ore or Bonny Light. Were these not to be the case one could just select one's Beta and relax in the full confidence that whatever investments one may choose they will all be just as good at meeting one's objectives. With all avoidable risk diversified out, one could have confidence that they would track the market index in the expected way to the sensitivity required. Of course, Betas, may be expressed in multiple dimensions and, as Berry, Burmeister, and McElroy (1998) argue, one might extend the nature of risks embraced, but on this and related theories we can be sure of a pareto-optimal solution even if it we cannot be sure whether it meets our preferences related to this complex of risk, were they to be determinable.

This is not the view that the writer (as decision maker) has taken but it is a consoling long-stop: if all portfolios are automatically optimal there is no point in spending sleepless nights worrying about the correctness of one's specific decisions! (It is also possible that it is "sufficiently true" to place doubt on whether fund managers and advisers can secure incremental value added for their clients sufficiently, in excess of their remuneration for attempting to do so).

The writer (as decision maker) whilst not using the CAPM uses Beta and depends to some extent on "leg 1", that an individual's diversified optimum portfolios track a straight line.

4.4 Issues of MPT

For all its apparent sophistication MPT presents simultaneously an over-simple view of the market, some philosophic difficulties, and, as a normative mechanism, begs important issues. The theory is only bi-variate seeking to balance return and a measure of its uncertainty. Moreover, and critically, were investments to be neatly labelled with their expected returns and Betas, or Beta vectors, in the way in which the theory assumes, would we then perceive ourselves as having a serious problem in making investment choices. It is the characterisation of investments in this manner, the specific reduction of other information into these terms, that would constitute the most valuable contribution in the analysis process. Were that to be available in reliable form, the investors wealth would be assured. The subsequent process of the assessed parameters into portfolio suggestions is a technically interesting issue but the incremental contribution it would make is of far less importance.

We may also have difficulties in the meaning and measurement of the terms Expected Return and Variance of Return, and the extent to which the latter, and the way it is compounded in a portfolio, constitutes a meaningful measure of risk. The first perhaps presents less difficulty of principle. We can conceive a stream of cash flows generated by the business in which the investor has an equity stake, we might formulate a basis for attributing to an investment a capital value on disposal at some distant horizon, we can conceive a method of aggregating them (perhaps, using a DCF technique with the discount mechanism being as subtle as we like) we can generate a present value, and we can compare this with what we might have to pay now to purchase those cash flows.

The difficulty comes in how market prices are introduced into both return and more particularly the variance of return. It is beyond reproach to say that a return on investment is the amount we sold it for less the amount we purchased it for, plus any net dividends, interest, or capital returns we received in the interim. It is reasonably unexceptionable also to say that over a long period an unrealised change in the value of the investment should be perceived as part of the return. It would not be acceptable to count this as increment to profit in the books of a company, but increment to asset value can realistically be considered return in the investment context. But what about the shorter term? A possible approach is that the long term is merely the sum of the short term and to adopt the same argument. However, I suggest that a shortening of the period at least causes a need for questioning of the validity of the concept. Let us consider what might be happening over a period of a month. If the share price drops (but the income earned by the company whose share constitutes the investment stays the same) the return is deemed to have dropped by an amount which will frequently swamp the fundamental return over the same period by many orders of magnitude. But why did the share price drop? It might have dropped because the expectation of future fundamental returns dropped and here is the first dilemma: is it right to assign all aggregate changes in the future expectation of return and place them in a small period of time as an actual? True if we were to close off the books by now selling future expectations we would have to "dump" all differences from the previous periods into the latest period but is this not a very different situation?

However, that is not the only issue. Future expectations of earnings may not have changed at all, but market's valuation of their worth, the discount factors, may have altered. Is it still appropriate to consider this as "return"? Progressively, the disconnection from the fundamental is eroded.

With Expected Return we could if we wish resolve the issue by taking a long-view, that is periods of several years. However, as we move on to consider variance, to which MPT seems to attach the same logic, further factors come into play. The dominant part of the return variance becomes the variance of inter period investment price differences, the number of observations required for reasonable assessments severely aggravates the problem. If reference periods are of short interval, these become distal from fundamental return uncertainty. Now, even temporary differences in market assessment, which owe nothing to changes in return, are translated into variance. Thus we might have an assessment made in one month being reversed in the next- the price rises and then drops back. The aberration has no long term consequences in terms of overall market valuation or return, but the methodology has recorded an increment to variance forever. Movements in sentiment are likely to be more important than true return variability. So the detachment from fundamental meaning intensifies. What do covariances now mean? Do we expect deviations from mean in successive periods to be independent? Unlikely. What is the stochastic character of the time series of deviations, what is the appropriate sub-division of the time division over which the betas of the return variance are calculated? In physics the atomic view of matter is necessarily different from the macro view. In the same way, in this area should we necessarily expect the logic that we attach to the long view to also apply to the short period view (where changes of subjective evaluation overwhelm physical income)?

The issue can be side-stepped by detaching the measured variance of a price time-series from the expression "Variance of Return", simply considering it as price volatility. By any standard, the way in which investment prices co-vary with the market (their market sensitivity) and with other prices, and the ability of portfolios to deviate from the market is risk; possibly the most important single risk. MPT arithmetic can be applied to it. It is used by the writer (as decision maker) in this way. It is also has an important advantage as it is observable, at least historically.

This leads to the next issue. To what extent is the past a reliable guide to the future? Good past business performance is likely to be some sort of an indicator of future success. Moreover, we can conceive that analysts might reliably project the past into the future and, after appropriate consideration of future-influencing management and economic issues, generate an unbiased estimate of Return by some means. Variance is more problematic. What is it that relates past variances to future ones? The answer is far from clear. We should not necessarily expect unusual characteristics to be sustained and bias will result. Indeed, Elton and Gruber (op cit, p142) report work of Blume and Levy which illustrates the tendency of investment Betas to regress to unity. Naturally, this is not an indefinitely sustainable phenomenon: the future produces new special circumstances. These cause what prospectively may be considered random variations in Beta, probably resulting in a very similar dispersion of Betas amongst all investments, to the dispersion that persisted before. The implication is that an existing portfolio, which seems relatively riskless on the basis of past performance, will typically become more risky and a risky portfolio less so. However, provided that a positive correlation between the past Beta and their future values exists, the concept is still exploitable. It is used by the writer (as decision maker) on this basis.

We should also note that there are wider dimensions of risk and other indicators of it, some of which might not translate into investment price variability (until realisation). For example, high financial gearing could imperil a company and there could be perceived vulnerability to mismanagement. This might result in a lowering of the price of the underlying share, but it need not necessarily cause variability to be reflected in the share price; and if it does, it might not be reflected in the variability in a proportionate way. Decision makers may also wish to insure against an unspecified but finite possibility of an unforeseen risk, which again may not be reflected in a time-series's variance.

However, the main limitation of the approach, in the view of this researcher, is that it does not tackle the most difficult aspects of the problem. It pre-supposes a two-dimensional objective. Whilst the body of published thinking and the models to do this are formidable, at root they do not directly tackle even three-dimensional statements of objective. This necessitates a reductionist approach in which the rich complex of multiple objectives and the many dimensions of investment attributes must be reduced to two (return and its variance). But it is the translation of facts concerning investments, to appropriate exploitable reductions and the translation of investor values to appropriate balancing of those reductions, that is the essence of the investment analysis. It is instructive that the financial press usually argues from basic facts in many dimensions (historic earnings growth, projected PE ratios, gearing, product portfolio, management strategy, sector prospects, management track record etc.) to normative conclusion (Buy, Sell), without overt regard to intermediate condensations on which MPT depends.

4.5 The Core Problem

The approaches mentioned (Qualitative and Quant) are separate approaches; even alternative paradigms. One loose, vague, unstructured, flexible, using many variables, open-ended qualitative data but limited quantitative data; the other firm, specific, structured, quite rigid and in, ultimate process, only involving two criteria variables, but a mass of quantitative data.

Professional financial analysts, or the company's that employ them, may tend to favour one approach over another. However, they will generally have access to and may employ a combination. Nevertheless, it appears that there is no intermediate integrated methodology in regular use in the industry that attempts to quantitatively analyse the many Attributes of investments, whilst sustaining their multi-dimensional quality, into the determination of efficient sets of individual investments and the generation of efficient portfolios.

This limitation has been recognised by Spronk and Hallerbach (1997). They write, "In this view [the extended framework they commend], when buying a security, an investor is actually buying an exposure to various attributes. The issue of multi-attribute portfolio selection is to balance the attributes of the individual securities on the portfolio level. That is ..the attributes of his portfolio must be fashioned in a way that suits his particular circumstances and attributes best". They discriminate between "Directly return-related attributes" and "Indirectly return-related attributes" which they relate to the Markowitz concept. However, the central issue is that the Attributes reflect objectives but are not representations of them. [Spronk and Hallerbach's use of the word "attribute" appears similar to my own, that is as characteristic related to objectives, and not necessarily in line with the Keeney and Raiffa usage]. Hallerbach (1994) had earlier developed a multi dimensional implementation of the principle and further papers pursuing the concept, distinct in approach from that adopted by this author, have been recently published or are about to be published by Hallerbach and Spronk and their associates (eg Hallerbach et al, 2003)

It is this problem that the writer has also sought to address, as a private investor seeking to apply a quantitative investment and portfolio formation approach to his own decision making. There was in any case one ancillary factor which militated against the use of traditional MPT. First, suitable, cheap, off-the-shelf, pre-processed data, suitable for applying MPT, was not available, and, given that most private investors are unlikely to have the skills or interest to apply the principles, is unlikely to become so.

I refer to this personal application, which is intended to serve as a practical example of the "core decision analysis technique" as the "core problem". The approach adopted may be thought of as an alternative to MPT and as a more flexible approach in its principle and better able to cope with smaller scale problems. The methodology differs in one other respect. Unlike MPT it is not specific to this application area. Indeed, the methodology may attract greater interest for applications outside this area. Its application in the investment area is intended to be an example of its applicability, not a delimitation.

Chapter 5 The Basic Technique

5.1 Introduction

The following sections seek to introduce the basic Dora-D technique (originally Decision Option Reduction Analysis using concepts from DEA), placing it in the context of some of the decision analysis and cognitive assumptions already discussed.

It starts by relating the structure to the taxonomy outlined in Chapter 2. In essence it can be stylised as the selection of a single decision from mutually exclusive options characterised in terms of the magnitudes of a bounded set of attributes. The selection is informed by qualitatively well understood but quantitatively vague objectives. Each attribute is related monotonically to the goodness or badness of the decision. The data would normally be able to be represented in a complete matrix.

The inspirational origins of the technique in, and its connection to, Data Envelopment Analysis, is discussed, highlighting important distinctions. First amongst these is that the approach here is centred on decision makers' values.

Next the approach is structured, highlighting the central analytic objective, the formation of an additive linear value function. The concepts of assessing each potential decision in terms of a value function, which shows it in the best possible light, is explained and Maximal Comparative Advantage (MCA) and Comparative Advantage Function (CAF) are introduced. Initial Option Reduction is described, in which no decision maker's "values" (or just the most certain pre-emptive constraints) are specified. The concept of reducing the Latitude of the value function by seeking statements of preference is introduced, a process which is effected during Subsequent Option Reduction.

An LP formulation for effecting the process is outlined. Alternative methods or Mechanics for structuring elicitation and representing preference within the LP structure are outlined. The need for "breaking ties" is raised and approaches to this in Final Reduction are suggested. The approach is illustrated by an example.

Observations are then made concerning ancillary technical matters that might of interest to a reader or analyst.

Potential applications are mentioned, and the Chapter concludes with a brief discussion of the method in the context of the decision cognition assumptions the writer has made.

Sutton and Green (2002) constitutes an anticipation of this part of this thesis, although this Chapter amplifies some points. The structure was originally described in Sutton (1999), a transfer paper associated with this work, which suggested it as a concept for developing firm decisions from vague objectives.

5.2 Generic problem structure

5.2.1 Categorising this approach

The technique developed here may be described as a multiple objective selection method. In basic form it is capable of handling many objectives (1d in the taxonomy), indeed, as many strategic objectives as are ever likely to be practically required and sufficient objective related attributes for most normal problems.

It has been argued in this thesis, and it is an assumption of this method, that our ancestors were not required by the exigencies of the environment in which they evolved, to articulate strategic multiple objectives, nor to quantify them, and accordingly we are not now innately able to quantitatively relate factors to such objectives in a clear manner, prior to analysis. I have suggested that one quantitative objective can often be defined relatively straightforwardly (eg in business, money-value objective scales such as Net Profit, Cost, DCF, Opportunity Cost, Marginal Contribution etc.) but additional objectives may become progressively less simple to express, the more that are included for consideration. Objectives become concepts which are representable by cocktails of quantifiable measures. However, such packages of proxy measures may not necessarily be reducible to an aggregate measure prior to the decision analysis, and, even if it seems possible, it may not be wise.

Nevertheless, I have asserted that people often have a good intuitive but qualitative grasp of what they wish to achieve, a clear perception of the factors that relate to those objectives, and a firm understanding of their connection to the goodness or badness of the decision under scrutiny. People, I have suggested, have Vague objectives: but they "know what they like". At least in principle, they can be persuaded to declare trade-offs between one objective related Attribute and another. This is summarised as 2c in the taxonomy and the technique is built around this level of objective articulation. The technique can also cope with harder statements such as 2a or 2b. Less precise statements can be improved using other elicitation techniques. (However, extensions of the technique, to be discussed later, can be used to develop improved decisions in cases where means and ends are intertwined or objectives are undeclared, requiring only that possibly relevant attributes are specified and that the decision maker is prepared to suggest solutions).

The technique deals with deterministic outcomes, 3a, as well as uncertain outcomes where these are defined by statistical parameters which are treated in the same manner as deterministic attributes (the last sub-category of 3b). It handles many defined factors or attributes, 4d; but open-ended problems, 4e, must be represented by bounded approximations.

In basic form it deals with linear relationships, 5a/6a, between attributes, objectives, and overall decision desirability. Indeed it builds a linear and additive value function. However, insofar as material non-linearities exist in a decision maker's values, it is assumed, using the concept of Qualified Self Awareness discussed in Chapter 2, that they will be conscious and therefore that non-linear attributes can be converted into adequately approximate linear ones, prior to analysis. Nevertheless, the conditions for an additive value function are assumed to apply, that is that all relevant attributes are mutually preferentially independent in the case of problems with three or more attributes and the Thompsen Condition applies in the case of two attribute problems (eg French, 1986, p110). I shall discuss later how decisions evaluated using lexicographic or satisficing algorithms (but where the decision maker's "true" value mechanism is assumed to be continuous) can be improved using the method. In basic form, the method assumes that decision value is linear and non-configural, but I shall also later discuss how disjunctive and conjunctive assumptions, or the possibility of them, can be accommodated.

The methodology primarily handles 7a type decisions, that is there is no explicit constraint, or that option attributes are defined relative to a usage of one (or a few) scarce resources (eg Plant Capacity/ £ of capex). However, I shall discuss later how problems with more complex constraint structures, usually formulated as MOLPs, can also be investigated using the approach.

The approach is primarily oriented to processing well defined metrically quantified attributes 8a. It can embrace binary categoric attributes within 8b. One can include some ordinal variables within the LP structure adopted, though this is demanding of analysis resources as, *inter alia*, a separate LP variable must be assigned to each point in the ordinal scale. It is also open to the user to treat ordinal scales, or modifications of them, as cardinal scales, if, in the opinion of the decision maker, steps approximate equal intervals of value. 8c can only be accommodated to the extent that the decision maker is able to assign metric quantities to qualitative statements.

The methodology suits it for decisions firmly within categories 9a and 10a.

In basic form the methodology is suited to 11c type problems with many, even large numbers of discretely defined options. But the main extension considered in this thesis is for problems of type 11g. 11f problems are also manageable, and their treatment is discussed later in this thesis.

The method is suited to 12a and 12c decisions.

The method is for decisions made independently of other parties (13a) and has no "Game" elements within it.

The approach is analysis intensive and therefore appropriate for 14b and 16b but it can be used for strategic level unrepeated decisions 15b and repetitions of already structured routine decisions 15a.

It requires that a decision maker is the custodian of the values for the decision (17a) or that, if the conflicting interests of more than one party is a feature, the analysis treats the multiple interest group as if they were a single decision maker. There is no conflict resolution mechanic within the Basic technique, though a ready adaptation of the technique would allow the identification of sets of options with are pareto-optimal with respect to the values expressed within the conflicting value group. The application of similar methodology to Group Decision Making and Social Choice is discussed later.

The method can inform an emergent process (18b) or lead more specifically to a recommendation for a firm explicit decision (18a). It is principally a 19a support device. On the other hand it can inform all modes of analysis emphasis summarised under 20, except 20f. It depends on the identification of options, relevant attributions and the quantification of those attributes being available or becoming available though other independent methodology.

I see the method as a hard quantitative technique within a soft process, but generally I would not expect it to be mixed with other modelling techniques. A variety of preference elicitation methods could be used. The enumeration of objectives, options and attributes could be front-ended by a variety of creativity stimulating approaches.

The technique is unequivocally in the camp of Stewart's "Value or Utility Based Approaches" (22a).

5.2.2 Antecedents of the technique

The basic formulation for the *prospective* evaluation of decision choices bears a structural similarity to that used in Data Envelopment Analysis (DEA), first introduced in mathematical programming form by Charnes, Cooper and Rhodes (1978) for the *retrospective* evaluation of efficiency. Stewart (1996) commented that DEA potentially allows the maximum extraction of (implicitly factual) information from decision data. Others including Belton (1992), Doyle and Green (1993) have also pointed-out the promise of DEA for insightful decision support. Doyle and Green (op cit) and Cook and Green (2000) have sought selection indicators, looking mainly endogenously within the data describing options. However, Bouyssou (1999) suggests dangers in using DEA for more than convex efficiency analysis without the introduction of preference information. Sarrico et al (1997) introduce value judgements into a decision situation, though their interest remains primarily in generating general measures of performance efficiency rather than estimating fully defined value or utility functions for the decisions of individuals.

Techniques cannot extract what is not there and raw DEA can only identify the potentially optimal options of the efficient set. Approaches that go further in reducing the options without the explicit introduction of additional information are introducing preference "information" within the technique itself. For example, it may be perfectly reasonable to adopt a procedure which isolates the efficient option which is "closest" to the (non-efficient) average of all efficient options. This "centre of gravity" option may have properties which favour its use in conjunctive decision making, or approximate to a least radical choice, or one having the "most evenly weighted" objectives. But if these effects are incidental rather than intended objectives, the decision maker has either surrendered the responsibility to reveal how he or she really wishes to resolve matters or is not aware that there are other rationally acceptable choices which might better secure his or her aims.

The formulation here uses some of the ideas of DEA, including the notion of finding weights which show the performance of options in their most favourable light. This not only highlights good options on the basis of options' *factual* content, but, far more importantly, provides a framework to elicit and use information that is exogenous to the options which enables a selection to be made between them. Specifically, it seeks to elicit the information reflective of a decision-maker's ambitions or what Simon (1965, p45) distinguishes as the *value* or *ethical* content of a decision. In this method we seek a progressive quantitative articulation of a decision-maker's objectives, which may be qualitatively well-understood, but which are initially quantitatively undetermined, by interactively building an explicit linear valuation function.

In the method, Linear Programming is used to parameterise an additive value function consistent with a decision maker's preference declarations; in this respect resembling Bell (1977).

5.2.3 Model structure and concept

Consider a set of n discrete decision options, each characterised by k attributes of magnitude a_{iS} (reflective of the factors which the decision maker considers to be relevant to the efficacy of a prospective decision), where i is a specific attribute and S is a specific option. Notwithstanding that a decision maker may, initially, only have expressed his or her objectives in qualitative terms, we may in principle (under linear assumptions which we comment on later) express the value, v_S, of every option S, by a weighted value function:

$$\boxed{v_S = \sum_{\text{all } i} w_i . a_{iS}} \tag{5.1}$$

The identification of the best option and a ranking of others follows directly, if one can articulate, with adequate precision, the valuation weights w_i through a value or preference eliciting process.

The use of weights in this way shares common ground with other Value or Utility Methods of decision analysis and also the multiplier representations of the Charnes, Cooper, Rhodes (CCR) model (op cit) and other models of DEA. However, there are crucial distinctions between the use of weights here (together with those generated by other decision analysis methods) and their use in DEA. In the first they are internal to the decision maker and representative of his or her *values* excluding the *facts* characterising the options. Moreover, generally, after normalisation only one such function, albeit imprecise, can ultimately represent those values. In pure forms of DEA they are entirely endogenous to the Decision Making Units, reflective only of their facts (not the user's values). Also, several sets of weights (usually one per extreme point or efficient DMU) may be meaningfully extant.

Notwithstanding our initial lack of quantitative information concerning attribute weights which reflect the decision maker's values, we can nevertheless find weights for each decision option that shows it in its most favourable light. We can use this interactively to aid the decision maker to refine a value function, whilst progressively reducing the list of options which remain potentially optimal, eventually to a single option. An explicit value function consistent with all expressed preferences can be developed as an economic expression of decision makers' values which can be fed-back to the decision maker to validate analysis conclusions. It can also be used for the evaluation of new options or to define a consistent linear order.

We start by defining a Maximal Comparative Advantage (MCA) for *each decision option.* This constitutes the valuation obtained for the decision using the set of attribute variable weights which maximises the option valuation, subject to that set of weights not giving rise to a valuation of greater than an arbitrary constant (eg, 1, as used hereafter) for any *other* available decision, and not violating any other valuation constraints that might be imposed. Under these conditions, the valuation function defined by the attribute weights in the form of equation 1, is referred to as the Comparative Advantage Function (CAF) for that option.

All decision options, which are not dominated by any other option or convex combination of options would emerge from such an analysis with MCAs greater than or equal to one. However, the validity of the methodology does not depend on derivations from the standard definition of dominance (eg Keeney and Raiffa (1976 p69)) discussed in Chapter 2. It requires only that for an option to have an MCA of less than one, at least one other option must have a value of 1 when evaluated using the same valuation function. Accordingly, an option with an MCA of less than one cannot be optimal under any feasible parameters of the valuation function as all other candidate functions will be no better.

In addition to generating a reduced set of options that includes all candidates for the nominal optimum (the Efficient Set), one can also obtain the set of corresponding criteria, the CAFs, which generate the MCAs of each efficient option. These are effectively candidates for the decision maker's value function within a cone of temporarily permitted criterion space. However, we must ultimately hone down that space until an optimum option and, if required, a corresponding value function and a complete ranking of options is identified.

I refer to the process by which the MCAs and CAFs are first calculated as Initial Option Reduction. The subsequent problem (Subsequent Option Reduction) is one of progressively reducing the "latitude" of the criterion space by seeking statements of preference which rule out regions of the hitherto permitted criterion space. Whether the latitude needs to reduce to nil is a dependant on the circumstances of the problem. Generally a unique optimum or a unique ranking will not need a unique value function, but a simple encapsulation of policy is often valuable and facilitates confirmation that the analysis has reflected the decision maker's intentions.

The broad methodology outlined here is referred to by the abbreviated acronym Dora-D; Decision Option Reduction Analysis using concepts from DEA. Basic single decision selection based on a linear value model, as introduced in this chapter, is referred to as Basic Dora-D. Variations to deal with decision portfolios and other special situations are discussed later.

5.3 Operationalising the concept.

5.3.1 The formulation for Initial Option Reduction

We can express the problem of finding the CAF and MCA of any decision option with limited information on a decision-maker's objectives or preferences as a Linear Program of the following form:

For each decision option $S \in \{1,\dots,n\}$

$$\text{Maximise } v_S = \sum_{\text{all } i} w_{iS}.a_{iS} \qquad (5.2.1)$$

Subject to

$$\sum_{\text{all } i} w_{iS}.a_{ij} \le 1 \qquad \forall j \in \{1,\dots,n : j \ne S\} \qquad (5.2.2)$$

$$w_{iS} \ge \alpha_i \qquad \forall i \in \{1,\dots,k\} \qquad (5.2.3)$$

(5.2)

Where $n =$ number of options

$k =$ number of attributes

$a_{ij} =$ value of an attribute i for option j. There is no restriction on the signs of a_{ij}.

$v_S =$ value of option S. Max $v_S =$ MCA of option S.
For simplicity it is assumed here that value increases with increasing magnitude for all attributes i, scaling by -1, if necessary, to achieve this.

$w_{iS} =$ weight assigned to attribute i of option S.
When v_S is maximised, w_{iS} are the parameters of the CAF.
As the value of an option increases with increasing a_{iS}, a_{ij} the corresponding w_{iS} have imposed positive sign.

$\alpha_i =$ finite and positive real number considered by the decision maker to be the minimum weight (zero permitted) that can be taken by an attribute i ($i = 1$ to m) in the valuation function for option j ($j = 1$ to n).

The close relationship of this structure to that of DEA may now be apparent. A CCR model in which the w_{iS} are all output weights and each DMU has a single input of magnitude 1 would be of identical form, subject to including an Andersen and Petersen (1993) adjustment in equation (5.2.2), and the inclusion of negative variables. The distinction between inputs and outputs, which characterises DEA, is not meaningful here

In DEA the metaphor that a DMU "chooses" weights that show off its performance in the best possible light, is sometimes used. Here a corresponding metaphor is that an option "proposes a case" for being selected by "nominating" a weighting function that favours it to the decision-maker. The decision maker "responds" by providing extra information on what he or she wants and the options "revise their cases" accordingly.

The LP is solved for each option. This identifies the MCAs and CAFs for that option, as well as the Efficient Peers (or Reference Set) of inefficient options. These are the options which have a value of 1 when evaluated using the CAF for the inefficient option.

The constraints (5.2.2) are referred to as Frontier Constraints to distinguish them from Value Constraints which are introduced shortly.

5.3.2 Subsequent Option Reduction

The nominal optimum choice is contained within the Efficient Set identified and one can concentrate subsequent attention on this. We now seek to progressively reduce the latitude of the criterion space.

This is achieved by eliciting the decision maker's preferences. Every statement of preference serves to eliminate all criterion space, and all hitherto efficient options ("candidates" for the optimum choice) inconsistent with it. Each preference is used to specify a linear constraint to be included in subsequent LPs. In subsequent runs impossible CAFs are eliminated. New MCAs are sought using revised CAFs which are consistent with the value declarations of the decision maker. The MCA for any option will thus be less than or equal to the MCA for the prior stage. Decision options which had MCAs equal or greater than 1 may have their MCAs drop below 1. These cease to be potentially optimal candidates.

Preference expressed in any of a number of forms can be incorporated within the calculational framework. The methodology is also sufficiently flexible to simultaneously accommodate more than one mechanic. The following are mechanics that could be used, (though the reader should note that they are included if they are mathematically manageable without regard to whether they would facilitate the most cognitively reliable statements of preference):

(a) Option Preference. The preference of one potential decision over another (strictly, the preference for the combination of attributes corresponding to one decision over those of another). This can serve to eliminate criterion space, whether or not one or both decisions are in the hitherto efficient set and whether or not the preference contradicts, or is consistent with, the ranking of the decisions by the CAFs so far calculated. A variation of this method is to seek a partial ordering of a limited number of options within the hitherto efficient set, from the decision-maker. It should be noted that this is not tautological: an explicit statement of preference of A over B may reveal an implicit preference for C over D. I originally favoured this approach. However, real option comparisons are not necessarily psychologically reliable even though this basic approach seems mathematically efficient.

(b) Weight Capping. The specification of an upper limit for the weight of a particular attribute (or a combination), if the decision-maker considers its contribution to the MCA of any option to be excessive. A method I now favour.

(c) Weight Thresholds. The lower bound equivalent of (b). If mixed with (b) their seems to be a greater risk of generating mutually inconsistent value constraints.

(d) Contribution limiting. Performing a similar function to Weight Caps and Thresholds, this limits the value of the contribution that an attribute can make to the MCA of an option. It should be used with caution as it can disqualify a particular set of weights for one option whilst allowing it for another. This is at variance with the objective of finding a valuation function which applies to all. I favour examining *contributions* but if a particular contribution appears excessive, limiting the corresponding *weight.*

(e) Attribute Trade-off Preferences. Several approaches can be adopted. I favour "Preference Bracketing". In this, a decision-maker, faced with a hypothesised adverse movement in one variable, specifies two levels of favourable movement of another; one as large as possible but which he/she is sure represents inadequate compensation and one as small as possible but which he/she is sure is more than enough. This approach is consistent with that commended by Larichev (1992) who favours only changing two variables simultaneously. It is one operationalisation of Fundamentally Decomposed Preference, which will be discussed in Chapter 7.

(f) Preferences between artificially generated efficient combinations of attributes. A Base Case is created, for example, corresponding to the average attribute levels for options in an efficient set. Several efficient attribute mixes are then generated using LP, in turn optimising an attribute subject to the others not being inferior to Base Case levels. The decision-maker is then asked to rank the generated cases, or indicate preferences between pairs.

(g) Rank ordering of attribute increments. Attribute "swings" are ordered by value, in a comparable manner to that used in the initial stages of the SMARTER technique (Edwards and Barron, 1994). Weights might also be compared directly. This seems a potent mechanic and is consistent with the cognitive assumption that man is a good comparator.

(h) Fundamentally Decomposed Preference (FDP) by:

- Efficient Larichev vector ordering. An integrated search elicitation methodology based on a combination of methods (f) and (g), owing its inspiration principally to ideas within ZAPROS (Larichev and Moshkovich, 1995) but also SMARTER (Edwards and Barron, op cit). This approach is referred to in this thesis as Larichev decomposition.

- Franklin Decomposition. A methodology based on Franklin's often quoted but rarely promoted 1772 "moral or prudential algebra" is also described later.

FDP, Larichev Decomposition and Franklin Decomposition are special cases of (f); they are fully discussed later. I consider that methods based on these mechanics have good prospects of deriving reliable expressions of decision makers' values. Provided feasible fundamentally decomposed choices are generated, it is possible to ensure that mutually inconsistent constraint sets are not generated.

The form of constraint equations to be included in the LPs to reflect preferences, corresponding to the above, are:

For Preferences between Options

$$\sum_{\text{all } i} w_{iS}.a_{iP1} \geq \sum_{\text{all } i} w_{iS}.a_{iP2}$$

Where $P1$ = a decision option that is no less preferred by the decision maker than $P2$ (5.3)

This follows from

$$\mathrm{P1} \succsim \mathrm{P2} \Leftrightarrow \{a_{1\mathrm{P1}},\ldots,a_{i\mathrm{P1}},\ldots,a_{k\mathrm{P1}}\} \succsim \{a_{1\mathrm{P2}},\ldots,a_{i\mathrm{P2}},\ldots,a_{k\mathrm{P2}}\} \Leftrightarrow$$
$$v(a_{1\mathrm{P1}},\ldots,a_{i\mathrm{P1}},\ldots,a_{k\mathrm{P1}}) \geq v(a_{1\mathrm{P2}},\ldots,a_{i\mathrm{P2}},\ldots,a_{k\mathrm{P2}}) \rightarrow \sum_{\text{all } i} w_{iS}.a_{iP1} \geq \sum_{\text{all } i} w_{iS}.a_{iP2}$$

Where $v(a_{1j},\ldots,a_{ij},\ldots,a_{kj})$ = the value to the decision maker of the attributes of option j. (5.4)

For Weight Caps or Thresholds

$$w_{iS} \geq W_i \text{ or } w_{iS} \leq W_i$$

Where W_i = Constant for attribute i. (5.5)

For Attribute Preference Bracketing

$$
\begin{array}{l}
w_{mS}.(a_{mB} - a_{mA}) \geq w_{nS}.(a_{nFl} - a_{nB}) \\
w_{mS}.(a_{mB} - a_{mA}) \leq w_{nS}.(a_{nFu} - a_{nB}) \\
\\
\text{Where} \quad a_{mB}, a_{nB} \text{ are arbitrary reference levels for attributes } m \text{ and } n \\
\qquad a_{mA} \text{ is an arbitrary level for attribute } m\text{, adverse with respect to } a_{mB} \\
\qquad a_{nFl} \text{ is the highest level of attribute } n \text{ for which the decision maker is} \\
\qquad\qquad \text{confident that } v(a_{mA}, a_{nFl}) \leq v(a_{mB}, a_{nB}) \\
\qquad a_{nFu} \text{ is the lowest level of attribute n for which the decision maker is} \\
\qquad\qquad \text{confident that } v(a_{mA}, a_{nFu}) \geq v(a_{mB}, a_{nB}) \\
\qquad v(a_k, a_p) \text{ is the value of any decision with variable attributes } a_k, a_p \\
\qquad\qquad \text{all other attributes constant.}
\end{array}
\tag{5.6}
$$

[For example a decision-maker in assessing the trade-off between Gearing (G) and Return (R), for an investment decision, may define a Base Case (G=50%, R=5%) and wish to assess the return reduction that is equivalent to a Gearing of 70%. H/she may be confident that value of (G=70%, R=6%) is less valuable than the Base Case and that (G=70%, R=7%) is more valuable than the base. If so we can bracket the weights $-10w_{GS} \leq w_{RS} \leq -20w_{GS}$; or $10w_{GS} \leq w_{RS} \leq 20w_{GS}$, if the Gearing scale is inverted].

For Rank Ordering of Attribute Increments

$$
\begin{array}{l}
b_{i1}w_{i1,S} \geq b_{i2}w_{i2,S} \geq b_{i3}w_{i3,S} \geq \ldots \geq b_{in}w_{in,S} \\
\text{Where} \quad ik = \text{the index } i \text{ of the attribute for which the value of the} \\
\qquad\qquad \text{increment } b_{ik} \text{ is ranked } k \text{ amongst all the attribute} \\
\qquad\qquad \text{increments considered.} \\
\qquad\qquad \text{eg if attribute 3 is ranked 4, } i4 = 3
\end{array}
\tag{5.7}
$$

For Fundamentally Decomposed Preference

See Chapter 7.

The inclusion of constraints on weights also resembles methods used in some DEA implementations (eg Charnes, Cooper, Wei and Huang (1989), Dyson and Thanassoullis (1988), Thompson, Langemeir, Lee and Thrall (1990), Wong and

Beasley (1990), Allen et al. (1997)) where they are also used to reflect user values. However, in DEA their role is ancillary; to moderate conclusions based on purely factual descriptions of DMUs. Here where the user's value frame is primary issue, they are central to the process. All constraints expressing such preference are referred to as Value Constraints, whatever mechanic is used in their specification.

The specification of preferences is an iterative and interactive process, in which the rank ordering of each successive set of MCAs and the CAF valuation functions for the Efficient Options inform the expression of further preferences and the inclusion of further LP constraints, circumscribing the latitude of the criterion space. It continues either until a single option remains or the decision-maker is unable to discriminate.

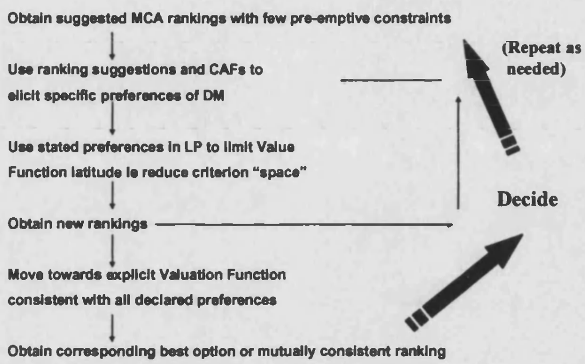
5.3.3 Final Reduction

Constraints should be added circumscribing the latitude of Equation (5.1) until only one option remains. At that stage the CAF can be normalised to give the optimum option a valuation of 1. A valuation function as per (5.1) would have been developed and the suffix S can be dropped.

If, despite every effort to elicit articulation of further preference, the decision maker is no longer able to express preferences or further delimit weights, a tie-breaking procedure may be invoked to generate an explicit value function and a corresponding ranking.

Under these circumstances, there is effectively "non-transitive indifference", at least in terms of expressible values. We can argue that the value of the remaining Efficient Options are all highly valued and an appropriate valuation function is one which values them all the most highly (and, if a feasible value function exists that allows it, of equal value). One operationalisation of this goal would be to find the function that maximises the value of the worst of them, subject to none having a value of more than one. Alternatively, one might maximise their mean subject to the same upper bound. The resulting weights can be validated by the decision maker as satisfactorily representing his or her objectives. If a single function can be found which allows more than one option to simultaneously have a value of 1, the remaining options can be construed as being on an efficient facet or edge; and the tie will persist. At this point it may be fair to assume immateriality.

Figure 1
The Option Reduction Cycle



5.4 An example

This example uses the same decision situation as Sarrico et al (1997).

Jenny is a student who expects outstanding results in her "A levels" (the last formal examinations in English secondary schools). She intends to go to University and wishes to prioritise her UCCA application. She undertakes an analysis based on the 9 classifications used in the Times Good University Guide 2000 to construct the League Table, summarised in Table 5.1.

Jenny puts particular store on teaching quality and the likely teaching attention implied by staff-student ratios. She is interested in how effective the institutions are in producing graduates with good degrees, though she notes a strong correlation between the proportion of students graduating with 1sts or Upper 2nds and A level intake scores. She is only interested in the extra achievement of the various institutions relative to the intake standard. She takes a similar view on completion rates, though she gives this a low priority as she is confident in her ability to apply herself and complete the course. She puts some priority in facilities, for which she considers rates of expenditure a reasonable proxy.

She is not especially interested in research standards, per se, but she puts value, though not a high one, on being part of a community with a high research reputation. Jenny had been ambivalent about the relevance of entry standards. She had thought that this could be a negative factor, as she favours the principle of extended access and she should meet the entrance standards of all the institutions in the list. Vanity wins and she decides to put a positive value on going to a university that people know has high entry standards. She is less concerned with "Destination", again because she felt that she had the personal qualities to open the necessary doors upon graduation. However, to the extent that it was a factor, the Times figure was inclusive of the effects of degree quality and she wished to synthesise a measure which excluded this.

Table 5.1 The Times 2000 League data

University	T	R	As	St	L	Fac	Deg	Des	Com	University	T	R	As	St	L	Fac	Deg	Des	Com
Aberdeen	86	66	72	53	70	78	76	95	86	Leicester	84	71	72	50	69	73	65	91	95
Abertay Dund'	76	23	30	50	72	62	59	86	90	Lincs & Humber	64	21	42	44	66	60	50	70	85
Aberystwyth	81	61	60	38	68	78	66	88	93	Liverpool	84	66	74	53	68	73	60	90	91
Anglia	76	20	42	47	61	67	66	86	80	Loughborough	94	68	70	47	69	84	68	95	93
Aston	87	58	70	42	69	79	74	96	93	LSE	93	97	93	40	79	67	80	92	94
Bangor	81	53	55	44	72	71	59	90	93	Luton	81	20	34	44	63	62	43	96	76
Bath	82	81	82	50	84	94	80	93	96	Manchester	87	77	81	47	73	72	74	96	93
Birmingham	89	73	82	57	68	79	78	94	94	Metropolitan, Le	70	22	53	32	60	62	56	88	82
Bournemouth	63	19	43	50	62	62	52	89	87	Metropolitan, M	82	27	47	36	61	58	52	87	86
Bradford	68	67	58	40	67	84	55	90	89	Middlesex	73	29	44	35	66	63	57	78	84
Brighton	82	32	49	40	66	69	59	89	87	Napier	75	21	47	44	64	69	65	91	71
Bristol	90	76	89	62	71	100	84	95	97	Newcastle	89	69	77	53	80	83	75	89	91
Brookes, Oxford	86	33	51	53	64	82	62	96	88	North London	74	27	29	36	66	70	49	85	70
Brunel	80	58	60	40	64	66	57	94	85	Northumbria	87	23	55	44	65	56	56	87	87
Caledonian, G	80	25	49	47	58	57	60	89	72	Nottingham	92	73	87	50	73	75	85	95	97
Cambridge	100	100	100	67	88	71	100	95	100	Oxford	97	96	98	62	100	65	91	99	99
Cardiff	83	75	77	44	67	77	70	96	89	Paisley	77	19	36	42	65	57	54	76	74
Cent England	73	23	48	33	58	65	51	82	77	Plymouth	80	32	51	53	66	61	59	84	92
Central Lancs	77	21	49	38	63	63	50	91	83	Portsmouth	75	32	51	35	61	66	53	84	86
City	77	47	67	57	67	73	60	91	84	Queen Mary, L	89	65	65	67	72	77	64	93	88
Coventry	79	25	45	38	64	68	48	84	80	Queen's, B'fast	87	59	79	47	67	82	63	95	93
De Montfort	72	33	45	40	62	68	68	85	77	Reading	86	72	68	57	68	77	68	82	92
Derby	67	21	43	31	64	67	50	87	83	Rob't Gordon	78	26	52	50	66	67	60	97	88
Dundee	83	61	65	62	68	77	67	91	84	Roy Holloway, L	84	72	71	53	68	85	74	95	87
Durham	91	76	85	38	69	82	73	90	99	Salford	77	57	52	35	62	62	51	91	83
East Anglia	85	73	72	38	66	76	74	90	94	Sheffield	92	74	84	44	66	75	72	95	89
East London	69	27	40	40	67	71	52	66	70	SOAS, London	95	76	73	57	91	69	77	82	83
Edinburgh	89	82	89	53	83	87	87	88	95	South Bank	73	23	37	44	64	69	44	76	74
Essex	87	80	58	50	70	83	61	91	93	Southampton	86	72	75	53	70	74	66	93	91
Exeter	83	64	76	47	67	69	76	95	95	St Andrews	93	76	84	62	73	82	88	96	95
Glamorgan	80	20	40	38	65	72	36	83	80	Staffordshire	75	25	45	36	63	67	57	87	87
Glasgow	90	63	79	53	71	72	83	91	86	Stirling	87	62	66	44	72	76	72	93	86
Goldsmiths', L	73	69	63	50	65	65	67	92	90	Strathclyde	89	60	68	44	64	74	67	94	87
Greenwich	75	25	39	38	69	73	47	84	79	Sunderland	76	25	41	42	60	72	56	87	78
Guildhall, L	72	24	41	33	65	62	42	84	72	Surrey	83	66	67	57	67	76	62	100	90
Hallam,	80	27	54	38	66	75	57	88	88	Sussex	79	77	71	50	72	73	67	94	88
Heriot-Watt	80	59	62	29	76	87	53	93	88	Swansea	82	59	63	47	65	72	73	88	89
Hertfordshire	67	25	44	40	69	66	63	94	89	Teesside	70	21	43	44	63	66	49	84	85
Huddersfield	66	29	45	36	61	64	59	89	84	Thames Val	69	17	31	26	61	61	43	83	84
Hull	88	62	66	53	70	63	69	96	90	Trent, Nott'	78	27	56	40	62	70	56	96	85
ICL	99	87	92	100	79	93	77	95	93	UCL	92	86	84	73	78	76	78	93	89
John Moores	74	27	49	42	64	62	47	87	85	Ulster	78	45	64	42	65	64	67	86	90
KCL	85	73	79	80	72	68	70	100	87	UMIST	82	80	76	57	66	76	61	100	85
Keele	80	66	67	44	59	65	74	91	90	Warwick	97	87	87	47	72	76	77	96	96
Kent	84	62	67	44	70	71	58	94	92	West of	89	25	56	44	61	65	52	89	85
Kingston	87	25	46	44	65	69	52	92	87	Westminster	78	27	44	44	69	68	60	83	82
Lampeter	75	59	46	35	67	69	49	85	89	Wol'hampton	74	20	43	40	60	77	59	87	82
Lancaster	90	82	72	42	70	83	72	90	90	York	97	81	83	57	69	76	70	90	97
Leeds	87	74	79	50	71	74	73	90	93										

Key T=Teaching assessment, R=Research assessment, As=A levels (entry standard), St=Student-staff ratio,

L=Library and computer spending, Deg=First and Upper Second degrees, Des=graduate destinations, Com=Completion

Table 5.2 Reprocessed Data and Results

University	Data									Maximal Comparative Advantage					Rank Order	
	T	R	As	St	L	Fac	DOG	CG	DG	Initial Reduction	Red'n 2 Ordered Weights	Red'n 3 Capped etc	Red'n 4 ICL= Cambridge	Final Reduction Maximise top ten	Times	"Jenny"
Aberdeen	62.2	59.0	60.6	36.5	28.6	50.0	76.4	32.13	71.65	0.978	0.743	0.732	0.732	0.731	26	21
Abertay Dundee	35.1	7.2	1.4	32.4	33.3	13.6	88.0	100.00	56.72	1.246	0.660	0.599	0.587	0.564	68	52
Aberystwyth	48.6	53.0	43.7	16.2	23.8	50.0	68.2	75.96	57.57	0.971	0.672	0.666	0.650	0.649	47	41
Anglia	35.1	3.6	18.3	28.4	7.1	25.0	97.5	43.65	51.36	1.022	0.683	0.550	0.550	0.494	73	64
Aston	64.9	49.4	57.7	21.6	26.2	52.3	74.3	63.78	76.28	1.014	0.747	0.744	0.733	0.732	34	20
Bangor	48.6	43.4	36.6	24.3	33.3	34.1	54.0	82.06	69.14	1.011	0.654	0.649	0.636	0.627	50	46
Bath	51.4	77.1	74.6	32.4	61.9	86.4	65.4	61.68	62.38	1.151	0.881	0.870	0.865	0.865	10	6
Birmingham	70.3	67.5	74.6	41.9	23.8	52.3	58.7	53.33	67.01	0.930	0.783	0.780	0.775	0.765	13	15
Bournemouth	0.0	2.4	19.7	32.4	9.5	13.6	49.4	71.64	71.39	0.922	0.415	0.403	0.387	0.374	90	87
Bradford	13.5	80.2	40.8	18.9	21.4	63.6	35.3	61.71	72.20	0.936	0.596	0.582	0.556	0.543	54	56
Brighton	51.4	18.1	28.2	18.9	19.0	29.5	64.0	64.33	66.03	0.872	0.570	0.552	0.547	0.547	57	55
Bristol	73.0	71.1	84.5	48.6	31.0	100.0	59.7	57.32	65.53	1.165	0.909	0.903	0.903	0.903	4	4
Brookes, Oxford	62.2	19.3	31.0	36.5	14.3	59.1	70.8	66.07	85.46	1.076	0.719	0.689	0.689	0.688	52	30
Brunel	45.9	49.4	43.7	18.9	14.3	22.7	38.2	42.58	83.08	0.865	0.551	0.548	0.538	0.513	53	61
Caledonian, G	45.9	9.6	28.2	28.4	0.0	2.3	67.3	1.74	65.27	0.845	0.601	0.464	0.443	0.395	71	82
Cambridge	100.0	100.0	100.0	55.4	71.4	34.1	80.2	56.43	53.29	1.169	1.089	1.058	1.000	1.000	1	1=
Cardiff	54.1	69.9	67.6	24.3	21.4	47.7	44.6	38.56	79.34	0.944	0.688	0.683	0.670	0.652	30	40
Cent England	27.0	7.2	26.8	9.5	0.0	20.5	38.8	23.82	50.43	0.591	0.320	0.312	0.307	0.304	89	93
Central Lancs	37.8	4.8	28.2	16.2	11.9	15.9	33.9	47.64	79.13	0.826	0.423	0.422	0.416	0.401	79	80
City	37.8	36.1	53.5	41.9	21.4	38.6	34.1	29.88	71.47	0.800	0.561	0.555	0.551	0.541	51	57
Coventry	43.2	9.6	22.5	16.2	14.3	27.3	33.2	39.99	58.93	0.664	0.432	0.409	0.407	0.405	80	78
De Montfort	24.3	19.3	22.5	18.9	9.5	27.3	100.0	27.47	46.73	1.051	0.608	0.504	0.504	0.465	67	70
Derby	10.8	4.8	19.7	6.8	14.3	25.0	42.7	54.95	66.71	0.782	0.378	0.370	0.352	0.347	93	90
Dundee	54.1	53.0	50.7	48.6	23.8	47.7	61.6	32.32	66.12	0.882	0.697	0.673	0.673	0.673	42	35
Durham	75.7	71.1	78.9	16.2	26.2	59.1	34.1	70.54	58.42	1.072	0.757	0.753	0.742	0.720	16	25
East Anglia	59.5	67.5	60.6	16.2	19.0	45.5	69.8	65.51	57.66	0.946	0.708	0.705	0.691	0.682	32	33
East London	16.2	12.0	15.5	18.9	21.4	34.1	53.4	4.36	0.00	0.610	0.401	0.381	0.380	0.304	94	94
Edinburgh	70.3	78.3	84.5	36.5	59.5	70.5	69.8	48.97	41.51	0.998	0.882	0.874	0.874	0.864	6	7
Essex	64.9	75.9	40.8	32.4	28.6	61.4	55.3	78.40	70.71	1.123	0.781	0.777	0.767	0.759	29	16
Exeter	54.1	56.6	66.2	28.4	21.4	29.5	67.0	64.81	71.65	0.992	0.706	0.701	0.689	0.671	37	36
Glamorgan	45.9	3.6	15.5	16.2	16.7	36.4	0.0	46.09	65.01	0.740	0.459	0.381	0.381	0.370	85	88
Glasgow	73.0	55.4	70.4	36.5	31.0	36.4	83.0	23.60	53.88	0.948	0.817	0.751	0.751	0.724	23	24
Goldsmiths', L	27.0	62.7	47.9	32.4	16.7	20.5	65.7	59.79	69.22	0.943	0.617	0.607	0.590	0.570	48	51
Greenwich	32.4	9.6	14.1	16.2	26.2	38.6	38.0	43.13	59.69	0.713	0.431	0.427	0.427	0.427	81	75
Guildhall, L	24.3	8.4	16.9	9.5	16.7	13.6	16.7	11.48	63.52	0.657	0.281	0.280	0.276	0.271	95	97
Hallam, Sheffield	45.9	12.0	35.2	16.2	19.0	43.2	49.1	62.41	64.46	0.845	0.534	0.532	0.525	0.525	63	59
Heriot Watt	45.9	60.6	46.5	4.1	42.9	70.5	21.0	52.66	83.04	1.037	0.641	0.635	0.615	0.610	49	48
Hertfordshire	10.8	9.6	21.1	18.9	26.2	22.7	84.7	76.77	78.49	1.073	0.540	0.527	0.509	0.509	72	63
Huddersfield	8.1	14.5	22.5	13.5	7.1	18.2	70.0	56.69	66.03	0.844	0.425	0.415	0.396	0.392	84	83
Hull	67.6	54.2	52.1	36.5	28.6	15.9	66.2	56.13	80.11	0.988	0.723	0.702	0.699	0.684	39	31
ICL	97.3	84.3	88.7	100.0	50.0	84.1	27.8	36.97	70.88	1.561	1.270	1.064	1.000	1.000	2	1=
John Moores, Liv	29.7	12.0	28.2	21.6	14.3	13.6	23.9	55.98	69.01	0.807	0.412	0.409	0.402	0.381	77	86
KCL	59.5	67.5	70.4	73.0	33.3	27.3	39.6	27.77	91.76	1.079	0.759	0.753	0.752	0.730	15	22
Keele	45.9	59.0	53.5	24.3	2.4	20.5	80.6	54.92	60.76	0.930	0.641	0.614	0.601	0.586	46	50
Kent	56.8	64.2	53.5	24.3	28.6	34.1	27.4	63.26	82.32	0.982	0.652	0.650	0.639	0.612	44	47
Kingston	64.9	9.6	23.9	24.3	16.7	29.5	45.1	57.09	80.70	0.993	0.649	0.562	0.561	0.554	62	54
Lampeter	32.4	60.6	23.9	12.2	21.4	29.5	36.1	76.33	61.27	0.909	0.527	0.521	0.504	0.484	58	66
Lancaster	73.0	78.3	60.6	21.6	28.6	81.4	63.1	48.82	59.19	0.939	0.760	0.759	0.750	0.750	19	18
Leeds	64.9	68.7	70.4	32.4	31.0	40.9	49.6	52.81	58.42	0.859	0.721	0.719	0.712	0.699	22	28
Leicester	56.8	65.1	60.6	32.4	26.2	38.6	39.7	69.09	67.65	0.979	0.702	0.698	0.688	0.664	35	38
Lincs & Humber	2.7	4.8	18.3	24.3	19.0	9.1	44.1	64.51	13.95	0.717	0.313	0.303	0.294	0.294	96	95
Liverpool	56.8	59.0	63.4	36.5	23.8	38.6	18.4	50.56	68.37	0.862	0.638	0.636	0.632	0.604	40	49
Loughborough	83.8	61.4	57.7	28.4	26.2	63.6	54.2	63.75	77.77	1.062	0.838	0.796	0.791	0.785	21	13
LSE	81.1	98.4	93.1	18.9	50.0	25.0	34.9	39.93	59.28	0.981	0.811	0.764	0.754	0.727	8	23
Luton	48.6	3.6	7.0	24.3	11.9	13.6	30.5	36.71	100.00	1.018	0.486	0.429	0.429	0.415	83	77
Manchester	64.9	72.3	73.2	28.4	35.7	36.4	47.9	50.38	78.28	0.934	0.746	0.743	0.732	0.713	18	26
Metropolitan, Leeds	18.9	6.0	33.8	8.1	4.8	13.6	47.4	38.59	66.22	0.761	0.383	0.358	0.345	0.335	86	91
Metropolitan, M	51.4	12.0	25.4	13.5	7.1	4.5	43.7	62.59	65.18	0.846	0.514	0.442	0.439	0.420	69	76
Middlesex	27.0	14.5	21.1	12.2	19.0	15.9	64.7	57.90	33.42	0.731	0.441	0.405	0.404	0.403	82	79
Napier	32.4	4.8	25.4	24.3	14.3	29.5	87.0	0.00	67.65	0.984	0.610	0.525	0.525	0.462	74	71
Newcastle	70.3	62.7	67.6	36.5	52.4	81.4	61.3	46.90	53.79	0.899	0.797	0.794	0.794	0.794	17	12
North London	29.7	12.0	0.0	13.5	19.0	31.8	55.6	17.76	61.27	0.747	0.428	0.412	0.412	0.384	88	84
Northumbria	64.9	7.2	38.6	24.3	16.7	0.0	44.0	57.02	62.12	0.632	0.649	0.507	0.491	0.477	64	68
Nottingham	78.4	67.5	81.7	32.4	35.7	43.2	68.7	59.75	64.76	0.991	0.817	0.816	0.809	0.801	12	10
Oxford	91.9	95.2	97.2	48.6	100.0	20.5	56.5	54.70	72.59	1.393	0.979	0.977	0.972	0.965	3	3
Paisley	37.8	2.4	9.9	21.6	16.7	2.3	65.0	25.92	29.51	0.696	0.526	0.450	0.427	0.350	91	89
Plymouth	45.9	18.1	31.0	36.5	19.0	11.4	60.8	82.76	50.51	0.957	0.606	0.546	0.542	0.534	56	58
Portsmouth	32.4	18.1	31.0	12.2	7.1	22.7	40.7	57.72	55.10	0.750	0.426	0.423	0.413	0.401	70	81
Queen Mary, L	70.3	57.8	50.7	55.4	33.3	47.7	61.6	49.01	74.62	0.941	0.760	0.755	0.755	0.753	25	17
Queen's, Belfast	64.9	50.6	70.4	28.4	21.4	59.1	18.3	52.81	61.59	0.983	0.684	0.683	0.676	0.652	33	39
Reading	62.2	66.3	54.9	41.9	23.8	47.7	58.6	62.04	37.42	0.898	0.699	0.697	0.694	0.693	31	29
Rob't Gordon	40.5	10.8	32.4	32.4	19.0	25.0	62.5	64.85	90.10	1.012	0.580	0.575	0.567	0.561	59	53
Roy Holloway, L	56.8	66.3	59.2	36.5	23.8	65.9	72.0	37.52	73.18	1.007	0.755	0.749	0.746	0.746	24	19
Salford	37.8	48.2	32.4	12.2	9.5	13.6	32.4	43.98	78.36	0.805	0.474	0.471	0.459	0.430	60	74
Sheffield	78.4	68.7	77.5	24.3	19.0	43.2	33.4	30.03	74.71	0.943	0.764	0.695	0.690	0.667	20	37
SOAS, London	86.5	71.1	62.0	41.9	78.6	29.5	77.5	18.39	30.53	1.023	0.926	0.912	0.909	0.804	14	9
South Bank	27.0	7.2	11.3	24.3	14.3	29.5	30.4	24.70	37.16	0.474	0.357	0.345	0.344	0.327	92	92
Southampton	62.2	66.3	64.8	36.5	28.6	40.9	36.1	49.34	73.09	0.885	0.703	0.701	0.695	0.675	27	34
St Andrews	81.1	71.1	77.5	48.6	35.7	59.1	86.8	55.07	65.57	1.115	0.919	0.895	0.895	0.894	7	5
Staffordshire	32.4	9.6	22.5	13.5	11.9	25.0	63.3	69.20	61.35	0.851	0.475	0.470	0.459	0.458	75	72
Stirling	64.9	54.2	52.1	24.3	33.3	45.5	76.2	39.44	68.60	0.961	0.708	0.705	0.705	0.705	41	27
Strathclyde	70.3	51.8	64.9	24.3	14.3	40.9	55.3	41.18	75.43	0.931	0.703	0.660	0.656	0.646	43	43
Sunderland	35.1	9.6	16.9	21.6	4.8	36.4	65.5	36.52	62.12	0.808	0.519	0.455	0.455	0.449	76	73
Surrey	54.1	59.0	53.5	41.9	21.4	45.5	40.7	54.92	97.88	1.099	0.720	0.715	0.706	0.684	38	32
Sussex	43.2	72.3	59.2	32.4	33.3	38.6	46.7	41.70	75.43	0.929	0.683	0.676	0.662	0.648	36	42
Swansea	51.4	50.6	47.9	28.4	16.7	36.4	85.7	55.62	52.22	0.962	0.702	0.650	0.646	0.645	45	44
Teesside	18.9	4.8	19.7	24.3	11.9	22.7	39.4	63.26	58.16	0.785	0.404	0.398	0.389	0.382	87	85
Thames Val	16.2	0.0	2.8	0.0	7.1	11.4	33.6	73.74	59.65	0.849	0.314	0.310	0.295	0.281	97	96
Trent, Nottingham	40.5	12.0	38.0	18.9	9.5	31.8	42.2	47.45	90.05	0.935	0.508	0.505	0.496	0.483	65	67
UCL	78.4	83.1	77.5	63.5	47.6	45.5	63.4	30.03	63.91	0.944	0.841	0.838	0.838	0.835	5	8
Ulster	40.5	33.7	49.3	21.6	16.7	18.2	63.7	58.57	50.60	0.818	0.542	0.538	0.528	0.520	55	60
UMIST	51.4	75.9	66.2	41.9	19.0	45.5	17.0	23.08	98.64	1.094	0.674	0.669	0.661	0.628	28	45
Warwick	91.9	84.3	81.7	28.4	33.3	45.5	42.0	55.58	73.99	1.014	0.919	0.829	0.823	0.799	9	11
West of England	70.3	9.6	38.0	24.3	7.1	20.5	28.9	47.45	71.39	0.877	0.703	0.502	0.501	0.485	61	65
Westminster	40.5	12.0	21.1	24.3	26.2	27.3	74.7	49.56	46.64	0.829	0.592	0.547	0.547	0.509	66	62
Wolhampton	29.7	3.6	19.7	18.9	4.8	47.7	72.8	50.78	59.82	0.861	0.515	0.477	0.477	0.476	78	69
York	91.9	77.1	76.1	41.9	26.2	45.5	29.4	64.63	60.72	1.029	0.919	0.797	0.796	0.769	11	14

Jenny recognised that the Times data were not *complete* (Keeney and Raiffa, 1976, p50), specifically that course variations were not included. She felt that a general analysis was a useful first step which she would amplify with some course focussed data later. She noted some attribute *redundancy* (for example the ability of an institution to produce graduates with good degrees should itself be related to teaching quality and staff-student ratios). She did not feel she would be entrapped into double counting provided she remained alert.

Jenny used the Times T, R, As, St, L, Fac variables unmodified (except for re-scaling as described below). In three cases she felt that the unmodified variables did not properly reflect the institutional value-added she wanted to use. Having noted that Degree Quality (Deg) was related to Entry Standards, she estimated the effect of this using a simple regression analysis and then removed it to create a new variable "Degree Quality Gain" which she used instead of Deg. She similarly developed new variables instead of Des, and Com, "removing" the effects of entry standards from Com, and Degree Quality from Des, as follows:

$$\text{Degree Quality Gain, DQG} = \text{Deg} - 0.0047\text{As}^2 - 0.0088\text{As} - 43.8 \qquad (5.8)$$

$$\text{Destination Gain, DG} = \text{Des} - 0.246\text{Deg} - 74.1 \qquad (5.9)$$

$$\text{Completion Gain, CG} = \text{Com} - 0.292\text{As} - 69.3 \qquad (5.10)$$

All variables were linearly transformed so that the value for the lowest institution was zero and for the highest equalled 100. This was done to simplify Jenny's relative weight perception. She felt that these approximated ratio scales but noted the later comments on Measurement Scales in this paper. The transformed variables used are included in Table 5.2.

Jenny ran the Initial Option Reduction finding for each option the CAF valuation function showing the option to its greatest Comparative Advantage and noted the resulting scores for each alternative, without any pre-emptive limitations on the latitude of each valuation function. Some 25 were efficient. She considered using Weight Capping to restrict the valuation latitude at this stage but she had a firm idea of her ordinal priorities. She resolved to restrict the weights so that the following weight-value relationships were maintained.

$$w > w > w > w > w > w > w > w > w \qquad (5.11)$$

for all institutions, S.

This proved to be a powerful filter reducing the number of options that remained efficient within the reduced valuation latitude to two; ICL and Cambridge. She examined the weights for the CAF valuation functions of leading contenders. She noted that ICL had been "given" high weights for the T and St attributes; too high in her view. Cambridge was also highly polarised. Oxford and Bath, whose comparative strengths were in attributes lower down in Jenny's batting order, had been given completely flat sets of weights. Both extremes were unreasonable in her view. With some experimentation she progressively imposed the following Caps, Thresholds and relative weight relationships, in addition to her previous constraints.

$$\boxed{\begin{array}{l} w_{T,S} \leq 0.005 \\ w_{T,S} \geq 1.2 w_{St,S} \\ w_{L,S} + w_{Fac,S} \geq 0.6\,(w_{T,S} + w_{St,S} + w_{DQG,S}) \\ w_{DG,S} \geq 0.0001 \end{array}} \qquad (5.12)$$

for all institutions, S.

This did not affect the leading contenders but Jenny was more satisfied with the shape of the valuation functions. However, the latitude of her implicit valuation function had not yet reduced to a single vector. She felt unable to distinguish between Cambridge and ICL and decided to force the LPs to give them equal value. This further reduced the function latitude but did not result in an explicit function. She therefore decided to minimise dispersion between the top ten institutions, finding the function which maximised the average MCA for these. This fixed the function to her satisfaction and she prepared her batting order on the basis of this. See Table 5.2.

The derived valuation function reflecting Jenny's priorities was

$$\boxed{\begin{array}{c} \text{Relative Value} = 0.00218 w_{T,S} + .00182(w_{St,S} + w_{DQG,S} + w_{L,S} + w_{Fac,S}) \\ + 0.00110(w_{R,S} + w_{As,S} + w_{CG,S} + w_{DG,S}) \end{array}} \qquad (5.13)$$

5.5 Some Observations

5.5.1 Weak Efficiency

Under some circumstances, options which are dominated will record MCAs of 1 at the Initial Option Reduction stage. I refer to these as Weakly Efficient, a concept introduced earlier. A Weakly Efficient option may be dominated, but can become co-optimal with an option which dominates it, if a decision maker chooses to assign zero weight to particular attributes. Accordingly, I suggest these are retained as potential optima and I define all decisions with MCAs of 1 or above (whether dominated but weakly efficient, or non-dominated) as members of the Efficient Set.

The justification is that it is no more reasonable to break ties by reference to "irrelevant" criteria within the considered set than on the basis of criteria outside it. Indeed, the existence of ties could properly initiate reconsideration of the value of hitherto excluded factors.

This is a different view from that implicit in DEA, which takes pains to exclude Weakly Efficient DMUs which are not also strongly efficient (ie those DMUs that are dominated but not strongly dominated). DEA seeks to guarantee Strong efficiency by the introduction of a Non-Archimedean Infinitesimal as a lower bound on weights. This is usually denoted by ε and is often referred to simply as Epsilon within textual discussion. Rather than this being represented in LPs by a small archimedean number, it is considered more rigorous to solve the equations using a special two stage algorithm. Allowing Weak efficiency during the refinement of the CAF latitude, therefore has the additional advantage of permitting the use of standard LP software.

After the incorporation of preferences, some Strongly Dominated options can be assigned values of exactly one. We are thus not entirely absolved from the problems of infinitesimals, as is discussed in the following section.

5.5.2 Preference within the context of an LP formulation

As mentioned in Chapter 2, LP is an approach which deals happily with "greater than or equal" constraints of the form $x \geq y$, not those of form $x > y$. However, when a person expresses a preference for one item, a, over another, b, it is likely that it is intended as a statement of Strict Preference, that is that the value of a, is greater than b. This may be written as $v(a) > v(b)$ or $v(a) \geq v(b) + \varepsilon$. However, LP can only simply deal with $v(a) \geq v(b)$ which is the value form of the Weak Preference relation a is "not less preferred" than b.

In once sense, this can be considered to be a conservative formulation: options are not eliminated unless so indicated by statements of weak preference. Nevertheless, it is possible, after stages of Subsequent Option Reduction, for options to have MCAs of 1 when a decision maker has made declarations of what he or she intends as Strict Preference and which on the basis of those preferences would be directly or implicitly, definitely "not preferred". There is a particular risk, when some efficient options are directly compared. However, this is not practically a severe issue. Such options can be retained temporarily and are clearly "tagged" in the AP formulation; they have MCAs of *exactly* 1. It will also be clear what expression of preference led to that condition.

Notwithstanding, if the issue seems relevant, one can take a quantitative view on the minimum quantum of extra value that a statement of preference implies. We can include expressions of the type $v(a) \geq v(b) + \gamma$. Here gamma, unlike epsilon, can be considered as finite and archimedean. Values of γ of approximately 0.01 (when 1 is the standard upper limit of value) will provide adequate computational distinction and represents a much finer scale of discrimination than a decision maker could competently distinguish in making a statement of strict preference. A higher figure of 0.05 could generally be used.

It should be understood that this issue is distinct from the issue of Weak Efficiency, already discussed. There the decision maker has not pronounced on the weights of attributes and he is at liberty to assign zero weight to one or more. Here, if he has made a statement of strict preference, he has pronounced a strict superiority of value.

5.5.3 Value Function form

Attribute measures should be linear with value or approximate to linearity. This is demanding in principle, but it has been argued that in many practical problems a linear additive value functional form will be developable, and that sufficient information can be elicited from the decision maker to transform non-linear components to adequately linear form. I have already suggested that the concept of Qualified Self-Awareness should generally make this practically achievable. (I except from this circumstances where the General Configural Model, might be applied, which will be discussed later). Within the basic Dora-D model, where the need for a transformation is apparent, but its parameters are unclear, a non-linear term can be introduced in the form of an additive function of more than one variable *provided* that the undetermined parameters are all multiplicative weights.

Thus the value contribution of a primary attribute a_{iS} could be represented in Equations (5.1) or (5.2) as a composite of the form

$$\boxed{w_{i1S} \cdot \mathrm{f}_{i1}(a_{iS}) + w_{i2S} \cdot \mathrm{f}_{i2}(a_{iS}) + \ldots + w_{imS} \cdot \mathrm{f}_{im}(a_{iS})} \tag{5.14}$$

For example, a polynomial form could be used. A non-linear interaction term, if relevant, can be introduced by the inclusion of an additional variable for the cross-product.

5.5.4 Convergence, redundancy and inconsistency

Each added preference constraint serves to reduce the latitude of the value function, if it intersects the hitherto feasible space. Under these circumstances the procedure converges towards a single function. However, there is no general guarantee of intersection and the previous feasible space may be left wholly on the feasible side of a new constraint (indicating redundancy) or on the infeasible side (indicating inconsistency). These possibilities become more severe as value latitude diminishes. The introduction of new constraints is best done in small sets to facilitate backtracking when necessary. Certain of the forms of preference elicitation can reduce the problem. For example, Capping is easy to backtrack and can be taken in smaller steps when indicated.

One might also use Method (f) or (h), Section 5.3.2 to generate efficient attribute "bundles" with characteristics that facilitate cognitively reliable elicitation, whilst reducing the risk of inconsistency. In certain instances feasibility can be guaranteed. An equation representing a preference between two choices within the hitherto feasible space will necessarily be feasible. I return to this issue later.

5.5.5 Measurement scale issues

Where the scales of measurement can be so constructed, it is useful for the attributes to be represented on ratio scales. Under such circumstances, the value function obtained will represent a ratio scale of value. If the origin of an attribute's ratio scale is mis-specified, or no natural zero exists, then the value scale ceases to be a ratio scale. Nevertheless, it remains a strategically equivalent interval scale that properly reflects the difference in value of each option from each other option.

5.5.6 Consistency of dimensionality of attributes

Though not always essential, it is valuable to maintain dimensional coherence between the attribute variables used in analyses. For example, in developing attributes to construct investment portfolios, I use two primary classes of variable. These are, firstly, variables indicative of money value per share and which might be assumed to be proportional to it (eg Earnings per share, Growth in Earnings per share, Cash Flow per share, Sales per share etc) and inherently dimensionless measures (eg Gearing, Relative Strength, Beta). The first group are transformed to dimensionless measures by dividing them by Price per share (obtaining, for example, Earnings/Price ("Earnings Yield"), Growth of Earnings/Price etc).

5.5.7 Including the option under consideration within the Comparison Set

Equation (5.2.2) is structured in a similar manner to Anderson and Petersen (1993). Excluding the case j=S from the Comparison Set is optional. The method works satisfactorily if it is included. Generally, I favour the AP-type exclusion, as:

It generally provides a non-degenerate solution with a unique CAF and a single optimum.

It provides an automatic unambiguous indication of the next best efficient option/s, assuming the CAF represents the decision maker's value function.

It provides a measure of the maximum degree of superiority (and hence a measure of potential materiality) of a particular efficient solution.

It distinguishes between strongly efficient options (for which the MCA is greater than 1) from weakly efficient options (for which the MCA is exactly equal to 1)

These, for me, outweigh the disadvantages of :

A slightly more complicated analysis procedure, as, for each option considered, the constraint for the previous option examined must be reinstated and that for the current option excluded.

A need in the final stages to re-normalise the CAF so that the optimal option has a comparative advantage of 1, when it is concluded that the CAF represents the decision maker's values.

5.6 Applications

The method above can be used for many single choice selection decisions, eg

(a) Mutually exclusive, defined investment choices
(b) Mutually exclusive market entry brand selection decisions
(c) Staff selection and promotion planning
(d) Contract/tender selection
(e) Project selection and capital investment decision making
(f) Facilities location decisions
(g) Corporate strategy selection

It could also be used, with little adaptation, for composite or multi component decisions involving interaction, where the characteristics of any given alternative can be explicitly enumerated (at least by computer) and the number of combinations is within bounds that could be represented by an LP constraint. One could, in principle, address problems with several thousand potential solutions.

More substantial adaptations are necessary once the number of solutions becomes very large or represented by continuous variables. The treatment of such problems is discussed Chapters 6 and 7.

5.7 Relationship with cognition issues

I suggest that this structure particularly lends itself to the process of hardening choice and developing quantitative objectives from well understood but Vague qualitative objectives. The assumption is that these should enable the clear identification of relevant and quantifiable attributes.

The process of "normalising" value function coefficients by the maximum valuation that can be generated by them is also helpful. Normalisation, say, to ensure that coefficients sum to unity, might in solution methods be arithmetically convenient but does not contribute to insight. In this formulation, contributions to value and relative contributions to value can be clearly seen against a clear yardstick. (If we retain the AP construction to the end of an analysis we confess that "1" is the value of the second best option. But this does not alter the transparency of the measures and, like Jenny, we can re-base the derived function).

The inductive or implication orientation is useful. As an impaired decision maker, I may not know my values. I may also not know whether a particular option is desirable in the light of my values. But if you can suggest a set of values which would cause a particular option to be optimal, presented in a way that I can discern contributions to value of particular attributes in particular options, I may be able to tell you that I do not like them and why.

Dominance and efficiency as mathematical descriptions are also blunt guides to discrimination. The latter will embrace the optimal, but includes the materially non-optimal. At the same time it excludes the near optimal. MCA, however, measures the "degree" of efficiency. Indeed, the reader will recall that the corresponding term in DEA is also used as a metric not just as a classification, following the concept of economics and engineering.

The model structure permits a variety of preference elicitation mechanics I believe that most expressions can be translated into LP constraints. Within this flexible framework, it is open to a decision maker and analyst to use any method they feel meaningfully exploits reliable expressions of preference. It is not incumbent on users to adopt the minimalist standards which the writer favours. Nevertheless, the methodology will accommodate preference expressions which, I will later argue, are the most basic that can be envisaged. These are the various forms of Fundamentally Decomposed Preference, later characterised as [1,1] choices.

Finally, in the contexts of lability of value, the related suggestion that people are inherently unable to discriminate or transmit very large amounts of value information, and the proposition that Qualified Self-Awareness allows relationships to be adequately linearised, the Dora-D technique would appear to be a more than adequate means of addressing the classes of problems targeted.

Chapter 6 Extending Dora-D to portfolios by Frontier Probing

6.1 Introduction

In this chapter the ideas introduced in Chapter 5 are extended to embrace portfolios. I start by reprising the characteristics of problems that can be handled using Basic Dora-D, noting that it requires explicit designation of options, but that there are significant examples of problems for which the combinatorial magnitudes or the definition of decisions in terms of continuous variables rule this out. Many of these can be described as portfolio problems.

I defer consideration of Project Portfolios but consider the handling of portfolios having the same structure as financial portfolios and use the term Financial Portfolios to embrace the generic class as well as the specific problem. I then structure this problem.

Such problems are characterised by combinatorially large or infinite numbers of options *and* lesser, but still unmanageably large, numbers of efficient options.

Frontier probing, a concept used in the attack of the core problem, is then conceptualised. This involves the insertion of explicit Frontier Constraints only when a violation of implicit constraints is observed. The method is illustrated with a worked example.

The chapter concludes by a discussion of the need for concavity in the function defining the interdependent portfolio attributes.

6.2 The difficulty with portfolios

The capability of Basic Dora-D is characterised by three special features which are common to many problems, but by no means all; even those with a hard bounded structure. These are:

- All options are discrete, ie capable of being explicitly and individually defined in terms of their attributes.
- They are mutually exclusive.
- There is a finite and manageable number of such options.

We should note at this point that mere scale is unlikely to constitute a practical impediment. Several thousand options could be accommodated; more than sufficient to deal with any realistically imaginable problem of this structure.

The difficulty is that many problems may be:

- Implicitly defined by decision variables which are continuous, or
- Consist of assemblages of interdependent sub-options or components that can be adopted together within defined constraints, or
- Both

Examples of such decision problems might include the location, number and capacity of a network of warehouses, the determination of an international marketing and production strategy involving both a determination of countries to be entered and products to be launched, the determination of a research programme consisting of selections from a number of candidate projects, the selection of investments or the principal design parameters of a new aircraft. These are examples of characteristics 11e, f, g of the taxonomy in Chapter 2. The first three of these may be considered selections of interdependent sub-options and the last two the selection of continuously defined parameters defining an option, in the penultimate example by specifying the proportion of components. All bar the last of these can properly be called the selection of a Portfolio (and the last could be described as the selection of a portfolio of features).

Not all problems of this type are beyond representation in the basic method. For example if there were 10 projects available to form part of a research programme, the one thousand combinations (precisely, $2^{10}-1$) could be quite readily handled. However, the one million combinations of 20 projects and the one billion of 30, present different propositions.

I will return to the issue of Research Portfolios of this type in Chapter 7 but will address first, and principally, portfolios which consist of continuous blends of portfolio components. These are not necessarily confined to decisions of the financial type but I will call them Financial Portfolios as this is an important example and, indeed, central to the Core Problem described in this thesis. Here the potential components could be a population of many hundreds and, moreover, they can be chosen in any proportions, not just on a binary in-out basis.

6.3 Structure of the Financial Portfolio problem

We can characterise the extended problem in the following way. There exists a set of n discrete potential portfolio components (index h) which are analogous to options in the Basic Method. Associated with these components are k additive attributes of magnitude a_{ih} (reflective of the factors which the decision maker considers to be relevant to the efficacy of the prospective decision), where i is a specific attribute and h is a specific potential component. A specific portfolio p is constituted by a proportion f_{ph}, $(\sum_{\text{all } h} f_{ph} = 1)$ of each potential component h.

We can define the additive attributes for any portfolio p by:

$$\boxed{A_{pi} = \sum_{h=1 \text{ to } n} f_{ph} a_{ih}, \qquad i \in \{1,\ldots,k\}} \tag{6.1}$$

We may in principle also associate any portfolio with m overall value related properties B_{pl} , where l is a specific property, which are not additive. Each such property may be a function over all h of the functions g_l which are themselves functions of the proportions of the components within the portfolio and may be (but need not be) parameterised by e additional attributes b_{hd} associated with the potential components.

$$\boxed{B_{pl} = G_l(g_{l1}(f_{p1}, b_{11}, \ldots, b_{1d}, \ldots b_{1e}), \ldots, g_{lh}(f_{ph}, b_{h1}, \ldots, b_{hd}, \ldots b_{he}), \ldots, g_{ln}(f_{pn}, b_{n1}, \ldots, b_{nd}, \ldots b_{ne}))} \tag{6.2}$$

In this generic form there is opportunity for some flexibility and complexity. However, in the writer's opinion, great complexity is generally unlikely to be called for. Indeed, on parsimony grounds, excessively parameterised models are likely to be suspect. e may be 1 or 2 (or there may be no b_{hd} at all). Examples are:

$$\begin{array}{|ll|}\hline B_{pl} = \sum_{\text{all } h} f_{ph}^2 b_h & (6.3.1) \\ B_{pl} = \sqrt{\sum_{\text{all } h} f_{ph}^2 b_h} & (6.3.2) \\ B_{pl} = \sum_{\text{all } h} f_{ph} \log_2 f_{ph} & (6.3.3) \\ B_{pl} = 1/\max\{f_{p1},\ldots,f_{ph},\ldots,f_{pn}\} & (6.3.4) \\ \hline \end{array} \qquad (6.3)$$

Similarly to the basic model, under linear assumptions v_P, of every option P can be represented by a weighted value function:

$$\boxed{v_P = \sum_{\text{all } i} w_i . A_{Pi} + \sum_{\text{all } l} w'_l . B_{Pl}} \qquad (6.4)$$

The effective computational task is the same as in the basic method; that is, for any portfolio, find that value function which shows the portfolio in the best possible light, subject to no feasible portfolio having an assigned value of greater than one, when evaluated using the same value function. Thus, paralleling the formulation in Basic Dora-D, we specify the task of finding the MCA and CAF of a portfolio (where no prior value indicating preference constraints have been specified), as:

$$\begin{array}{|llll|}\hline \multicolumn{4}{|l|}{\text{For each potential portfolio } P \in \mathbb{P}} \\ \text{Maximise} & v_P = \sum_{\text{all } i} w_{iP} . A_{Pi} + \sum_{\text{all } l} w'_{lP} . B_{Pl} & & (6.5.1) \\ \text{Subject to} & \sum_{\text{all } i} w_{iP} . A_{pi} + \sum_{\text{all } l} w'_{lP} . B_{pl} \le 1 & \forall p \in \mathbb{P} & (6.5.2) \\ & \sum_{\text{all } h} f_{ph} = 1 & \forall p \in \mathbb{P} & (6.5.3) \\ & w_{iP} \ge \alpha_i & \forall i \in \{1,\ldots,k\} & (6.5.4) \\ & w'_{lP} \ge \alpha'_l & \forall l \in \{1,\ldots,e\} & (6.5.5) \\ & & & \\ \multicolumn{4}{|r|}{\text{Where } \alpha_i, \alpha'_l = \text{a positive (archimedien) number or zero}} \\ \multicolumn{4}{|r|}{\mathbb{P} = \text{the set of all feasible portfolios}} \\ \multicolumn{4}{|r|}{\text{Other variables and parameters are as defined above}} \\ \hline \end{array} \qquad (6.5)$$

The reader should note that, whilst in Basic Dora-D I favour the exclusion of the option under consideration from the Comparison Set of other options, it is generally here neither practicable nor meaningful to exclude the portfolio under consideration, P, from $\mathbb{P}$. Constraints of the form of (6.5.2) are referred to as Frontier Constraints in distinction to Preference Constraints which may also be included.

Alternatively, or additionally, one may specify constraints on the B_{pl} or directly constrain the f_{ph}, eg:

$$\boxed{\begin{array}{ll} c_L \le \sum_{\text{all } h} f_{ph} \log_2 f_{ph} \le c_U \text{ or} & (6.6.1) \\ f_{ph} = \{0,[c_1,c_2]\} & (6.6.2) \end{array}} \qquad (6.6)$$

On the face of it such structure suggests formidable solution problems. There is an infinite number of portfolios for which to find MCAs and there is an infinite number of alternative portfolio value constraints involved in determining for each of those solutions. It is the non-linearity of the B_{pl} in the f_{ph} that presents the problem. Were there to be no such feature, this problem would condense to the Basic Method.

6.4 Introducing Frontier Probing

The characteristic of having an infinite number of candidate portfolios to evaluate is an issue that we cannot directly resolve, but here I outline an approach to limiting the otherwise infinite number of constraints on the valuation of the alternative portfolios in the Comparison Set, against which a test portfolio can be assessed.

It is a trivial observation that constraint equations only need to be included in formulations if they would otherwise be violated. Indeed, the simplex method of linear programming may be considered as simply a device for determining which constraints are relevant for the particular objective function being optimised. It ceases to be trivial if one can identify, from within the infinite or unmanageable number of potential constraints, a manageable number which are relevant or, as I will now refer to them, critical.

Such critical constraints might be found by seeking violations and introducing constraints where observed. Nothing is gained if one must explicitly search an infinite number of constraints. However, it may be possible on any occasion simply to identify the "most critical" constraint , to make it explicit, and repeat this process until the measured violation of successive "most critical" constraints are no longer material. The iterative procedure below solves this problem for the structure outlined above.

In this procedure an almost completely relaxed set of Frontier Constraints is initially specified: metaphorically, the frontier is poorly guarded and allows illicit violation. Non-linear programming is used to find "most critical" constraints by seeking constraints from the infinite set which gives rise to maximum violation: that is, probed to find how far into illicit territory it is possible to go. An explicit constraint is then interposed: in the metaphor, blocking-off that portion of the frontier that was successfully penetrated. This process can be formalised as follows:

Partition the infinite set of Frontier Constraints specifying the maximum values that *each* feasible portfolio may have, $\mathfrak{C}$, into two disjoint sub-sets;

specified constraints, a finite set, $\mathfrak{C}_S$, and

unspecified constraints, an infinite set, $\mathfrak{C}_U$,

ie

$$\mathfrak{C}_S \cup, \mathfrak{C}_U = \mathfrak{C}, \mathfrak{C}_S \cap \mathfrak{C}_U = \varnothing \tag{6.7.1}$$

Specify a test portfolio P.

Find the constraint C_M within set $\mathfrak{C}_U$ that has maximum violation

given that v_P is maximised for the test portfolio s.t. $\mathfrak{C}_S$,

ie

$$C_M = \{C_i \in \mathfrak{C}_U : (\mathrm{V}(C_i) = \max_{\forall C_j \in \mathfrak{C}_U} (\mathrm{V}(C_j)) : (\max v_P : \mathfrak{C}_S)\} \tag{6.7.2}$$

Where $\mathrm{V}(C_x) =$ the violation of constraint C_x,

measured in the same units as the constraint definition.

C_M is the constraint corresponding to the portfolio with maximum value using the weights corresponding to $\max v_P$, and $\mathrm{V}(C_M) = \max_{\forall C_j \in \mathfrak{C}_U} (\mathrm{V}(C_j))$. Its violation is the assessed value of that portfolio, using the same weights, less 1.

Form new sets of specified constraints $\mathfrak{C}'_S$, and unspecified constraints, $\mathfrak{C}'_U$

by adding C_m to $\mathfrak{C}_S$, and removing C_m from $\mathfrak{C}_U$, ie

$$\mathfrak{C}'_S = \mathfrak{C}_S \cup C_m \text{ and } \mathfrak{C}'_U = \mathfrak{C}_U \setminus C_m \tag{6.7.3}$$

(6.7)

The sets $\mathfrak{C}'_S$, $\mathfrak{C}'_U$ are then renamed respectively $\mathfrak{C}_S$ and $\mathfrak{C}_U$ and the procedure is repeated. The specification of a new $\mathfrak{C}_S$ alters the $\mathrm{V}(C_M)$ in an unsteady way, when v_P is maximised within the new $\mathfrak{C}_S$. The procedure is therefore repeated until $\mathrm{V}(C_M) \leq \delta$ over several iterations, where δ is a number such that violations are considered immaterial. It is then, subject to observations later in this section, stopped. At this point there are no relevant constraints within $\mathfrak{C}_U$.

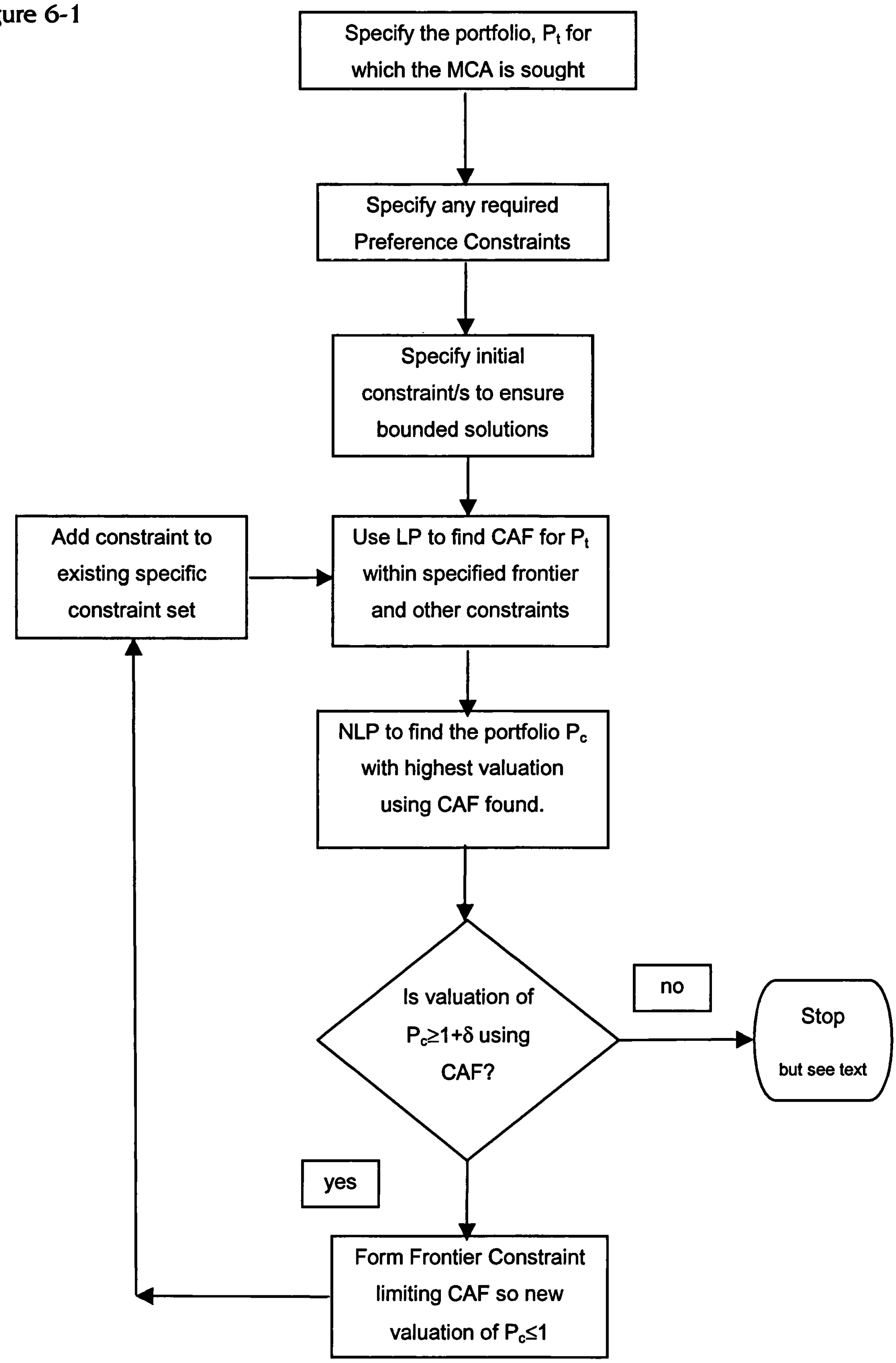
It should be noted that this involves only a one way movement from a partition of unspecified constraints into a set of specified ones. Not all the constraints migrating in this way typically remain critical and accordingly there may be immaterial constraints within the explicit set $\mathfrak{C}_S$. However, the essential issue is that the total number of explicit frontier constraints within this set will be finite (and should be moderate). Although there remains an infinite number within $\mathfrak{C}_U$, the point when none are material is reached quite quickly. At least one constraint (just one is sufficient if it involves non-zero finite weight of all potential components) is required to be in the initial set $\mathfrak{C}_S$ to prevent an unbounded solution to the first calculation of $\max v_P : \mathfrak{C}_S$.

The heuristic of selecting the constraint with maximum violation on each occasion may be supposed to generate a reasonably effectual set of material constraints (though it is unlikely that it would constitute the most parsimonious set). It may be imagined as generating a minimalist set of tangent hyperplanes which collectively form an envelope of the continuous hyper-surface defining efficient portfolios in the relevant region.

This recursive approach particularly lends itself to the portfolio structure already outlined and it is illustrated in Figure 6-1. The weights of attributes that are nominally optimal, within the specified frontier constraints, for a particular test portfolio can be used to evaluate any other portfolio. Any portfolio found which has a valuation of above 1, using this set of weights, indicates a violation with respect to the unspecified constraints. Mathematical Programming is used straightforwardly to find the highest valued portfolio and hence, most violating portfolio, from amongst these. This portfolio corresponds to the most critical constraint C_M. That opportunity for violation is "blocked" by adding C_M explicitly to the specified set $\mathfrak{C}_S$ and the frontier is "probed" again for other violations. The process continues until a "stop" judgement is made.

The procedure enables the identification of the MCA, CAF and the Efficient Peer portfolio of any portfolio P_t it is desired to test. Although described without the incorporation of Preference or Value Constraints, such constraints and any other solution restricting constraints may be simultaneously incorporated. The ability to accommodate Preference Constraints within the formulation is suggested as an attractive feature of this approach.

Figure 6-1



The routine illustrated in Figure 6-1 finds the MCA and CAF of any pre-specified initial portfolio under quite complex conditions. By finding the portfolio with the maximum MCA with the CAF that shows the Test Portfolio in its Best Possible Light, within the identified critical constraints, one identifies a very close approximation to its efficient peer. It should be emphasised that, although in most structures it will not be the exact peer, it will *always* be fully efficient, and this quality does not depend on the cut-off point; *every* critical constraint found corresponds to an efficient portfolio. Efficient portfolios which are near peers are incidentally identified in the analysis

Some words of caution are appropriate. As constraints are introduced and the CAF is refined, the calculated valuation of the test portfolio reduces on *each* cycle and relatively quickly tends to an asymptotic value. However, although the extent of violation δ found for the most critical constraint on each cycle generally moves towards zero, it does not follow that in a subsequent cycle, using a different evaluation function, δ cannot be higher. The user can legitimately strike a level of immateriality for δ but must confirm that it is not exceeded for a number of cycles (I suggest a number of cycles comparable to the number of included attributes). Users can also pragmatically observe how the characteristics of the critical constraints change. These tend to be closely related as the degree of violation diminishes.

The reader should also note that whilst one part of each cycle is an LP (ie finding the CAF of the Test Portfolio), the generation of a critical portfolio will generally be a non-linear problem. Towards the end of an analysis series, the user should take care to ensure the software package used does not lock into a sub-optimal solution, by testing different NLP initial solutions. (Using What's Best, I found that if initial component proportions were somewhat greater than zero, and less than one, for all components, problems were rare).

It will be noted that in this formulation we do no more, at this stage, than find the efficient peer and the value of *one* explicitly stated initial portfolio. Despite the number of efficient portfolios being orders less than the number of possible portfolios, they remain infinite in number. However, in a decision situation the gain made is important and the function developed can form the basis of value elicitation to generate an optimum, by introducing elicited preference constraints to progressively reduce CAF latitude, just as in the basic method.

Moreover, in many circumstances an initial solution may be a subjective but sound nominee for selection as a practical portfolio and indicative of a decision maker's values. Accordingly, its efficient peer might be an appropriate superior (in the absence of further information, optimal) selection. Frontier probing then effectively answers a decision problem stylised as, "What is the optimal portfolio corresponding to the CAF of this suggested portfolio, subject to conformity with the following additional statements of preference, ..?". If the preferences fully constrain the CAF, the initial nomination does not matter. If there is latitude, the initial solution will fix a specific CAF and optimum solution, whether a decision maker chooses to express no, few or many additional preference constraints.

The use of "pet" solutions as indicators of value will be further discussed in Chapter 7.

6.5 An example

To illustrate the method, consider a situation where we seek to form portfolios from 50 potential components. Associated with each of these are differing magnitudes of 5 additive attributes, A1,..,A5. These combine pro rata with the fraction of each component in the portfolio as discussed, to generate the magnitude of that attribute within the portfolio. Attribute magnitudes for each component are as in Table 6.1.

There is one further interdependent attribute, B_p, associated with a portfolio constituted by these, which is a non-linear function of the proportions of each component included in the portfolio; ie:

$$\boxed{B_p = 1 - \sqrt{\sum_{\text{all } h} f_{ph}^2}} \tag{6.8}$$

		Table 6.1 Attribute Magnitudes for Potential Components					
		Attribute					Final
		A1	A2	A3	A4	A5	Mix
Component	1	0.421617	2.577996	0.210297	1.886558	1.627006	2.2%
	2	0.85464	1.894861	0.153858	-0.41216	0.261113	1.4%
	3	3.028773	-0.62774	1.211986	1.58146	1.41028	2.8%
	4	-0.63005	-0.18004	0.88057	-0.15807	1.706016	1.1%
	5	1.431091	2.446579	-0.27216	2.176979	1.332321	2.6%
	6	2.074086	1.639907	2.557087	0.178121	-0.20893	2.0%
	7	0.64463	-0.92866	0.636306	-0.28376	-1.35312	1.4%
	8	-0.73108	1.475175	1.016333	1.476973	-0.71422	1.7%
	9	2.678422	0.446666	0.218628	0.397301	-0.07177	2.3%
	10	1.079067	-0.19506	0.421254	1.785324	1.037148	2.3%
	11	1.950481	0.732582	1.338229	-0.47254	2.537337	1.7%
	12	0.694562	0.692959	0.338443	-1.37695	-0.19584	1.0%
	13	2.505223	1.501461	0.962622	1.094424	1.472007	2.5%
	14	1.076458	1.85493	0.87425	1.607017	2.274748	2.3%
	15	1.904263	0.15342	0.26923	0.430366	-0.27613	2.1%
	16	1.734171	3.087236	1.648946	2.128951	2.515755	2.7%
	17	1.484014	1.149537	1.362469	0.760075	0.256682	2.1%
	18	-0.09558	1.384864	1.945445	-0.19757	0.654959	1.2%
	19	4.401074	0.716213	0.727346	1.815355	-0.37855	3.3%
	20	1.232452	1.992793	0.76283	0.983743	0.23082	2.1%
	21	1.352845	2.235073	1.96903	-2.14729	1.552977	0.9%
	22	0.12337	0.874019	-0.40968	1.842647	1.874049	2.1%
	23	0.046149	0.090196	1.428993	0.514869	1.235833	1.6%
	24	1.193947	2.291828	-1.14614	1.620964	2.459553	2.3%
	25	0.361031	0.102281	2.004507	0.061588	-1.8107	1.5%
	26	-0.52377	-0.90184	2.316901	-0.03278	1.543198	1.2%
	27	1.656519	1.133541	0.900118	1.331432	0.392523	2.3%
	28	-0.41924	2.074902	0.284926	0.678089	0.595952	1.5%
	29	0.711911	0.957339	2.470967	2.706289	1.160065	2.6%
	30	0.352565	0.867616	0.810494	2.366384	1.061582	2.4%
	31	2.224805	2.029007	0.147053	-0.25618	-0.0734	1.9%
	32	3.232609	0.780105	1.939599	1.148453	1.876405	2.7%
	33	-0.06123	1.13717	0.503823	-0.07232	1.478602	1.3%
	34	1.328928	0.576033	0.054676	0.168756	1.69341	1.8%
	35	0.415365	1.051544	-0.29837	0.555523	1.438069	1.7%
	36	0.386348	1.448873	0.860977	0.582385	-0.45513	1.7%
	37	1.080141	0.129197	0.328509	3.783854	0.659016	3.1%
	38	0.255068	0.612664	1.622913	1.279648	1.201358	1.9%
	39	0.926688	3.608504	1.231666	2.216106	0.895043	2.5%
	40	-0.15733	1.451582	-0.27148	0.597861	0.338063	1.5%
	41	1.267021	1.353172	1.818345	-0.33365	1.959049	1.6%
	42	1.781267	1.676099	3.601664	0.86476	-0.05241	2.2%
	43	2.338321	1.408118	0.630579	0.359811	1.161383	2.2%
	44	1.722503	-1.03117	0.228658	1.234731	0.868464	2.3%
	45	0.460344	-0.09516	-0.2822	0.33186	1.366229	1.6%
	46	1.802199	-0.19054	-0.92718	-0.26498	0.968363	1.8%
	47	1.14768	2.85797	2.887111	1.974551	0.841484	2.4%
	48	1.988671	1.409864	0.554847	0.776106	2.591152	2.2%
	49	2.022536	1.276706	0.816877	1.386428	0.914178	2.5%
	50	0.279482	0.055751	1.207294	1.104264	-0.63149	1.9%

We must choose a portfolio for examination (the Test Portfolio) which I define to consist of 0.1 of each of the components 1 to 10. This portfolio has the following portfolio level attributes:

$$\boxed{B_T = 0.684 \qquad \mathbf{A}_T = \{1.085, 0.855, 0.703, 0.863, 0.503\}} \tag{6.9}$$

The initial task is to find the efficient peer of this portfolio.

We must also define an arbitrary portfolio to serve as the basis for a bounding constraint. This may be any real portfolio having positive magnitudes in all attributes, it is not ultimately material. In this case I use the portfolio constituted by all portfolio components in equal proportions. Its attributes are:

$$\boxed{B_t = 0.859 \qquad \mathbf{A}_t = \{1.141, 1.062, 0.891, 0.836, 0.864\}} \tag{6.10}$$

the corresponding constraint is:

$$\begin{array}{|l|}\hline 0.859w'_T + 1.141w_{1T} + 1.062w_{2T} + 0.891w_{3T} + 0.836w_{4T} + 0.864w_{5T} \le 1 \\ \qquad \text{Where } w'_T = \text{weight of Portfolio attribute } B_T \\ \qquad\qquad w_{iT} = \text{weight of attribute } A_{iT} \text{ derived pro-rata from the} \\ \qquad\qquad\qquad \text{attributes of the components forming portfolio } T \\ \hline \end{array} \tag{6.11}$$

Using LP, we now seek the weights which maximise the value of the test portfolio, ie:

$$\begin{array}{|rl|}\hline \text{Maximise } v_T = & 0.684w'_T + 1.085w_{1T} + 0.855w_{2T} + 0.703w_{3T} + 0.863w_{4T} + 0.503w_{5T} \\ \text{Subject to } & 0.859w'_T + 1.141w_{1T} + 1.062w_{2T} + 0.891w_{3T} + 0.836w_{4T} + 0.864w_{5T} \le 1 \\ & w_{iT} \ge 0 \qquad\qquad \forall i \in \{1,\ldots,5\} \\ \hline \end{array} \tag{6.12}$$

The weights which maximise this are:

$$\boxed{w'_T = 0 \qquad \mathbf{w}_T = \{0,0,0,1.197,0\}} \tag{6.13}$$

We now seek the portfolio with the maximum value using the weight set:

$$\begin{array}{|llll|}\hline & w'_T = 0 & \mathbf{w}_T = \{0,0,0,1.197,0\} & (6.14.1) \\ \text{ie:} & & & \\ \text{Maximise} & v_P = & 0.B_P + 0.A_{P1} + 0.A_{P2} + 0.A_{P3} + 1.197A_{P4} + 0.A_{P5} & (6.14.2) \\ & A_{pi} = & \sum_{h=1 \text{ to } 50} f_{ph} a_{ih}, \qquad \forall i \in \{1,\ldots,5\} & (6.14.3) \\ & B_p = 1 - \sqrt{\sum_{\text{all } h} f_{ph}^2} & & (6.14.4) \\ & \sum_{\text{all } h} f_{ph} = 1 & & (6.14.5) \\ & f_{ph} \ge 0 & \forall h \in \{1,\ldots,50\} & (6.14.6) \\ \hline \end{array} \tag{6.14}$$

In principle, this is an NLP though the solution is trivial in this instance. The maximum value portfolio consists entirely of the component having the highest value of attribute 4; that is potential component 37. The value of this portfolio calculated using the weights given is 4.528, which is a massive violation of the virtual constraint requiring that no weights shall cause any portfolio to have a value of greater than 1. The attribute values of a portfolio consisting entirely of potential component 37 are:

$$\boxed{B_t = 0 \qquad \mathbf{A}_t = \{1.080, 0.129, 0.329, 3.784, 0.659\}} \tag{6.15}$$

This implies the need for an explicit frontier constraint:

$$\boxed{0.w'_T + 1.080w_{1T} + 0.129w_{2T} + 0.329w_{3T} + 3.784w_{4T} + 0.659w_{5T} \leq 1} \tag{6.16}$$

to ensure that no weights can be suggested which will let that portfolio violate again. We then again find the weights which maximise the value of the test portfolio, using the now extended LP:

$$\boxed{\begin{array}{rll} \text{Maximise } v_T = & 0.684w'_T + 1.085w_{1T} + 0.855w_{2T} + 0.703w_{3T} + 0.863w_{4T} + 0.503w_{5T} & (6.17.1) \\ \text{Subject to} & 0.859w'_T + 1.141w_{1T} + 1.062w_{2T} + 0.891w_{3T} + 0.836w_{4T} + 0.864w_{5T} \leq 1 & (6.17.2) \\ & 0.w'_T + 1.080w_{1T} + 0.129w_{2T} + 0.329w_{3T} + 3.784w_{4T} + 0.659w_{5T} \leq 1 & (6.17.3) \\ & w_{iT} \geq 0 \qquad\qquad i \in \{1,\ldots,5\} & (6.17.4) \end{array}} \tag{6.17}$$

The procedure is continued, recalculating the weights of the test portfolio and progressively adding in critical frontier constraints, as tabulated in Table 6.2. The weights of the eventual CAF of the test portfolio and the attribute values corresponding to its efficient peer correspond to the last row of Table 6.2. The mix of components of the portfolio is shown as "Final Mix" in Table 6.1. The MCA of the test portfolio is 0.802, which we can interpret as its value relative to its efficient peer.

We can go further by adding value constraints reflecting decision maker's preferences as in the Basic Method and (or to add further Preference Constraints, as it is also legitimate to include such constraints *ab initio*). Nevertheless, if the Test Portfolio were to be a well-considered reflection of his or her values, it might be acceptable to halt the procedure here. In this instance the test portfolio was an arbitrary selection with no preference-indicating value constraints. A high weight was assigned to variable B, and this might be expected given the character of the initial portfolio.

Table 6.2 Illustrating Frontier Probing

	Part 1 - The Optimal Attribute Weights of Test Portfolio						Value of T'st P'folio with Given Weights	Part 2 - Attribute Values of Highest Value Portfolio with Given Weights						Value of this P'folio with Given Weights	Violation
	Attribute B	Attribute A1	Attribute A2	Attribute A3	Attribute A4	Attribute A5		Attribute B	Attribute A1	Attribute A2	Attribute A3	Attribute A4	Attribute A5		
Cycle 1	0	0	0	0	1.1967	0	1.0326	0	1.0801	0.1292	0.3285	3.7839	0.6590	4.5281	3.5281
Cycle 2	0	0.8637	0	0	0.0177	0	0.9525	0	4.4011	0.7162	0.7273	1.8154	-0.3785	3.8332	2.8332
Cycle 3	0.7667	0.1340	0	0	0.2260	0	0.8647	0.6262	2.2738	0.9918	1.0198	2.3131	0.8886	1.3076	0.3076
Cycle 4	0.9153	0.1877	0	0	0	0	0.8296	0.7057	2.7724	0.8445	0.9847	0.9770	0.7928	1.1663	0.1663
Cycle 5	0.9682	0.0830	0	0	0.0886	0	0.8286	0.7938	1.9257	1.1860	0.9609	1.6251	1.0151	1.0724	0.0724
Cycle 6	1.0786	0	0	0	0.0885	0	0.8139	0.8291	1.2135	1.2158	0.8906	1.5402	0.9725	1.0306	0.0306
Cycle 7	1.0626	0.0480	0	0	0.0394	0	0.8127	0.8410	1.5700	1.1324	0.8971	1.2090	0.9308	1.0166	0.0166
Cycle 8	1.1125	0.0057	0	0	0.0459	0	0.8065	0.8509	1.2152	1.1212	0.8845	1.1896	0.9129	1.0081	0.0081
Cycle 9	1.1076	0.0300	0	0	0.0178	0	0.8052	0.8542	1.3718	1.0920	0.8955	0.9893	0.8941	1.0048	0.0048
Cycle 10	1.0800	0.0280	0	0	0.0395	0	0.8030	0.8490	1.3868	1.1219	0.8914	1.1704	0.9179	1.0020	0.0020
Cycle 11	1.1099	0.0159	0	0	0.0305	0	0.8025	0.8543	1.2774	1.1038	0.8898	1.0745	0.9009	1.0012	0.0012
Cycle 12	1.0931	0.0274	0	0	0.0291	0	0.8022	0.8524	1.3664	1.1064	0.8930	1.0783	0.9052	1.0005	0.0005
Cycle 13	1.1083	0.0221	0	0	0.0232	0	0.8019	0.8548	1.3176	1.0965	0.8926	1.0241	0.8962	1.0003	0.0003
Cycle 14	1.1016	0.0218	0	0	0.0290	0	0.8019	0.8536	1.3212	1.1041	0.8915	1.0699	0.9022	1.0001	0.0001

6.6 Concavity of value of compound attributes

It is desirable that functions defining values of portfolio attributes incorporating non-linear terms in the proportions of potential components, should be concave, ie:

$$\boxed{\begin{array}{l} \qquad\qquad B(\mathbf{x}_3) \geq p.B(\mathbf{x}_1) + (1-p).B(\mathbf{x}_2) \\ \text{Given that} \quad \mathbf{x}_3 = p.\mathbf{x}_1 + (1-p).\mathbf{x}_2, \quad \forall \mathbf{x}_1, \mathbf{x}_2 \in \{\text{all feasible } \mathbf{x}\}; \forall p \in (0,1) \end{array}} \tag{6.18}$$

I conjecture that, under these conditions, either the procedure converges to a unique efficient peer portfolio or, if there is more than one efficient portfolio described by a vector, then the convex combination of any of them will also be efficient.

In financial portfolios diversity is desirable; implying that the value of a portfolio of any convex combination of other portfolios will be more valuable than, or equal to, the weighted average (found using the same proportions) of the values of the constituent portfolios. Functions meeting the above condition are necessary to reflect this.

Practical portfolios problems calling for a convex value form would be very unusual but could be meaningful in some situations (a premium could be placed on polarisation, for instance). In these circumstances the technique could still be used. However, local optima would exist. The technique should find at least one locally efficient peer portfolios of a test portfolio and (if a series of alternative starting solutions were experimented with) possibly all the material ones. To reduce the possibility of perverse solutions in such cases, it may be desirable to impose an additional constraint that the magnitude of any attribute shall not be less than the magnitude of that attribute in the test portfolio.

Frontier probing is the basis of the solution method used to address the Core Problem, which is described in Chapter 8.

Chapter 7 Other methodological extensions

7.1 Introduction

In this chapter I examine further features which can be used in association with Dora-D to ameliorate the problems of the impaired decision maker, to cope with more complex valuation or to simplify analysis. I also discuss further extensions to cover additional problem structures.

I start by highlighting some alternative methods for handling complex decision problems. I highlight two, Decomposition and Holistic Integration, for further exploration within the chapter. Looking first at decomposition, I observe that first one can break down decision selections into pairwise choices. Such binary choices themselves represent situations of varying complication and, as a prelude to their simplification, suggest a method of classifying them using an "[m,n] Complexity Indicator".

I move on to discuss how choices can be partitioned into groups of sub-choices of reduced complexity depending on mutual preferential independence. I mention the limits to simplification in partitioning problems and discuss the circumstances in which expressions of preference, relating to sub-choices, can imply a preference for one of the two options in an undecomposed pair.

Using the defined Complexity Indicator I refer to the most structurally simple case, classified [1,1], which I refer to as a Fundamentally Decomposed Choice. This figures in the discussion in a variety of ways later in the chapter.

I then discuss Franklin's Prudential Algebra as an example of decomposition. Based on his straightforward conceptualisation, I develop a modernised algorithm. Here I attempt to partition the problem of selection between a binary pair of options into series of partitions, limited to three of the most structurally simple choice types, for which I suggest we are most likely to be able to express reliable preference. I do this in a way which is designed to maximise the prospects of drawing a firm conclusion regarding the whole, from views expressed about the partitions. This I call Franklin Decomposition.

I also mention an approach which I call Larichev Decomposition, which only makes use of [1,1] choices. These are developed for a particular decision but are derived from the decomposition of sets of efficient options rather than individual options. I go on to describe a methodology for doing this in a way which allows the preferences expressed to be converted into value constraints in Dora-D. I also describe how the information declared in expressing preferences between Franklin Decompositions, can be used to reduce value Latitude and other potential optima, not just the options subject to the decomposition.

I then take the alternative perspective and show how holistic selections might be improved in a Dora-D framework.

I also talk about how the scale of a selection problem could possibly be reduced by only considering options which an impaired decision maker might reliably discriminate in value terms. The Representative Efficient Set is introduced. This concept reflects the ideas of Principal Components Analysis (though depending on completely different mechanisms).

I then conceptually consider how four further types of problem structure can be accommodated within the approach being presented. First is the problem of multi-attribute decisions under constraints, the type of problem that might otherwise be formulated in MOLP terms. An illustrative example is presented.

The second is a consideration of how configural valuation can be brought within the ambit of the Dora-D structure. Particular consideration is given to the treatment of the Modified Minkowski Metric introduced in Chapter 2.

I then consider the analysis of project portfolio selection problems, using as an example a problem already examined by other authors. I finally examine the translation of "voting" data of the type generated in group decision making or social choice into Dora-D structure. Cook and Kress (1990) has tackled this problem with a data envelopment approach. The formulation suggested is little different, but is somewhat closer to the principles of Chapter 5, and, offers alternative insights.

7.2 Outlooks of Decision Analysis revisited

Within the model of an impaired decision maker with vague objectives, one can perhaps envisage and discern three viable mechanisms for decision making and scrutiny:

(a) Decomposition
(b) Simplification
(c) Holistic Integration

One can perceive the essence of decomposition as the partition of the problem into more cognitively manageable bites in a conscious, or largely conscious, process. Simplification, inevitably forced upon us by our bounded rationality, is a recasting process, which may be imagined as a form of decomposition in which some elements are discarded as unimportant. Holistic integration is a leap through unconscious or partly unconscious process directly to an inspired or intuitive solution, but subject to conscious validation.

We can also reasonably assume that in an unaided process each of these may be undertaken adequately or poorly, but, in any case, imperfectly. Moreover, formal decision analysis assistance will not enable perfection but may improve all. In the first case a decision maker can be aided to cognitively reliable or more complete decompositions; forms of expression could be invoked that can be more reliably analysed, partitions that can be more safely discarded might be identified, or deductions may be made more reliably from the decomposed problem than a decision maker would him or herself make. In the last we might, perhaps, induce understanding of a decision maker's values that led to a specific proposition and deduce a better suggestion using the information.

We can also envisage a composite analysis which starts with an assumption of holistic integration and improves the decision making process by decomposition.

I discuss how ideas relating to decomposition and holistic integration can be effected or exploited within the Dora-D framework.

7.3 Classifying complexity of paired choices

Decomposition as a decision analysis process is a mechanism for breaking down a complicated multiple factor preference problem, about which the decision maker may only be able to make unreliable direct statements, into a series of other preferences which are easier, within his or her cognitive capabilities, to discriminate meaningfully. Then to reconstitute such preferences into a preference inference relating to the whole.

A selection between pairs of options is a straightforward first stage decomposition to which all disjoint multiple option situations can be reduced. Much of the succeeding discussion is therefore centred on paired comparisons. In this discussion I make a distinction between an option and a choice. An Option, which is a special case of a Choice, constitutes a package of attributes corresponding to an actual decision possibility. A Choice constitutes a package of attributes about which a decision maker might be asked to venture a value opinion, but is not necessarily an available implementable alternative. A decision maker cannot express a preference *for* a set of attributes but can express a preference *between* binary sets of attributes, that is to say between choices.

Let us assume that the magnitude of all the relevant attributes of two choices differ in magnitude, but are defined on a scales oriented so that increases in the magnitude of any attribute, are always perceived as being more valuable. Let us further define the magnitude of the attribute for any choice as consisting of a base level b_i, being the lower of the magnitudes for attribute i (of n) for each choice, and an additional amount e_i.

We can then express the pairs of choices, for which preferences are to be expressed, in terms of attribute sets expressed thus:

$$\boxed{\begin{array}{r}\{b_1+e_{11},\ldots,b_i+e_{i1},\ldots,b_n+e_{n1}\},\{b_1+e_{12},\ldots,b_i+e_{i2},\ldots,b_n+e_{n2}\} \\ e_{i1},e_{i2}\in\{\mathbb{R}:e_{i1}\geq 0,e_{i2}\geq 0,e_{i1}e_{i2}=0\}\end{array}} \tag{7.1}$$

The last expression ensures that only one of e_{i1},e_{i2} can be non-zero

We are dealing here with only two choices. It is therefore legitimate to express the preference as a deviation from a base case $\{b_1,\ldots,b_i\ldots,b_n\}$, that is:

$$\boxed{\begin{array}{c}\{b_1+e_{11},...,b_i+e_{i1},...,b_n+e_{n1}\}\succ\{b_1+e_{12},...,b_i+e_{i2},...,b_n+e_{n2}\}\\ \leftrightarrow\{e_{11},...,e_{i1},...,e_{n1}\}\succ\{e_{12},...,e_{i2},...,e_{n2}\}\end{array}} \qquad (7.2)$$

We can therefore equivalently describe the pairs of choices as:

$$\boxed{\begin{array}{l}\{e_{11},...,e_{i1},...,e_{n1}\},\{e_{12},...,e_{i2},...,e_{n2}\} \qquad e_{i1},e_{i2}\in\{\mathbb{R}:e_{i1}\geq 0,e_{i2}\geq 0,e_{i1}e_{i2}=0\}\\ \qquad \text{Where } e_{hk}=\text{ magnitude of an attribute excess } h \text{ favouring option } k\in\{1,2\}\\ \qquad\qquad\qquad =0 \text{ if } k \text{ is not favoured}\end{array}} \qquad (7.3)$$

Each of these sets thus consists of positive numbers and zeros. If there is a zero in the first set there will be a positive number in the corresponding position of the second set and vice versa.

We can, perhaps, use this as a basis for classifying the complexity of choices. Within the class of binary expression can we identify "easier" choice problems? One way of ranking choice complexity might be through the number of attributes that differ in the binary choices presented. We may thus say that a problem varying in 10 attributes is more complex than one varying in 3. However, mere variety is not a sufficient indicator. A problem varying in ten dimensions becomes trivial if all the attributes favour a particular choice, as that choice dominates the other.

I suggest alternatively that we can characterise a choice by a pair of numbers indicating the number of positive $e_{i.}$ associated with the first option and the number associated with the second, eg [5,3]. The sum of the elements of this pair will be the number of attributes of unequal magnitude (n). I will call this pair the "Complexity Indicator". We can claim that a choice structure is more complex than another, if one or both elements of its Complexity Indicator is greater, and neither is less, than the elements of the Complexity Indicator of the other. Thus a [6,4] or [6,3] choice is inherently more complex than a [5,3] (or [3,5]) choice. The latter will also be referred to as a "Reduced Order Choices". Reduced Order refers to the characteristic where a binary [m',n'] choice has $m < m'$ and $n \leq n'$ when compared with a binary choice classified [m,n]. Higher order is the converse.

We can perhaps also argue, though from less solid ground, that a more polarised choice involving the same total of varying attributes is simpler than a less polarised one. Certainly, [4,0] is simpler than [2,2]; the first of these describes a dominating situation and involves no choice at all. However [3,1] might also be cognitively easier to deal with than [2,2]. Attention can be concentrated on a single attribute and a decision maker may feel better able to judge whether all other factors collectively outweigh the attribute or are outweighed by it, than to make the two versus two choice. If the reader is prepared to bear with this assumption, the three most simple non-trivial structures are [1,1], [1,2] and [1,3], The first two should be unexceptionable; [1,2] is of higher order than [1,1] but of lower order than *all* other possibilities. It is simply assumption that [1,3] is cognitively simpler than [2,2], its only contender. I will shortly discuss a decomposition methodology, dependent on cognitive competence in the discernment of preference of choices characterised by up to [1,3] complexity.

I will also give special consideration to the [1,1] case.

7.4 Decomposing options and choices into paired sub-choices

But the above formulation is not only a basis for classification.

Let us make the additional assumption that attributes are mutually preferentially independent (and, if not, recast them so that they are). We can partition elements from the above *single* pair of sets into *two* pairs of sets. We can do this by moving positive elements from a choice in the original pair (and the corresponding zero element of the other choice of the pair) into one or other of the two new pairs, placing zero's in both elements of the other pair, in such a manner that the sum of the corresponding elements of the two partitioned sets, is equal to the magnitude of the elements of the original sets. (We could of course meet the second condition by transferring part of an element to one and the residual to the other but there is no merit in anything but an exclusive transfer to one or the other). There may be many allowable partitions.

Thus for example we can partition a choice

{5,3,2,0,0} and {0,0,0,4,4}

into

{5,0,2,0,0} and {0,0,0,4,0}

and

{0,3,0,0,0} and {0,0,0,0,4}

or more generally:

$$\boxed{\{e_{11},\ldots,e_{i1},\ldots,e_{n1}\},\{e_{12},\ldots,e_{i2},\ldots,e_{n2}\}} \tag{7.4}$$

into

$$\boxed{\begin{array}{l}\{l_{111},\ldots,l_{i11},\ldots,l_{n11}\},\{l_{121},\ldots,l_{i21},\ldots,l_{n21}\}\\ \text{and}\\ \{l_{112},\ldots,l_{i12},\ldots,l_{n12}\},\{l_{122},\ldots,l_{i22},\ldots,l_{n22}\}\\ \qquad\qquad \text{Where } e_{ij} = l_{ij1}+l_{ij2},\ \ l_{ij1},l_{ij2}=0\end{array}} \tag{7.5}$$

It should be noted that where there is a zero in any position in the original set there will be a zero in that position in *both* partitions. Moreover, as a positive element in one of the original sets must be matched by a zero element in the corresponding position in the other, it follows that if a set representing a partitioned choice has a positive element, the corresponding element in all the other partitioned sets must be zero, as is the case in the example.

We can then say that:

$$\begin{array}{ll}\text{If } & \mathrm{E}_1 = \{e_{11},\ldots,e_{i1},\ldots,e_{n1}\}\ \ \mathrm{E}_2 = \{e_{12},\ldots,e_{i2},\ldots,e_{n2}\}\\ & \mathrm{E}_1' = \{l_{111},\ldots,l_{i11},\ldots,l_{n11}\}\ \ \mathrm{E}_2' = \{l_{121},\ldots,l_{i21},\ldots,l_{n21}\}\\ & \mathrm{E}_1'' = \{l_{112},\ldots,l_{i12},\ldots,l_{n12}\}\ \ \mathrm{E}_2'' = \{l_{122},\ldots,l_{i22},\ldots,l_{n22}\}\\ \text{Then} & \mathrm{E}_1' \text{ and } \mathrm{E}_1'' \text{ are partitions of } \mathrm{E}_1\\ \text{and} & \mathrm{E}_2' \text{ and } \mathrm{E}_2'' \text{ are partitions of } \mathrm{E}_2\end{array} \tag{7.6}$$

Let us now consider how preference implications can be drawn from such a structure.

$$
\begin{array}{|ll|}
\hline
\text{Let } \{a_1,0,0,0\} \succ \{0,0,a_3,0\} & (7.7.1) \\
\text{and } \{0,a_2,0,0\} \succsim \{0,0,0,a_4\} & (7.7.2) \\
\quad \{a_1,0,0,0\} \succ \{0,0,a_3,0\} \leftrightarrow \{a_1,a_2,0,0\} \succ \{0,a_2,a_3,0\} & \\
\qquad \text{from preferential independence} & (7.7.3) \\
\quad \{0,a_2,0,0\} \succsim \{0,0,0,a_4\} \rightarrow \{0,a_2,a_3,0\} \succsim \{0,0,a_3,a_4\} & \\
\qquad \text{from preferential independence} & (7.7.4) \\
\therefore \{a_1,a_2,0,0\} \succ \{0,0,a_3,a_4\} \quad \text{from (7.7.3) and (7.7.4)} & (7.7.5) \\
\qquad \text{It therefore follows} & \\
\quad \{a_1,0,0,0\} \succ \{0,0,a_3,0\} \cap \{0,a_2,0,0\} \succsim \{0,0,0,a_4\} & \\
\qquad \rightarrow \{a_1,a_2,0,0\} \succ \{0,0,a_3,a_4\} & (7.7.6) \\
\text{Moreover any or several } a \text{ can stand proxy for multiple elements} & \\
\text{eg } \{a_1,0,0,0\} \Rightarrow \{b_1,b_2,0,0,0,0\} \text{ and } \{0,a_2,0,0\} \Rightarrow \{0,0,b_3,b_4,0,0\} & \\
\qquad \text{permitting generalisation to} & \\
\qquad (E_1{}' \succ E_2{}') \cap (E_1{}'' \succsim E_2{}'') \rightarrow E_1 \succ E_2 & (7.7.7) \\
\hline
\end{array}
\qquad (7.7)
$$

That is to say that if the partitioned choice E_1' is preferred to E_2' and partitioned E_1'' is preferred to E_2'', then unpartioned E_1 is preferred to E_2. In the example

{5,0,2,0,0}$\succ${0,0,0,4,0} and {0,3,0,0,0}$\succ${0,0,0,0,4}

$\rightarrow${5,3,2,0,0}$\succ${0,0,0,4,4}

We can also similarly decompose value into independent additive components:

Assume there exist commodities v that can be held in association with attributes and over which a decision maker can express combined preference. We can define such a compound of attributes and commodities by a set

$$\{a_1,\dots,a_i,\dots,a_n;v_1,\dots,v_i,\dots,v_n\}$$

Assume there is a $v_i = v(a_i)$ such that the decision maker is indifferent between an attribute and corresponding commodity, ie:

$$\{0,\dots,a_i,\dots,0;0,\dots,0,\dots,0\} \sim \{0,\dots,0,\dots,0;0,\dots,v(a_i),\dots,0\} \quad \forall i \in \{1,\dots,n\} \qquad (7.8.1)$$

Subject to mutual preferential independence we can say

$$\{a_1,\dots,a_i,\dots,a_n;0,\dots,0,\dots,0\} \sim \{0,\dots,0,\dots,0;v(a_1),\dots,v(a_i),\dots,v(a_n)\} \qquad (7.8.2)$$

If the commodities are identical and conserved a decision maker will be both indifferent to their position in the set and indifferent between a sum of quantities in one position and the same quantum distributed, ie:

$$\{0,\dots,0,\dots,0;0,\dots,v_k,\dots,\dots,v_l,\dots0) \sim \{0,\dots,0,\dots,0;v_k,v_l,\dots,0,\dots,0,\dots,0)$$
$$\sim \{0,\dots,0,\dots,0;v_k+v_l,0,\dots,0,\dots,0,\dots,0) \qquad (7.8.3)$$

Thus

$$\{a_1,\dots,a_i,\dots,a_n;0,\dots,0,\dots,0\} \sim \{0,\dots,0,\dots,0;v(a_1),\dots,v(a_i),\dots,v(a_n)\}$$
$$\sim \{0,\dots,0,\dots,0; \sum_{i=1 \text{ to } n} v(a_i),0,\dots,0,\dots,0\} \qquad (7.8.4)$$

We can call the commodity "value" and rewrite the above

$$v(a_1,\dots,a_i,\dots,a_n) \rightarrow \sum_{i=1 \text{ to } n} v(a_i) \qquad (7.8.5)$$

(7.8)

The value of attribute differences for pairs of choices may be similarly disaggregated.

From this we can conclude that:

(a) We can simplify problems for which our cognitive facility may not allow a direct comparison of preference, by decomposing them into partitions of reduced order, from which a decision maker may make more reliable binary preference statements.

(b) We can relate binary preference to additive value allowing the attachment of some sort of scale of value, notwithstanding the previous objections to the objectivity of a unique scale of cardinal value, without an external standard.

The dependence of the existence of an additive value function on mutual preferential independence is a well known finding of decision analysis theory (eg see Keeney and Raiffa, 1976, p104), though they note its origin to Debreu (1959, p56). Keeney and Raiffa's style of illustration is somewhat different from the above.

However, the idea of relationship to a "conservable commodity", perhaps allows judgement of the validity of such concepts in the light of cognitive assumptions to be made more readily.

Two observations are necessary concerning the usefulness of partitioning. Firstly, direct decompositions are only useful if partitions can be matched up in a manner such that *all* partitions for one option are not inferior to one or more partitions of the other option, with all partitions in the second group being accounted for. Secondly, there are many ways of partitioning choices and, with care, partitions may be constructed in way which enables this to be established.

7.4.1 The idea of "Fundamentally Decomposed Preference", and Larichev Decomposition

The reader will note that any binary choice other than [m,1] can be partitioned into choices of reduced order and reduced complexity. All choices can thus be reduced ultimately into a series of [m,1] partitions: m may often be a small integer.

However, the [1,1] characterisation is of particular interest. Although he did not classify choices in this way, Larichev (1992) considers this is a preference form that allows psychologically reliable elicitation. Larichev and Moshkovich (1995) depend only on this in their ZAPROS technique, designed to evaluate options involving a limited number of discrete levels of several attributes, (though a rather different form of problem from that considered in this thesis).

We can in any case also view this choice structure from an alternative perspective. It is the basic, the most elementary, building block. If a decision maker cannot make meaningful distinctions between choices of this form, there would seem to be little else *structurally* that can be done. If a decision maker is not capable of distinguishing binary choices of this simplicity, we cannot find more revealing expressions which could be distinguished. Moreover, if a decision maker is not able to express a preference between such a pair of choices, we can more safely interpret this as implying choices of approximately equal value. We are less able to assert this with more complex choices where a failure to discriminate can be due to incapacity to express preference, rather than it not existing. Because of its bald simplicity we may suppose that bias in the expression of preference is also less likely, (though the framing of the expression of choices could still influence preference elicitation, even in these straightforward cases).

It is for this reason that I refer to the reduction of choices to such [1,1] expressions of preference as "Fundamentally Decomposed Choice" and the concept as "Fundamentally Decomposed Preference". I refer to any reduction of an *overall* decision problem into fundamentally decomposed binary choices for selection by a decision maker as Larichev Decomposition. In naming it so I should stress that this naming (as well as Franklin Decomposition, referred to shortly) is intended to indicate and honour the provenance of the ideas and distinguish alternative methods. Neither person, as far as I am aware, explicitly described their approaches as decomposition nor sought to classify decision decomposition.

I would comment parenthetically that in the decision structures that Larichev and Moshkovich (op cit) consider, they are not generally able to generate full linear orders when depending only on such relationships (and the assumption of mutual preferential independence). However, allied with a structural relationship between attribute magnitude and value, we can be more ambitious. If we can find the right [1,1] comparisons, it may be possible to define the relevant value structure fully and economically. I will describe shortly how this might be used in the Dora-D framework.

7.4.2 Prudential Algebra and Franklin Decomposition

Benjamin Franklin in writing to Joseph Priestly in 1772 described what he called "moral or prudential algebra". I reproduce the full text of his letter in Appendix A. The letter is often quoted but perhaps not taken sufficiently seriously.

What Franklin effectively recommended was a procedure for a decomposing a single complex binary decision option into a series of simpler preference choices. Thus an [m,n] decision option could perhaps be decomposed to an [r,s] choice and an [m-r, n-s] choice. These can in principle be further decomposed. If each of these decompositions favours a particular option, (or a binary choice is viewed by the decision maker as presenting a choice of equal value), then that decision option should be preferred in its entirety. Franklin balances off pros and cons seeking to find attributes of one option which balance the "respective weights" of attributes of the other. He then "strikes out" those attributes and addresses his attention to those that remain until he can see "where the balance lies". It is instructive that he was prepared to balance three attributes against two, implying that he felt no cognitive inhibition in the evaluation of type [3,2] choices. However he possessed one of the most brilliant minds of his generation and I suggest that such choices should, if possible, be avoided. Franklin in seeking and eliminating balancing or, more strictly, approximately balancing, combinations, proposes a powerful pragmatic tool which should still be taken seriously as a decision aid today. It, nevertheless I suggest, can be improved by limiting complexity of choices posed to [1,1], [2,1] and [3,1] types and not seeking "balance" but merely that the decomposed choices on one side are all preferred to their alternatives.

I refer to the process of decomposing *individual* decisions in this style as Franklin decomposition (distinguishing it from Larichev decomposition which uses *only* Fundamentally Decomposed Preference but also focuses on the entirety of the decision situation, not necessarily individual decision pairs). We can turn the Franklin concept into a modernised decomposition algorithm, depending only on strong preference relationships and the three choice types above. The algorithm which can be looked upon as a special knapsack algorithm, is outlined below:

1. Consider two options. Hypothesise that one option A is preferred to the other B. Rank those attribute differences that favour A in order of perceived value, call these $A_1, \ldots, A_m$. Rank those attribute differences that favour B in order of perceived value, call these $B_1, \ldots, B_n$. Produce a single overall ranking, placing attributes of B below those attributes of A which are valued more highly and above those which are less highly valued.
2. Consider whether the highest ranked (or the remaining highest ranked) A_x is preferred to the highest ranked (or remaining highest ranked) B_x . If A_x is preferred to B_x , go to 3, if B_x is preferred to A_x, go to 5. If there are no remaining B_l but A_l remain, the hypothesis is validated. If there are remaining B_l but no A_l the hypothesis fails and the alternative that option B is preferred to A should be examined. Stop
3. Seek amongst the list of B_l (excluding B_x) the highest ranked B_y such that $A_x \succ B_x + B_y$. If none, then note the relationship $A_x \succ B_x$, exclude that A_x and B_x from further consideration, and repeat procedure from 1. If found, continue.

4. Seek from amongst the list of B_i (excluding B_x and B_y) the highest ranked B_z such that $A_x \succ B_x+B_y+B_z$. If none, then note the relationship $A_x \succ B_x+B_y$, exclude A_x, B_x, and B_y from further consideration. If found, note the relationship $A_x \succ B_x+B_y+B_z$, exclude A_x, B_x, B_y, B_z from further consideration. Repeat procedure from 1.
5. Find amongst the list of A_i (excluding A_x) the *lowest* ranked A_w such that $A_x+A_w \succ B_x$. If found, then note the relationship $A_x+A_w \succ B_x$, exclude that A_x, A_w and B_x from further consideration, and repeat procedure from 1. If none continue.
6. Find amongst the list of A_i (excluding A_x) the *highest* ranked A_y. Seek also the lowest ranked A_w such that $A_x+A_y+A_w \succ B_x$. If found, then note the relationship $A_x+A_y+A_w \succ B_x$, exclude that A_x, A_y, A_w and B_x from further consideration, and repeat procedure from 1. If not found, there are no three A_i remaining which outweigh B_x; the hypothesis fails. The alternative that option B is preferred to A should be examined. Stop

The algorithm in effect packs up to three "packages" B_j into one "knapsack" A_i, or shares the load of one "package" B_j amongst three "knapsacks" A_i, with minimum wasted "capacity". A worked example is shown in Figure 7.1.

It should be noted that both the hypotheses can fail. This may arise from one of three reasons:

(a) The overall preference cannot be determined within preference declarations of complexity [1,3] or less.

(b) The preferences are too close for the decision maker reliably to discriminate the strong preferences required.

(c) The heuristic is non-optimal. An alternative decomposition might discriminate.

With a little added complexity, the method could be extended to cope with (b). I will leave aside the resolution of other issues in this thesis. The purpose of the suggestion of this technique was not primarily to proposes another "stand-alone" decision aid for but to serve as an introduction to how such decomposition can be used in the Dora-D framework.

Figure 7.1 An example of Franklin Decomposition

In a decision situation involving 2 options A and B, the attribute differences are ordered by the decision maker's value of the attribute difference. Those favouring option A are designated A1 through A11, and those favouring B, B1-B10.

The decision maker orders his perceived valuation of the attribute differences as follows:

A1≻B1≻A2≻B2≻B3≻A3≻B4≻A4≻A5≻B5≻A6≻B6≻B7≻B8≻B9≻A7≻A8 ≻A9≻A10≻B10≻A11

Using the algorithm the decision maker pronounces

A1≻B1+B4+B10

Leaving

A2≻B2≻B3≻A3≻A4≻A5≻B5≻A6≻B6≻B7≻B8≻B9≻A7≻A8 ≻A9≻A10≻A11

Then A2≻B2

Leaving B3≻A3≻A4≻A5≻B5≻A6≻B6≻B7≻B8≻B9≻A7≻A8 ≻A9≻A10≻A11

Then A3+A11≻B3

Leaving A4≻A5≻B5≻A6≻B6≻B7≻B8≻B9≻A7≻A8 ≻A9≻A10

Then A4≻B5+B8

Leaving A5≻A6≻B6≻B7≻B9≻A7≻A8 ≻A9≻A10

Then A5≻B6

Leaving A6≻B7≻B9≻A7≻A8 ≻A9≻A10

Then A6≻B7 Leaving B9≻A7≻A8 ≻A9≻A10

7.4.3 Using Larichev Fundamentally Decomposed Preference in Dora-D

I have thus far in the description of the Basic Technique and its portfolio extension, presented the technique as a flexible mechanism that can exploit a variety of means by which preference information might be revealed. Nevertheless I have also argued that processing techniques should not just be processors of data taken willy-nilly at face value, but they should be compatible with reliable forms of elicitation from decision makers whose impaired cognitive systems were designed for another time. Some mechanics would seem to be reliable. Ordering of swing weights should be reliable because placements effectively involve only [1,1] type preferences. Capping should also be reliable. Though it does not express a binary preference, it depends on starkly revealed implications.

The expressed preference between real options, *if reliable*, can also be used with the technique, but the decision maker must be able to express meaningful choice. Unprocessed options will generally be [m,n] choices, which are difficult in principle, particularly when n and m exceed 3. They may not always be difficult in practice. Options within the efficient set will often have radical characteristics which can be used as basis of discrimination. Some options within a provisional ranking may also be obviously "out of order". However, many choices will present difficulty. It is also possible for mutually incompatible preferences amongst options to be expressed leading to LP infeasibility.

In this section I propose a method of generating artificial choices, bundles of attributes, for consideration by the decision maker, which:

(a) Depend only on [1,1] Fundamentally Decomposed Choices.

(b) Is intended to maximise discriminatory power.

(c) Largely avoids the risk of infeasibility.

The principle is to generate a set of efficient choices each differing from a base case only in the magnitude of one attribute. These choices are then ordered. The [1,1] quality is met by definition. Good discriminatory power is met by the insisting that, as far as possible, choices should be as close to efficiency as possible and the incidental implication that if [1,1] comparisons are made then the vectors being compared are mutually orthogonal with respect to the attributes. It is thought that this implies that the vectors can be ranked without risk of infeasibility, provided no other preference constraints have already been incorporated. However, as I shall discuss, a full set of such choices cannot always be ranked without risk of infeasibility, if other preference constraints have already been specified. (To demonstrate, one should note that a set of such constraints does not always reduce value latitude. It follows that a repetition of an already incorporated set of [1,1] choices might in this situation be proposed for decision maker ranking. A different ranking of any two of them would then generate an infeasible constraint). Nevertheless, in general, except where a previous set failed to secure reduction, some feasible comparisons should be found. In tests, problems did not arise until substantial reduction (ie where the number of options were comparable or smaller than the number of attributes) had already been achieved.

The procedure is described below in terms of its use with the basic Dora-D model but can also be used with the portfolio extension.

7.4.4 Finding [1,1] Choices

The reader will recall the basic formulation outlined in Chapter 5, which is reproduced here. The definitions remain as outlined there.

$$\boxed{\begin{array}{ll} \multicolumn{2}{l}{\text{For each decision option } S \in \{1,\dots,n\}} \\ \quad \text{Maximise } v_S = \sum_{\text{all } i} w_{iS}.a_{iS} & \\ \quad \text{Subject to} & \\ \qquad \sum_{\text{all } i} w_{iS}.a_{ij} \le 1 & \forall j \in \{1,\dots,n : j \ne S\} \\ \qquad\qquad w_{iS} \ge \alpha_i & \forall i \in \{1,\dots,k\} \end{array}} \tag{7.9}$$

Initial Option Reduction is first carried out, as previously described. This identifies initial efficient options, and their CAFs. The attributes of each efficient option can also be found. We now identify the best artificial choice which is dominated by all of them. We will call this the Best Dominated Choice (BDC). This is the vector of nadir attribute magnitudes; that is a choice corresponding to a package of attribute magnitudes, such that the magnitude of any attribute is equal to the lowest of the magnitudes for that attribute amongst all the efficient options, ie:

$$
\boxed{
\begin{array}{ll}
\text{Let} & \{a_{1J},\ldots,a_{iJ},\ldots,a_{kJ}\} \qquad \forall J\{1,\ldots,N\} \\
& \quad = \text{sets of attribute magnitudes for efficient option } J \\
& \quad\quad \text{of } N \text{ efficient options} \\
\text{Then} & \{a_{i1},\ldots,a_{iJ},\ldots,a_{iN}\} \qquad \forall i \in \{1,\ldots,k\} \\
& \quad = \text{sets of magnitudes of attribute } i \text{ over the} \\
& \quad\quad N \text{ efficient options} \\
\text{Then if} & \overline{a}_i = \min\{a_{i1},\ldots,a_{iJ},\ldots,a_{iN}\} \qquad \forall i \in \{1,\ldots,k\} \\
& \{\overline{a}_1,\ldots,\overline{a}_i,\ldots,\overline{a}_k\} = \overline{\mathbf{A}} \\
& \quad = \text{set of attributes defining the Best Dominated Choice (BDC).}
\end{array}
}
\tag{7.10}
$$

We now seek an artificial choice with the highest possible value A_I for each attribute in turn whilst all other attributes remain at the values of the Best Dominated Choice, and a set of weights W_{iI} corresponding to it, such that the valuation of the artificial choice is equal to one and the value of none of the real options exceeds one. Moreover, each such choice should be within the hull defined by the efficient real options.

This could be set up as an MP, however, the solution is trivial. The weight assigned to all attributes that remain at the magnitudes of the Best Dominated Choice, will be zero (ie $W_{iI} = 0 \;\; \forall i \neq I$). This most favours the artificial choice, as all real efficient options have higher attribute magnitudes for those attributes and we seek to give them the lowest possible value relative to the artificial choice. The level of A_I is the maximum level of that attribute that occurs in any of the real efficient options; its weight, W_{II}, is its reciprocal. This will give the artificial choice an MCA of 1. If option J has the maximum a_{IJ} amongst all options for that attribute, then $A_I = a_{IJ}$. We can then define our synthesised artificial choices for comparison as:

$$
\boxed{
\begin{array}{ll}
& \mathbf{A}_I = \{\overline{a}_1,\ldots,A_I,\ldots,\overline{a}_k\} \qquad \forall I \in \{1,\ldots,k\} \\
\text{Where} & A_I = \max\{a_{I1},\ldots,a_{IJ},\ldots,a_{IN}\}
\end{array}
}
\tag{7.11}
$$

Each artificial choice, $\mathbf{A}_I$, is efficient (generally, weakly efficient) within the terms of the definitions used in this thesis; it is usually dominated by the real option J from which it derives. We will refer to each of these as the Maximal Efficient Choice (MEC) for attribute I.

It will be seen that every MEC has a $[1,1]$ relationship with respect to every other MEC, and accordingly we can conceptualise the set of MECs as a fundamental decomposition of the efficient options from which they are derived. Under linear assumptions a valuation of these choices, together with the BDC, would enable valuation of any option in the feasible space by a positive weighted sum.

We would then ask the decision maker to rank their comparative preference for each of these choices. Desirably we obtain a complete linear order, though we can ignore relationships where the decision maker is unable to express a preference and exclude such from the LP. The preferences should be those of strict preference. In the early stages of analysis, the representation of these as weak value inequalities is unlikely to be material. However, as the latitude of criterion space reduces the remaining real efficient options to a few, ties may arise which can be avoided by strong constraints. Therefore one can either express preferences as:

$$\begin{aligned}
& v(\mathbf{A}_{(i1)}) \geq v(\mathbf{A}_{(i2)}) \geq \ldots \geq v(\mathbf{A}_{(ik)}) \qquad im \in \{1,\ldots,k\} \\
& \rightarrow v(\mathbf{A}_{(i1)} - \overline{\mathbf{A}}) \geq v(\mathbf{A}_{(i2)} - \overline{\mathbf{A}}) \geq \ldots \geq v(\mathbf{A}_{(ik)} - \overline{\mathbf{A}}) \\
& \rightarrow (A_{(i1)} - \overline{a}_{(i1)}).w_{(i1)} \geq (A_{(i2)} - \overline{a}_{(i2)}).w_{(i2)} \geq \ldots \geq (A_{(ik)} - \overline{a}_{(ik)}).w_{(ik)}
\end{aligned} \tag{7.12}$$

Where $v(\mathbf{A}_{(im)})$ = value of the set of attributes $\mathbf{A}_{(i1)}$, for maximal efficient choice im, which corresponds to attribute im, being the mth ranked amongst the MEC.
$w_{(im)}$ = Weight of attribute im.

or, in strong form:

$$\begin{aligned}
& v(\mathbf{A}_{(i1)}) \geq v(\mathbf{A}_{(i2)}) + \alpha \geq \ldots \geq v(\mathbf{A}_{(ik)}) + (k-1)\alpha \qquad im \in \{1,\ldots,k\} \\
& \rightarrow v(\mathbf{A}_{(i1)} - \overline{\mathbf{A}}) \geq v(\mathbf{A}_{(i2)} - \overline{\mathbf{A}}) + \alpha \geq \ldots \geq v(\mathbf{A}_{(ik)} - \overline{\mathbf{A}}) + (k-1)\alpha \\
& \rightarrow (A_{(i1)} - \overline{a}_{(i1)}).w_{(i1)} \geq (A_{(i2)} - \overline{a}_{(i2)}).w_{(i2)} + \alpha \geq \ldots \geq (A_{(ik)} - \overline{a}_{(ik)}).w_{(ik)} + (k-1)\alpha
\end{aligned} \tag{7.13}$$

Where α=in principle, a non-archimedean infinitesimal but, practically, a finite number which is inconsequential relative to the ability of a decision maker to discriminate.
Other definitions as above.

These manipulations are of greater simplicity than they might appear. They reduce to finding the difference between the highest and the lowest magnitude of each attribute amongst the efficient set of options (within such value constraint that may already have been specified); placing these in order of preference; and then using them as coefficients for the weights in LP constraints. These vectors correspond to the ranges or swings of Edwards and Barron (1994) though they are used within Dora-D in a different way.

The set of constraints can be conceptualised as a hyperplane "hinged" along each axis in such a manner that it cannot go past the horizontal for any hinge. We might expect that this degree of restriction would substantially reduce CAF latitude and the number of options that remain potentially optimum. Also, were the choices to be real options, we could guarantee eliminating at least as many options as preferences expressed. These provide grounds for supposing that the approach could constitute a potent elimination mechanism. However, it provides no guarantees. Applied successively, it seems to reduce options to a single option or to a group on an edge or facet of the criterion space (these are options for which a CAF exists such that they simultaneously have an MCA of one). However, I am not clear whether this is mathematically inevitable.

7.4.5 An example of option reduction using Larichev Decomposition in Basic Dora-D

As an example I use the 5x50 Set 1 data (see Chapter 9), without prior constraints. I assume that a decision maker acts consistently, on the basis of an underlying preference of equal weights of the five attributes. This information is "hidden" from the analyst except to the extent that it is revealed by the expression of preference. The Basic Method is used to identify the efficient options amongst the 50 candidates. There are 10 of these. From amongst these 10 *only*, the highest and lowest magnitudes for each attribute are identified. The difference between these is also calculated. These are:

Attribute	Highest Magnitude Incidence	Lowest Magnitude Incidence	Difference
1	4.40	0.71	3.69
2	3.61	0.13	3.48
3	3.60	0.33	3.27
4	3.78	-0.47	4.26
5	2.59	-0.38	2.97

The decision maker places these attribute differences in the order that he or she would prefer, all other differences being zero. Of these differences the decision maker considers that a difference of 4.26 for attribute 4, is preferred to 3.69 for attribute 1, is preferred to 3.48 for 2, is preferred to 3.27 for 3, is preferred to 2.97 for 5. Thus:

$$\boxed{\begin{array}{l} 4.26w_4 \geq 3.69w_1 \geq 3.48w_2 \geq 3.27w_3 \geq 2.97w_5 \\ \text{(or if the other preference had been expressed} \\ 2.97w_5 \geq 4.26w_4 \geq 3.69w_1 \geq 3.48w_2 \geq 3.27w_3\ ) \end{array}} \tag{7.14}$$

(As an alternative example, should the decision maker have valued a difference of 2.97 for attribute 5 more highly than the others he or she would have amended the order by saying 2.97 for attribute 5 is more valuable than 4.26).

Having inserted these constraints we now move to subsequent option reduction and run the routine again. We now obtain 4 efficient options which are consistent with the constraints imposed. The highest and lowest incidence of the various attributes is again found from amongst these four, as above, ie:

Variable	Highest Magnitude Incidence	Lowest Magnitude Incidence	Difference
1	4.40	1.08	3.32
2	3.09	0.13	2.96
3	2.89	0.33	2.56
4	3.78	1.82	1.97
5	2.52	-0.38	2.89

These attribute differences are ordered in like manner and further corresponding and complementary constraints are developed. The LPs are run again, this time reducing the number of feasible efficient options to one.

In 10 trials with similarly structured data (50 options and 5 attributes), the number of initially efficient options (ranging from 10 to 18) was reduced to 1 in 2 subsequent rounds in two cases, and in one case to 1 in 3 subsequent rounds. There were 4 instances in which a reduction to 2 efficient options was achieved in 2 subsequent rounds, with a further round demonstrating that no further reduction could be achieved with the method. In one case reduction to 4 options was achieved in 2 subsequent rounds, with reduction to 2 and confirmation that no further reduction was possible taking 3 further rounds. In two cases reduction to less than 5 options was not achieved using the pure method. Supplementing it by Franklin decomposition (see below), ultimately reduced the efficient options to 2, in one instance, and 3, in the other. In all instances the true optimum was confirmed to be within the efficient set and in practice was the option with the highest measured MCA(AP). The latter seems fortuitous and did not happen with tests on the same data using pure [1,1] Franklin decomposition.

The excess of the MCA above 1 is a measure of the opportunity cost of not choosing an option when it is the "true" optimum. In most instances, the level of this parameter for the second highest MCA in an irreducible efficient set, was well within the practical ability of a decision maker reliably and stably to discriminate value (eg under 1.05). However, if this is not achieved supplementary tie-breaking methodology is required (though it should be recognised that final short-lists from this method tend to be on facets which cannot be broken using the methods described for the basic method). Often a failure to break ties was associated with the true values of the options being close but, again, not always. A high MCA for the second option is a good indicator of the need for the further investigation using a supplementary approach.

The reader should note that in using Larichev Decomposition with portfolios, a BDC cannot readily be found. A Reference Portfolio is used in its place. The methodology is otherwise identical. This is discussed further when considering the application in Chapter 8.

7.4.6 Using Franklin Fundamentally Decomposed Preference in Dora-D

When options are reduced to two, the formulation described above reduces to a Franklin decomposition, but one using only [1,1] choices. It is open to the analyst and decision maker to confine their attentions *ab initio* to the comparison of pairs of real options in this way. Prioritising of the attribute differences obtained, serves to constrain valuation latitude within the Dora-D framework and to reduce potentially optimum options, in a methodology which in other respects replicates the above procedure. The preferences adduced for the pairs of options decomposed in this way, might be used directly to eliminate one or other of the decomposed pair but, within Dora-D, the same information is also used to examine, and possibly eliminate, other hitherto efficient options.

It is, in principle, possible for a decision maker to compare *any* options in this way, provided one option does not dominate the other. However, the comparison of efficient pairs does directly highlight and seek an opinion on those features of potentially optimal solutions that make them potentially optimal. One might expect more potent discrimination. Accordingly this is recommended.

There is also merit in avoiding arbitrary selection between efficient options. I commend the comparison of the two options, not already compared, having the highest MCAs under AP conditions. This may be loosely rationalised in terms of putting the two presently leading contenders for the optimum, head-to-head in a closer comparison. Alternatively, it may be thought of as comparing the two contenders whose valuations are least constrained by the other and, ergo, constitute the "most contrasted" pairing.

It is also possible to switch between Larichev and Franklin. As pure-Larichev can stick without complete reduction, this is also desirable.

Generally speaking, the Pure Franklin approach obtained similar ultimate reductions to those obtained using the Larichev approach followed by switching to Franklin, when a group of three or more efficient options could not otherwise be reduced. In one case out of ten tested, the pure Franklin approach reduced to a single option whilst Larichev decomposition followed by Franklin did not reduce below 2. In two other cases Larichev Decomposition reduced the problem to a single option whilst Franklin reduced to two, in one case, and three, in the other, The number of cycles required in the pure Franklin approach tended to be greater and only in one case was this not true. Larichev was invariably either more effective than, or equal to, Franklin in eliminating options in the early cycles. Moreover, whilst it is possible to switch to Franklin when Larichev "sticks", one essentially has to start again in order to use Larichev after Franklin.

I recommend Larichev followed, if necessary, by Franklin. Although, the ideas underlying Pure Franklin decomposition might be more accessible to some decision makers.

7.5 Limits to Reduction, and Infeasibility of Larichev and Franklin Decomposition in Dora-D

The application of neither of the above methods guarantees reduction to a single option, though in tests no case was observed of failing to reduce to no more than 3 options. In all cases examined all reduced options constituted an efficient facet. It is open to an analyst and decision maker to adopt another methodology to reduce further, eg Capping or Preference Bracketing. It may also be possible to derive more refined Fundamental decompositions.

It is a feature of both of the decomposition mechanics expounded here, that a binary preference can be expressed between any *two* choices generated by the methods, without risk of LP infeasibility, notwithstanding that other preferences might already be incorporated. This is because each choice generated, whilst not generally a practical option, is a technically feasible solution within the decision space. It will always be possible to express a feasible preference between points in the hitherto feasible criterion space and that feasible preference gives rise to a feasible constraint.

I also conjecture that, when no prior preferences have been specified, all the vectors representing the MECs can be ranked in any order, without disrupting feasibility. This is because, being mutually orthogonal vectors, each preference will involve a "new" attribute and can be introduced without contradicting a previously specified preference. It is not guaranteed that feasibility will be maintained when *all* choices within the set of Maximal Efficient Choices are ranked if there are existing constraints; I have already mentioned a counter-example. I also conjecture that if not all the options are on a facet, and the ranking of all the choices in a set of virtual options is infeasible, reduction may nevertheless be achieved by introducing the leading elements of the ranking only. Accordingly, one may be able to reduce options to a facet under all circumstances, using the mechanic.

This issue is pragmatically examined in the test simulations.

7.6 Choice as a value statement. Using Dora-D to assist Holistic Decision Making.

Let us assume that a rationally-intended decision maker, apparently arbitrarily, asserts that a particular solution to a multiple-objective decision problem is *the best* solution. What should an adviser do? Clearly, he or she has a duty to ensure that the decision maker has considered as many identifiable options as might realistically embrace the optimum and that the decision maker is properly informed about facts relating to the options that he or she considers are relevant. The adviser may also explicitly present alternatives, which she considers strong contenders, for special consideration by the decision maker and offer models and heuristics which provide insight. Quietly she might also satisfy herself that the judgements of the decision maker properly represent all the decision stakeholders. However, if the assertion of the decision maker is sustained notwithstanding, then the *only* basis for disagreement is if a *demonstrably* superior decision exists on the basis of the option facts assumed by the decision maker. The decision maker is sovereign on issues of value whether she chooses to declare them or not; and moreover may be cognitively competent to make such a holistic judgement.

A demonstrably superior decision then exists only if another option dominates it; that is, in the terms used in this thesis, provided there is an option which is superior to the nomination, whatever valuation weights are placed on the attributes determining decision goodness. If the nominated decision is non-dominated, the decision is not just a candidate for the best solution it is, in the circumstances I describe, at the moment of the assertion, the *true* optimum. The solution cannot be gainsaid except on the basis of an alternative judgement of value.

But if it is dominated how should one proceed. We can assert two essential condition for the true optimum:

(a) It should be efficient

(b) It should dominate the nominated solution

However, there will be several or many of such solutions in many practical situations. How might one proceed?

We can argue that in nominating the particular solution suggested, the decision maker was effectively asserting an implicit valuation function which valued the nominated option more than any other. He or she was wrong. However, the nomination was intended to be efficient. I suggest we can, in the face of this failure, alternatively assert that he or she was *nevertheless* governed by a value function which would give the nominated option the highest possible value, consistent with an arbitrary upper limit on the value of its dominant peer/s.

Under the circumstances discussed for the basic and portfolio models, there generally will be one function which maximises the valuation of such a nominated solution, subject to none having a value of more than one; that is the CAF. The MCA of the nominated solution will be less than one. Nevertheless, the nomination maps to a fully determined value statement from which the relative value of any other option can be determined. This value function minimises the degree of violation of the nominated option's optimality. In the absence of further information it would be reasonable to assume that this function represents the value function of the decision maker.

In the case of portfolio decisions defined by continuous decision variables, the mapping would be to a single optimum. In the discrete case, there will still usually be one such function, but it may not correspond to a single efficient peer. Notwithstanding, it can be looked upon as being in the "centre" of the criterion space which define better options.

In these circumstances, the decision maker will be unable to maintain that the nominated decision is superior to the option or options found in this way (provided all relevant *facts* have been revealed). However, the decision maker may legitimately adopt any other value function within the feasible space. If none is forthcoming, I suggest there is no demonstrably better criterion that can be adopted, whilst sustaining the assumption that only efficient solutions dominating the original nomination should be entertained.

We can thus provide a simple policy prescription to a decision maker who has a "pet" project. If it is efficient choose it. If it is not, then choose its efficient peer (or one of them in the discrete case). As usual she may modify the value function at will.

This provides an operationalisation of the ideas mooted by Kasenen, Wallenius and Wallenius (2000), though their structure attempted to characterise "pet" projects by the scope of attributes necessary to achieve non-domination.

It can also be argued that an intuitive decision maker may unconsciously bring into play attributes which are not overtly considered relevant, using weights which overwhelm overt considerations. I believe that "qualified self-awareness" should enable such attributes to be elicited and the above mechanic may be useful for this purpose. It can facilitate the identification of additional attributes, bringing them within the ambit of conscious analytic examination. There is little externally observable distinction between the inspiration of a superior talent and the mere self-indulgent adoption of an undefended whim by someone with power. The difference lies in the ability to bring it within the realm of conscious appreciation. A rationally intentioned decision maker should seek this.

7.7 Exploiting Infacility. Generating Representative Efficient Options and Formalised Indifference

It is a predicate of this work that our impaired decision maker will discern variations of a single objective with precision, but he or she is unlikely, within his or her innate cognitive capabilities, to discriminate value variations in a multiple objective situation with anything like the same finesse. The weights she attaches in the formation of an overall objective will neither be precisely specifiable nor temporally stable. Thus though a decision maker can and should be sensitive to a very small variation from an efficiency ideal, he or she is unlikely to be able to discriminate much larger differences in value *between* efficient solutions, even though they might be very different in value when measured with respect to a single objective. He or she may have no better than 3-bit ability to discriminate. Indeed, I have gone further, believing that objectives themselves are usually vague.

The selection of decisions in multiple attribute situation seems often to be based on an assumption that the decision maker has, contrary to this view, an inbuilt fine scale of multiple attribute value which generates complexity of procedure and a concomitant need to explore a large number of options. However, whilst this will not lead to "wrong" answers much of the effort may be gratuitous. Although the number of efficient options may be very many, a selection between a few may be quite sufficient.

Could we generate an economic short-list of non-spurious choices? One might (and this researcher would) believe that a decision maker could not meaningfully discriminate between two *efficient* options if the second has measured value of at least 90% (let us use this arbitrary, possibly conservative, figure) of the first, when measured using the CAF of the first, and the first has measured value of 90% of the second, when measured using the CAF of the second. If one then identifies a set of efficient options such that *any* feasible CAF will result in at least one of that set of options having an MCA of more than 90%, then no option outwith the set could be designated (within the discriminatory competence of the decision maker) as superior to all within it. If we then concentrate on discrimination between them, we should have done all that is within the decision maker's competence to do. I shall call such identified options a Representative Efficient Set.

An alternative way of expressing the above condition is that no efficient option excluded from the Representative Efficient Set shall have an MCA of greater than 1/0.9 (for simplicity say 1.1) when measured using a CAF such that the valuation of all options *within the identified set,* measured using the same valuation function, shall be no greater than one. If such exists, it must be added to the identified set.

In the basic discrete case this means that all efficient options having an MCA of greater than 1.1, when measured under *normal* Andersen-Petersen conditions (with all other options in the Comparison Set), are required to be in the representative set. However, as then demanding restrictions are placed on the CAF, further options must be added to the Representative Efficient Set until the condition outlined in the previous paragraph ceases to be violated. In the Basic structure where all options are explicit, this approach is demonstrably practicable.

Using the 5x50 Set 1 data, there were 10 efficient options. Of these 8 had MCAs of above 1.1 and 5 above 1.2. Neither of the two options initially excluded under the 1.1 cut-off were reintroduced into the Representative Set under the refined procedure. One of the five excluded at a 1.2 cut-off was reintroduced. In this example, and another using Set 2, only a small gain was made.

In the financial portfolio extension the situation is more complex. One might conjecture that whilst there may be an infinite number of efficient solutions, the number of Representative Efficient Options will be finite. Nevertheless, two issues arise. This researcher has not found a systematic search procedure for generating a Representative Efficient Set in the portfolio case, which one can be sure does not miss relevant options. (NLP can be used but find local optima). Moreover whilst the number may be finite it may still be too large to usefully use.

The writer performed two searches using portfolio data based on the Core problem. In the first of these he sought specific options maximising MCA, introducing constraints when any was found with an initial MCA in excess of 1.1. The measured MCAs for the last 10 solutions found, out of 100 identified options before ceasing the search, were:

1.225 1.311 1.570 1.509 1.362 1.312 1.282 1.165 1.322 1.367

This high and fluctuating level suggested that there could be hundreds more options before all those in excess of 1.1 were eliminated.

In the second test, the threshold was set at the high figure of 2.0. Twenty-five options were still found, although there could well have been further elusive options. The reader should note that the thresholds were applied to the MCA when newly identified. (The introduction of subsequent constraints tends to suppress the MCAs already identified). Both these tests suggests that for the portfolio problem the development of a comprehensive representative set may not be helpful or practicable. Nevertheless used in conjunction with some of the other tools proposed, the concept may still be useful. It would enable one to set a standard of Formal Indifference. This may be considered as a standard of closeness such that, even though a decision maker may declare preference, we must doubt the decision makers competence to resolve a distinction; whether he or she really cares about the policy difference it makes, or would express the same preference tomorrow. Of course an analyst may accept such a preference where declared, provided it is consistent with previous declarations, but he or she should not do so expecting it to add very much.

7.8 [1,1] Decomposition in association with Dora-D to analyse decision problems structured as Multiple Objective Linear Programming problems

One may use the concept of virtual frontier constraints, in association with a Dora-D formulation, to solve MOLP formulations of decision problems. In such formulations one may consider each alternative Objective as an Attribute. The task is then to find weighted sums of those Objectives such that no option within the decision space, defined by the linear constraints on system variables, can be assigned a value of greater than one; ie:

$$
\begin{array}{|ll|}
\hline
\text{Maximise } v = \sum_{\text{all } i=1 \text{ to } n} w_i.O_i & \\
\text{Subject to } O_i = \sum_{\text{all } j=1 \text{ to b}} p_{ij} x_j & \forall i \in \{1,\ldots,n\} \\
\sum_{\text{all } j=1 \text{ to b}} c_{kj} x_j \le C_k & \forall k \in \{1,\ldots,r\} \\
\sum_{\text{all } i=1 \text{ to } n} w_i.O_{im} \le 1 & \forall m \in \{\text{all vertices}\} \\
w_i \ge 0 & \forall i \in \{1,\ldots,n\} \\
x_j \ge 0 & \forall j \in \{1,\ldots,b\} \\
\text{Where } O_i = \text{Objective } i, \text{ a linear function of decision variables } x_j & \\
O_{im} = \sum_{\text{all } j=1 \text{ to b}} p_{ij} x_{jm} & \\
\quad = \text{ value of objective } i \text{ at vertex } m & \\
x_{jm} = \text{value of variable } x_j \text{ at vertex } m, \text{ total unknown} & \\
C_k = \text{system constraint} & \\
p_{ij} = \text{coefficient defining weight of variable } j \text{ in objective } i & \\
c_{kj} = \text{coefficient describing the contribution of variable } j \text{ to constraint } k & \\
w_i = \text{ weight of objective } i \text{ in combination of objectives} & \\
& \\
\text{To be solved for all } w_i \text{ and } x_j \text{ simultaneously} & \\
\hline
\end{array}
\tag{7.15}
$$

As with the Financial Portfolio, method it is not necessary explicitly to specify each $\sum_{\text{all } i=1 \text{ to } n} w_{im}.O_{im} \le 1$ constraint. Frontier Probing can be used. When a value of $v = \sum_{\text{all } i=1 \text{ to } n} w_i.O_i$ is found of greater than one, the O_i correspond to a new efficient vertex in criterion space. A new constraint can be inserted and the maximisation repeated. (One or more arbitrary constraints must be inserted initially to suppress unbounded solutions). The formulation is a non-linear program but readily solves with standard inexpensive software (eg What's Best).

Left to its own devices the routine will identify a number of optima (nominally "local" optima but actually co-valid pareto optima) and then "lock into" one. The routine can then be "jogged" into generating new optima by using randomly generated vectors of w_i variables as a new starting points. The routine can be stopped when an arbitrary number of new starting points have failed to generate any new optima. This procedure should generate an extensive sample of efficient points but *not* a comprehensive census. It may miss points corresponding to small enclaves in the starting solution space. Such elusive optima are unlikely to be materially superior to identified options even given a valuation function corresponding to their CAFs, for real decisions. As they are revealed only in the fine structure of starting solutions, value differences for solutions in contiguous areas should be small.

An indication of the prospective significance can be established by considering the extent to which a different list is generated if the analysis is repeated from the beginning. The method was used to identify optima corresponding to the following problem from Steuer (1977):

$$
\begin{aligned}
&\text{Maximise } 15x_1+5x_2+12x_3-8x_4+2x_5+7x_6-17x_7-x_8-14x_9+9x_{10} \\
&\text{and } -17x_1-19x_2-x_3+3x_4-4x_5-18x_6+14x_7+4x_8-16x_9-11x_{10} \\
&\text{and} -3x_1+3x_2+13x_3+8x_4+15x_5+18x_6+17x_7-19x_8+14x_9-19x_{10} \\
&\text{Subject to} \\
&18x_1+11x_6+6x_8+3x_{10}\le 100 \\
&4x_1+19x_3+15x_4+x_5+11x_6+13x_7\le 100 \\
&8x_2+3x_4+11x_8\le 100 \\
&12x_5+x_6+5x_7+3x_9+4x_{10}\le 100 \\
&13x_1+4x_3+9x_5+7x_6+3x_7+13x_8+12x_9+3x_{10}\le 100 \\
&4x_3+19x_5+8x_9+9x_{10}\le 100 \\
&8x_1+3x_2+18x_3+3x_5+2x_7+2x_9+5x_{10}\le 100 \\
&x_1+9x_4+13x_9+19x_{10}\le 100 \\
&x_j\ge 0 \ \forall j=\{1,\ldots 10\}
\end{aligned}
\tag{7.16}
$$

In two runs the optimal points (in criterion space) were as shown in Table 7.1.

Alternatively, the approach might also be used interactively to generate solutions consistent with preferences declared between one solution found and another that immediately preceded it, short-cutting the need to identify an extensive set of efficient options before choosing. However, whether one attempts an extensive or interactive identification, the approach retains the characteristic of other MOLP that this writer questions. It is doubtful that it is cognitively less taxing to choose directly between the [m,n-m] choices implied by the comparison of such optima, than to make a prior determination of weights from which a single optimum can be straightforwardly identified.

Table 7.1 Example of Solution Differences in Repeated Dora-D MOLP Simulations

Efficient Points, Run 1		Efficient Points from Run 2 not found in Run 1		Efficient Points from Run 1 not found in Run 2
186.1, -434.7,105.5	3	34.1, -124.9, 227.8	7	144.5, -162.6, -12.6
185.6, -421.8, 64.0		-130.8, 107.7, 130.8		66.7, -15.4, 105.1
184.1, -431.0, 115.1				57.1, 19.0, -47.0
178.3, -373.4, -70.6				
174.1, -359.2, 82.2				
163.0, -380.8,179.7				
148.2, -362.2, 215.4				
144.8, -171.7, 16.8				
144.5, -162.6, -12.6				
140.7, -137.5, -82.6				
135.1, -413.6, 251.8				
133.0, -389.8, 255.2				
132.1, -411.4, 259.2				
122.2, -117.8, 86.2				
120.4, -97.5, 23.5				
102.8, -93.4,132.8				
96.6, -362.4, 265.3	4			
70.5, -152.3, 217.7				
69.0, -27.8, 132.6	5			
66.7,-15.4, 105.3				
60.5, -17.1, 132.0	6			
57.1, 19.0, -47.0				
-107.9, -207.9, 270.8	1			
-113.4, 81.0, 202.8	2			
-136.7, 131.4, 18.3				
-170.4, 29.6, 233.3				

However, we may use the principles of decomposed choice developed above to generate, for each succeeding pair of options, a set of n (=the number of objectives), [1,1] Franklin choices, which can be placed in order of preference. The constraints thus generated can be included in the next attempt to generate an efficient vertex, ensuring that any newly generated efficient vertex will be consistent with this and other similarly generated preferences. When no new vertices can be found, it is necessary to reassess all those previously revealed, eliminating those which can no longer be optimal. [1,1] Franklin comparisons can be made between all solutions not eliminated, and not already directly compared. The translation of these comparisons into valuation constraints may further reduce the potentially optimal vertices or, with decreasing likelihood, allow the generation of new ones.

This procedure may not reduce to a single option but should result in a set of options which cannot be discriminated within the confines of [1,1] choice expression. It is also possible that the decision maker's true optimum could remain unidentified but the writer conjectures that material failures would be rare.

In a simulation of finding the optimum solution to the Steuer problem above, when the decision maker's hidden value objective was an equally weighted function, seven efficient vertices were revealed in the order shown in Table 7.1. [1,1] Franklin comparisons were made as they emerged, that is 1 with 2, 2 with 3, 3 with 4, 4 with 5, 5 with 6, and 6 with 7. Comparisons were also made of 2 with 5, and 2 with 6. Options 2, 5 and 6 remained after this. Option 6 was the true optimum confirmed by ordinary linear programming. Option 2 had a relative value of .972, and Option 5 one of .991.

7.9 Configural Dora-D

I have argued that the concept of qualified self awareness, generally precludes the need for assumptions of non-linearity which are not consciously recognised by the decision maker, at least as a possibility. I have excepted configural valuation from this view, recognising that a decision maker may behave configurally, without articulating a prior intention, and may subsequently argue the desirability of so doing.

I discuss here how configural issues can be approached within the methodology for selection problems, which in other respects correspond to the structure tackled in the basic model. I have commented that the Minkowski metric $V(X)=(\sum a_i x_i^r)^{1/r}$ has been suggested as a transformation for configural valuation of positive attributes, allowing graduations of disjunctive valuation with r values of greater than 1, and conjunctive valuation with r below it. I have pointed out that the simpler transformation $V'(X)=\sum a_i x_i^r$ (which I refer to as the General Configural Model) will produce strategically equivalent valuations. Of course many other configural models could be proposed including, for example, those with cross-product terms. Nevertheless, the modified Minkowski metric appears to be a powerful and versatile approximation.

To illustrate, I postulated a situation in which there were 100 options defined by combinations of every x_1 and x_2, each of these running from 0.1 to 1.0 in intervals of 0.1. A true valuation function of $v = x_1.x_2$ was assumed. A Modified Minkowski function of x_1 and x_2 (ie $v = k.(x_1^r + x_2^r)$, which correctly replicated the leading options when ranked according to their true valuation, was then sought. With a power parameter of 0.1 all options were ranked without error.

Four possible approaches to the configural problem, based on the basic methodology but using the General Configural Model, suggest themselves:

(a) Simultaneous Dora-D Parameterisation
(b) Conservative Configural Formulation
(c) Supplementary Variables
(d) Multiple Parallel Models

7.9.1 Simultaneous Dora-D parameterisation

This approach is mentioned for completeness. An attempt was made to use mathematical programming to estimate r, simultaneously with attribute weights. In practice the What's Best optimiser was not well behaved, identifying true optima with poor reliability. The method is complicated, unreliable, and unnecessary.

7.9.2 Conservative Configural Dora-D formulation

When I embarked on the simulations to test the previous method, I expected that potential optimum options would have an upper and lower bounded range of Modified Minkowski parameters, over which a particular solution was efficient. During the analysis it appeared that whilst such options were in general efficient up to defined upper bounds of r, they appeared to remain efficient in the other direction to very small values of r. This suggested an intriguing and potentially valuable empirical property, that if an option was efficient under a particular set of circumstances, it might also within the General Configural Model be efficient under *all* more conjunctive circumstances. This could be exploited in a simpler approach to that outlined above and was investigated further.

For each of the ten sets of data designated in Chapter 9 "5x50data_2810" (modified as used in Simulations 12-14), efficient options were established using $r = 0.2$, 1.0 and 2.0, without pre-emptive preference constraints. For some data sets, other intervening values of r were also examined. In *all* data sets there were *no* instances occurring where an option that was efficient for a particular value of r was not also efficient for all lesser values of r. For example, all options that were efficient for the disjunctive valuation with $r = 2$ were also efficient for the radically conjunctive situation with $r = 0.2$. (The converse was not true, typically there were several options that were efficient under the conjunctive assumption that were not efficient under the disjunctive one).

This condition appears to be a general one and seems to be true for any positive concave transformation of positive attribute magnitudes. I suggest a basis for a proof in Figure 7.2, though I have not contrived a concise one.

The implication of this property is that one can address the solution of the General Configural Model, when the Minkowski parameter is unknown, using a fixed parameter formulation. If one assumes a sensible conservative transformation, all options which would be optimal over any less conservative parameter, will be revealed as efficient and reduction can take place from that base; for example, using Larichev and Franklin reduction. (As people may have a less intuitive grasp of the interpretation of weights in configural conditions using other mechanics, I commend this). The cost is an additional option elimination load but, as will be seen in Chapter 9, this can be small even for large parameter swings. If disjunctive configurality is suspected, the linear model suggests itself as a conservative model. If a conjunctive model is possible, the issue is less clear cut. However, even a parameter as low as 0.2, which radically discounts marginal increments at higher attribute values, may not severely add to the reduction load.

Figure 7.2

To demonstrate that if an option is efficient under a particular valuation of its attributes it will remain efficient under a concave transformation of the same valuation.

If $\mathbf{x}_0$ is efficient there will exist convex combinations of the remaining options that it dominates, ie:

$$\exists\, \mathbf{x}_0 \geq \sum_{i=1 \text{ to } n} f_i \mathbf{x}_i \rightarrow x_{0j} \geq \sum_{i=1 \text{ to } n} f_i x_{ij} \quad \forall j$$

Where $\mathbf{f} = (f_1, \ldots f_i, \ldots f_n)$, $(\sum_{i=1 \text{ to } n} f_i = 1;\ f_i \geq 0\ \forall i)$

For any such j, the x_{ij} can be regrouped into two disjoint subsets to generate new variables x_{1j} and x_{2j}

such that $\sum_{i=1 \text{ to } n} f_i x_{ij} = t.x_{1j} + (1-t).x_{2j}$,

Where $t = \sum_{i=1 \text{ to } n} f_i.n_1$, $\quad n_1 = 1$ if i is within subset 1,= 0 otherwise

Thus $x_{0j} \geq t.x_{1j} + (1-t).x_{2j} = x'_{0j}$

For $\mathbf{x}_0$ to remain efficient under a positive concave transformation g requires that under transformation g

$\mathrm{g}(x_{0j}) \geq t.\mathrm{g}(x_{1j}) + (1-t).\mathrm{g}(x_{2j})$ whenever $x_{0j} \geq t.x_{1j} + (1-t).x_{2j}$

and we require to demonstrate this.

From para 2

$x_{0j} \geq x'_{0j} \rightarrow \mathrm{g}(x_{0j}) \geq \mathrm{g}(x'_{0j}) \rightarrow \mathrm{g}(x_{0j}) \geq \mathrm{g}(t.x_{1j} + (1-t).x_{2j})$

If g is concave, by definition

$\mathrm{g}(r.y_1 + (1-r)y_2) \geq r.\mathrm{g}(y_1) + (1-r)\mathrm{g}(y_2), \qquad 0 \leq r \leq 1$

$\therefore \mathrm{g}(t.x_{1j} + (1-t).x_{2j}) \geq t.\mathrm{g}(x_{1j}) + (1-t).\mathrm{g}(x_{2j})$

$\therefore \mathrm{g}(x_{0j}) \geq \mathrm{g}(t.x_{1j} + (1-t).x_{2j}) \geq t.\mathrm{g}(x_{1j}) + (1-t).\mathrm{g}(x_{2j})$

$\rightarrow \mathrm{g}(x_{0j}) \geq t.\mathrm{g}(x_{1j}) + (1-t).\mathrm{g}(x_{2j})$

for all j and all possible x_{1j}, x_{2j} meeting the required condition.

All points originally dominated, remained dominated in transformed form. Accordingly, an efficient option defined by positive attributes, will remain efficient under a concave transformation.

This includes the power transformation, $\mathbf{y}_0 = \mathbf{x}_0{}^p, 0 < p < 1$.

7.9.3 Supplementary configural variables

It is open to an analyst to introduce configural variables as separate attributes within models. This might be appropriate if the Modified Minkowski approximation is unsuitable, for example if specific value interactions of the cross-product type are assumed.

7.9.4 Method of Multiple Parallel Models

The Best Possible Light concept that underlies Dora-D, does not need to be the best possible light for parameters chosen within a single model. The best of one or more alternative representations may be chosen. Options may thus be evaluated according to more than one model, and selected as potential optima if they have CRVs of one, or greater than one, for *any* of the alternative formulations. Such models would be run independently.

However, the elicitation of preference needs to be interdependent and constraints derived during the reduction process must be fed back into all models. Any model going infeasible, after the introduction of constraints corresponding to well considered preferences, or no longer generating efficient solutions, would be deemed to be ruled-out as a description of the decision maker's values.

Alternative models can be general or quite specific. For example, they could be several archetypal models with fully determined weights.

7.10 Project Portfolios

Cook and Green (2000) developed a methodology to select a group of projects to form a portfolio, (we may also use the word "programme"), in a multiple criteria situation under resource constrained conditions, using a Data Envelopment Analysis Approach. Their method is essentially value independent (or more properly the value selection is embedded within the methodology), although they do suggest the use of "assurance region" extensions which can be used to delimit decision makers' values. In consequence only a single optimum is derived. However, there are, in reality, very many efficient solutions and the selection from them is, desirably, dependent on the values of the decision maker.

Here I develop an alternative procedure, based on the Frontier Probing methodology described in this thesis. I illustrate the method by reanalysing the problem that they examined, which was in turn drawn from data from Oral, Kettani and Lang (1991).

The reader will recall the general discussion of portfolios and the financial portfolio formulation in Chapter 6. The Project Portfolio problem differs as follows:

(a) There is no non-linear value component.

(b) There is no fixed sum proportional inclusion of portfolio constituents.

(c) The inclusion of portfolio constituents, individual projects, is determined by binary integer "in-out" variables.

(d) One or more resource constraint will exist, restricting project inclusion.

The formulation becomes:

$$\boxed{\begin{array}{lll}
\multicolumn{3}{l}{\text{For each potential portfolio } P \in \mathbf{P} \subset \mathbb{P}} \\
\text{Maximise} & v_P = \sum_{\text{all } i} w_{iP}.A_{Pi} & \\
\multicolumn{3}{l}{\text{Subject to}} \\
& \sum_{\text{all } i} w_{iP}.A_{pi} \le 1 & \forall p \in \mathbf{P} \\
& f_{ph} = \{0,1\} & \forall h \in \{1,\ldots,n\}, \forall p \in \mathbf{P} \\
& w_{iP} \ge \alpha_i & \forall i \in \{1,\ldots,k\} \\
\text{Where} & \mathbf{P} = \{p \in \mathbb{P} : \sum_{\text{all } h} f_{ph} r_{jh} \le R_j, \forall j \in \{1,\ldots,m\}\} & \\
& A_{pi} = \sum_{h=1 \text{ to } n} f_{ph} a_{ih} & \forall i \in \{1,\ldots,k\} \\
& a_{ih} = \text{magnitude of benefit attribute } i, \text{of } k; \text{ for project } h, \text{of } n. & \\
& r_{jh} = \text{magnitude of resource}, j, \text{ of } m; \text{ for project } h. & \\
& \mathbb{P} = \text{all possible combinations of projects} & \\
& \mathbf{P} = \text{all combinations of projects within resource constraints} & \\
& P = \text{the portfolio under investigation} & \\
& p = \text{any combination of projects irrespective of resource feasibility} & \\
& f_{ph} = 1, \text{ if project } h \text{ is a constituent of portfolio } p & \\
& \quad = 0, \text{ otherwise} & \\
& w_{iP} = \text{coefficient of attribute } i \text{ in value function} & \\
& \quad \text{or CAF of portfolio } P & \\
& \alpha_i = \text{a positive archimedean number or zero} &
\end{array}} \tag{7.17}$$

We face the same difficulties with regard to the scale of this problem that we did for financial portfolios (combinatorially large numbers of potential programmes for testing, each with a combinatorially large numbers of frontier constraints associated with the Comparison Set) and we can tackle it using Frontier Probing in the same way. For one or any number of Test Portfolios, P, we can find its efficient peer in identical manner. Thus, find the value function which maximises the value of the test portfolio. Then find the portfolio, *within* $\mathbf{P}$, with the highest value under that function, positively ensuring that the conditions for $\mathbf{P}$ are met. This is accomplished by adding the resource constraints $\sum_{\text{all } h} f_{ph} r_{jh} \leq R_j$ to the LP.

If a portfolio, meeting the resource constraints, violates the implicit frontier constraint, insert an explicit frontier constraint preventing it. Repeat the process until there are no material violations. This develops the MCA and peer portfolios of the Test Portfolio.

One could then develop a value function using the type of preference eliciting and representation mechanics explored elsewhere in this thesis. However, I suggest that this is an area where the concept of "choice as a value statement" (discussed above) can be employed. It is probable that a decision maker in this area (eg, a Director of R&D) will have a "good" starting plan which embeds the organisation's values. Finding the Efficient Peer of such a plan could well constitute a very reasonable solution or, if not, the basis for one.

The method also provides a convenient means for suggesting what Cook and Green (op cit) call a "robust core". We can do this by defining as a new Test Portfolio, a Test Complement, which contrasts maximally with it. This is a portfolio in which all f_{ph} of value 1 in the Test Portfolio are rewritten as zero, and all those of zero rewritten as 1. The projects common to both the Efficient Peers of both the Test Portfolio and the Test Complement may be said to be the "robust core".

To illustrate the use of the formulation, we make use of the data used by Cook and Green and by Oral et al. This is shown in Table 7.2. This describes the benefits of 37 prospective projects against 5 criteria. In the terms used in this thesis these are Attributes which when weighted define the value of the project. Also associated with a project is a cost which we treat as the single constrained resource, limited by budget.

We define the task here as one of finding the Efficient Peer of a nominated programme. I use here the Cook and Green solution as a Test Portfolio, recognising that they sought a slightly different objective, to maximise benefit *per unit of cost* whilst remaining within budget, whereas the task as formulated here is simply to maximise the value of outputs *within a budgetary constraint.*.

Table 7.2- R&D Programme Data

Proj. No.	1. Indirect economic contrib.	2. Direct economic contrib.	3. Technical contrib.	4. Social contrib.	5. Scientific contrib.	Proj. cost
1	67.53	70.82	62.64	44.91	46.28	84.20
2	58.94	62.86	57.47	42.84	45.64	90.00
3	22.27	19.68	6.73	10.99	5.92	50.20
4	47.32	47.05	21.75	20.82	19.64	67.50
5	48.96	48.48	34.90	32.73	26.21	75.40
6	58.88	77.16	35.42	29.11	26.08	90.00
7	50.10	58.20	36.12	32.46	18.90	87.40
8	47.46	49.54	46.89	24.54	36.35	88.80
9	55.26	61.09	38.93	47.71	29.47	95.90
10	52.40	55.09	53.45	19.52	46.57	77.50
11	55.13	55.54	55.13	23.36	46.31	76.50
12	32.09	34.04	33.57	10.60	29.36	47.50
13	27.49	39.00	34.51	21.25	25.74	58.50
14	77.17	83.35	60.01	41.37	51.91	95.00
15	72.00	68.32	25.84	36.64	25.84	83.80
16	39.74	34.54	38.01	15.79	33.06	35.40
17	38.50	28.65	51.18	59.59	48.82	32.10
18	41.23	47.18	40.01	10.18	38.86	46.70
19	53.02	51.34	42.48	17.42	46.30	78.60
20	19.91	18.98	25.49	8.66	27.04	54.10
21	50.96	53.56	55.47	30.23	54.72	74.40
22	53.36	46.47	49.72	36.53	50.44	82.10
23	61.60	66.59	64.54	39.10	51.12	75.60
24	52.56	55.11	57.58	39.69	56.49	92.30
25	31.22	29.84	33.08	13.27	36.75	68.50
26	54.64	58.05	60.03	31.16	46.71	69.30
27	50.40	53.58	53.06	26.68	48.85	57.10
28	30.76	32.45	36.63	25.45	34.79	80.00
29	48.97	54.97	51.52	23.02	45.75	72.00
30	59.68	63.78	54.80	15.94	44.04	82.90
31	48.28	55.58	53.30	7.61	36.74	44.60
32	39.78	51.69	35.10	5.30	29.57	54.50
33	24.93	29.72	28.72	8.38	23.45	52.70
34	22.32	33.12	18.94	4.03	9.58	28.00
35	48.83	53.41	40.82	10.45	33.72	36.00
36	61.45	70.22	58.26	19.53	49.33	64.10
37	57.78	72.10	43.83	16.14	31.32	66.40

Source Oral et al (op cit)

The procedure goes through 7 cycles finding violating constraints which are progressively inserted. The measured value of the violating programmes using the

Table 7.3-Summary of Results of R&D Programme Optimisation.

		Cycle 1	Cycle 2	Cycle 3	Cycle 4	Cycle 5	Cycle 6	Cycle 7
Test CAF								
Attr 1		0	0	0	0.0008	0.0011	0	0.2393
Attr 2		0	0.0014	0	0	0	0.0007	0.7179
Attr 3		0	0	0	0	0	0	0
Attr 4		0	0	0.0025	0.0009	0.0002	0.0008	0.0438
Attr 5		0.0165	0	0	0	0	0	0
Value of max violation		10	1.339	1.269	1.057	1.022	1.026	1.001
Max violation portfolio	Test Portfolio (Cook & Green)							
Project 1	1	0	1	1	1	1	1	1
Project 2	0	0	0	1	1	0	1	0
Project 3	0	0	0	0	0	0	0	0
Project 4	0	0	0	0	0	0	0	0
Project 5	0	0	0	1	1	0	0	0
Project 6	1	0	1	0	0	0	1	1
Project 7	0	0	0	1	0	0	0	0
Project 8	0	0	0	0	0	0	0	0
Project 9	0	0	0	1	0	0	0	0
Project 10	0	1	0	0	0	0	0	0
Project 11	0	1	0	0	0	1	0	0
Project 12	0	0	1	0	0	1	0	0
Project 13	0	0	0	0	0	0	0	0
Project 14	1	1	1	1	1	1	1	1
Project 15	1	0	0	1	1	1	1	1
Project 16	1	0	1	1	1	1	1	0
Project 17	1	0	1	1	1	1	1	1
Project 18	1	1	1	0	0	1	1	1
Project 19	0	1	0	0	0	0	0	0
Project 20	0	0	0	0	0	0	0	0
Project 21	0	1	0	0	1	0	0	0
Project 22	0	1	0	1	1	0	0	0
Project 23	1	1	1	1	1	1	1	1
Project 24	0	1	0	1	0	0	0	0
Project 25	0	1	0	0	0	0	0	0
Project 26	1	0	1	1	1	1	1	1
Project 27	1	1	1	0	1	1	1	1
Project 28	0	0	0	0	0	0	0	0
Project 29	0	1	1	0	0	0	0	1
Project 30	0	0	0	0	0	0	0	0
Project 31	1	0	1	0	1	1	1	1
Project 32	1	0	1	0	0	1	0	1
Project 33	0	0	0	0	0	0	0	0
Project 34	1	0	1	0	0	1	1	1
Project 35	1	1	1	0	1	1	1	1
Project 36	1	0	1	0	1	1	1	1
Project 37	1	0	1	0	0	1	1	1

CAF of the Test Portfolio drops from initially (an imposed) 10 to 1.0001 during this sequence.

The main results are summarised in Table 7.3. The measured MCA of the Test Portfolio was 0.982 and aggregate cost 999.4. The final solution only differed from the Test, (Cook and Green's solution) by excluding Project 16 and including 29. This is hardly surprising as the difference in our respective objectives was a subtle one and the Cook and Green solution was efficient with respect to their criteria. It should be noted that the portfolios developed at the end of cycles 2, 5 and 6, proved to be co-optimal with that derived at the final cycle. The portfolios formed in cycles 3 and 4 were efficient but, not in the end, efficient peers of the Test Portfolio, and hence not optimal with respect to the developed CAF. Only the portfolio developed in cycle 1 was not efficient.

In practice it would be open to a decision maker to adjust or constrain the CAF, for example if the heavy weights on attributes 1 and 2 (which maximise the value of the Test portfolio) is not to his or her taste.

I also sought the efficient peer of the Test Complement. The Test Complement was an infeasible portfolio with respect to the budget. Its efficient (and feasible) peer is the set of 16 projects {1, 2, 14, 15, 16, 17, 18, 21, 22, 23, 26, 27, 31, 34, 35, 36}. The robust core is thus the 12 projects {1, 14, 15, 17, 18, 23, 26, 27, 31, 34, 35, 36}.

I also used the example solution of Oral et al as a Test Portfolio. This was {1, 5, 14, 16, 17, 18, 21, 23, 26, 27, 29, 31, 34, 35, 36, 37}. The MCA of this portfolio is 0.965 and its efficient peer programme is {1, 2, 9, 14, 16, 17, 18, 21 23, 26, 27, 31, 32, 34, 35, 36, 37}; 5 and 29 drops out to be replaced by 2 and 9. Whilst project 15 of the robust core is excluded from this solution, a co-optimal programme is in fact obtained when *all* members of the robust set are forced in.

7.11 Data Envelopment and Social Choice

A data envelopment approach can also be applied to social choice and group preference evaluations, as in Cook and Kress (1990). This fits comfortably within the framework developed.

To do this it is convenient to take a different perspective from that traditionally adopted, originally by Arrow (1951), where the focus is the manifestation of social preference for entities, through the preference ranking of those entities, by individuals. The alternative view in this framework is that society places value on the "opinion" of Representatives, though not necessarily equal value, and that the valuation of options by those representatives in aggregate, is taken to be the value to society or the organisation. However, the values of representatives are largely hidden and are only manifest in the rankings they assign to entities.

Alternative "constitutions" could be envisaged. The following "meritocratic" one is a possible operationalisation:

1. A ranking of all options by all representatives is declared, but no other information on their valuation is available.
2. A ranking of representatives is declared.
3. The aggregate of the value to representatives is strategically equivalent to the valuation of society or the organisation.
4. Any value may be attributed to any option nominated by a representative, except that:
    a. No assignment of value to rankings by representatives may be made which values option ranked m, by representative ranked n, more highly than the value of option m-1, by representative n, or than the value of option m, for representative n-1
    b. Where a representative is indifferent between options it shall be assumed that in the view of the representative they are of equal value and shall be attributed a value no higher than the option or options that ranks next highest.

If any values for rankings by individuals can be found within these constraints, which result in the valuation of an option being superior to all other options, that option is a potential group optimum.

A democratic constitution, where the same valuation is given to particular rankings for all representatives, can also be imposed.

We may then seek value weightings for each rank level which shows each option in its Best Possible Light. This may be formulated in similar structure to basic Dora-D, for the more complex constitution as:

For each decision option $S \in \{1,...,n\}$

$$\text{Maximise } v_S = \sum_{\text{all } i}\sum_{\text{all } k} w_{ikS}\, r_{ikS}$$

Subject to

$$\sum_{\text{all } i}\sum_{\text{all } k} w_{ikS}\, r_{ikj} \le 1 \qquad \forall j \in \{1,...,n : j \ne S\}$$

$$w_{ikS} \ge w_{i(k+1)S} \qquad \forall i \in \{1,...,m\}, \forall k \in \{1,...,n-1\}$$

$$w_{ikS} \ge w_{(i+1)kS} \qquad \forall i \in \{1,...,m-1\}, \forall k \in \{1,...,n\}$$

$$w_{ikS} \ge 0 \qquad \forall i \in \{1,...,m\}, \forall k \in \{1,...,n\} \tag{7.18}$$

Where n = number of options

m = number of representatives

w_{ikS} = valuation weight assigned to option ranked k by the "representative" ranked i when evaluating option S

r_{ikj} = 1 when option j is ranked k by representative i

= 0, otherwise

Under the simpler democratic constitution, the suffix i is omitted and the equations $w_{ikS} \ge w_{(i+1)kS}$ are redundant.

The democratic version of the above is identical in intent and similar in formulation to Cook and Kress (1990). However, the option under consideration was included in the Comparative Set by them and is excluded in the methodology here. They also include what they call a "discrimination intensity function" to ensure material differences in the valuation weights for different valuation ranks, which is not necessary in this variant.

It is thought that, under formulation (7.18) weights, can usually be found which will give *all* options a value of not less than 1. Efficient options are therefore those for which the LP finds solutions for which v_S is *greate*r than 1. In this instance the possibility of absolute superiority must be demonstrated.

The values of v_S which can still be described as constituting the Maximal Comparative Advantage of options, provide an indication of the materiality of options which have valuations of greater than one, (thereby constituting an advantage for excluding the option under consideration from the Comparative Set). Other procedures remain necessary to reduce the remaining set. For example, further reduction might be achieved by securing consensus amongst the delegate group regarding bounds to the rank weightings.

The methodology was tested for the "democratic" constitution using the data outlined in Dyer and Miles (1977) which they used to aid the selection of trajectory pairs for the Mariner project. In this, some 32 alternative pairs were ranked in value by 10 science teams. In the Dyer and Miles study, cardinal valuations were also developed, but here I use only the ordinal data. The teams' rankings are indicated in Table 7.4.

Table 7.4- Ranking by Mariner Science Teams of alternative Trajectory Pairs

Team	1	2	3	4	5	6	7	8	9	10
Trajectory Pair										
1	5	28	13	21	17	23	14	15	22	19
2	5	24	21	24	21	4	7	19	9	18
3	9	14	4	27	5	27	28	27	6	24
4	9	6	25	20	2	27	27	26	6	25
5	9	9	28	17	4	11	6	6	1	6
7	9	32	32	32	18	32	30	25	22	26
8	9	20	8	5	11	20	11	7	14	10
9	9	12	15	13	25	9	19	16	14	13
10	2	11	24	8	19	2	20	24	14	8
11	9	13	7	3	28	4	26	31	32	27
13	8	18	21	27	11	31	31	30	22	28
15	2	29	15	18	8	16	16	20	9	22
17	9	4	13	6	11	23	9	14	14	9
18	9	8	8	15	21	11	24	17	22	29
19	9	26	8	27	25	11	25	29	22	30
20	5	10	31	21	32	20	22	27	22	16
21	9	14	3	31	21	27	29	11	6	31
22	9	29	8	18	31	4	17	23	22	23
23	9	24	15	24	30	4	8	22	22	20
24	2	17	25	27	1	27	32	32	22	32
25	9	7	28	24	8	1	5	5	9	1
26	9	2	8	11	7	16	3	1	1	2
27	9	1	30	16	3	16	1	4	3	3
28	1	23	25	14	11	23	13	17	9	7
29	9	5	18	6	8	3	2	2	3	4
30	9	22	1	12	25	16	10	21	14	12
31	9	3	5	8	6	8	4	3	3	5
32	9	19	18	21	28	11	18	13	14	14
33	9	26	6	10	19	9	21	12	14	15
34	9	16	18	3	11	20	15	8	9	17
35	9	21	2	2	11	11	12	9	14	11
36	9	31	21	1	24	23	23	10	22	21

The results of the analysis are shown in Table 7.5.

Table 7.5. Efficient Trajectory Pairs and corresponding MCAs

	MCA	MCA	MCA
Trajectory Pair	As model	Pair 31 eliminated	31&29 eliminated
26	2.000	2.000	2.000
27	1.200	1.200	1.200
29	1.143	1.250	-
31	1.125	-	-
All other pairs	1.000	-	-

Interestingly, pairs 31, 29, and 26 were ranked in the top three by all the collective choice rules explored by Dyer and Miles, with pair 27 appearing in 4th place for two rules. Pair 31 emerged top using their rank sum rule, though I would have been tempted to eliminate this pair as not providing sufficient advantage versus other options, even under BPL conditions, to justify retention. In fact the pair with the highest MCA above, pair 26, was selected for implementation.

With the meritocratic constitution (arbitrarily ranking science teams by their table order) there was greater freedom of weights and only 6 pairs were immediately eliminated; 3 pairs had MCAs of 1.001-1.2; 13 had MCAs of 1.201-1.5; 7 of 1.501-2.0; 2 of 2.01-3; and 1 was unbounded. Whilst the MCAs could be used to determine a ranking it would be desirable to further circumscribe the latitude of the weights.

The variant here was also applied to Cook and Kress's data (1990,p1309). The MCAs were a=0.813, b=1.273, c=1.046, d=1.1364, e=0.688, f=0.688. b, c and d are the same potential optima identified by Cook and Kress. The limited circumstances of optimality for c are indicated by its low MCA.

Readers should note that scores under rank valuation methods may be affected by irrelevant alternatives and might profitably be reworked with "no-hopers" eliminated.

This approach could in principle be used as a first screening mechanic in elections conducted under the Alternative Vote System. However, the method would lack the methodological transparency to ordinary voters, necessary for public elections.

It might be possible to incorporate features such as the blank "slots" approach adopted in a different context by Cook and Kress (1985) to reflect intensity of preference; (I am not aware whether they have sought to combine their two techniques). The possible impact of such a feature is unclear.

Chapter 8 Using Dora-D in developing a personal financial portfolio

8.1 Introduction

In this chapter I seek to illustrate the use of the approach in a practical application- the Core Problem. This is investment decision making- specifically my share decision making. I start by explaining my attitude to the problem and my approach to explaining this as, simultaneously, decision maker, analyst and researcher.

I describe my data sources and my Vague Objectives as decision maker. I then describe in detail the application of the Basic method to a share purchase analysis conducted in 1998. This involves an explanation of the Attributes I used and how I derived them, indicating problems I perceived and how I attempted to address them. I then describe a series of runs from Initial Option Reduction and subsequent reductions in which I employ a number of elicitation and representation mechanics, eventually homing in on a single CAF. I discuss the methodological conclusions I drew at the time.

I remind the reader of the limitations of the basic method in a portfolio situation and go on to discuss a more recent analysis based on May 2002 data using the Extended Model. I discuss the nature of the risk I am seeking to ameliorate and my tastes and attitudes concerning the valuation and representation of risk. I outline the risk measure incorporated.

I also describe how I use Beta, and a simplification that the Capital Asset Pricing Model allows me to make. I describe modifications to Attribute definitions relative to the 1998 analysis. I also discuss the issues of using a static model for sequential decision making and describe the approach that I chose to take in my role as analyst. I also discuss formulation short-cuts that can help to speed the NLP if it is taking too long.

Before going on to the actual analysis, I describe my initial share portfolio and some practical aspects which need to be acknowledged and taken account of.

I outline the issues involved in the selection of a Reference Portfolio and the choice made.

In this analysis I make particular use of the Larichev Decomposition and Attribute Weight Capping mechanics. My preferences between decomposed choices and their representation within the MP are discussed. The many analysis cycles involving capping adjustments are then outlined. This led to a single CAF which could be said to be my value function as decision maker. It also defined a theoretical portfolio. However practical problems had to be addressed and a series of other analyses were performed before an implementable plan was created. I describe these.

8.2 The background to my problem

As mentioned in Chapter 4, I have an interest in stocks and shares. This is married with a belief or hope that, over the long run, I should be able to do as well as the manager of a fund whose assets I (and my fellow stakeholder, my wife) might alternatively purchase, at least after management charges are added in. This supposition was partially attributable to occasional comments in the money advice press concerning the ability, or lack of it, of Managed Funds to outperform Trackers. Additionally I considered that the market is mainly constituted by Funds or professional advisers to people and organisations operating in a similar way, and, *de facto*, their average performance approximates to that of the market. This is not to suggest that there are not fund managers who are superior. But the information necessary reliably to identify them is not available to me. Moreover, it is difficult to distinguish luck from performance in this area.

I digress at this point. Whether I am right or wrong in this view is of a personal interest to me and my wife but is irrelevant to this thesis. This is not an exercise in the quality of my decision making judgement, nor do I claim a system designed to forecast the movement of share values in a way which beats the market. What is at issue is an aid to articulating objectives, judgements of the importance of issues, and the translations of these into decisions. This theme occurs throughout many of the succeeding sections. I will be explaining the use of technique as Researcher, as Decision Maker and as Decision Analyst supporting the Decision Maker. In the role of Decision Maker I claim for myself the same sovereignty over values that all decision makers can assert, that wise decision theorists recognise in the approaches they commend, and to which good analysts respond in problem identification, their models, and in the information they seek concerning fact and the mind of the decision maker.

The distinction I ask the reader to make between the writer as researcher and the writer as analyst is rather more difficult. All good analysts have to innovate or develop technique to effect solutions to a decision maker's problem, moreover both speak the language of models and technique. The borderline here is therefore inevitably unclear and where it is ambiguous it is appropriate that the researcher within the trichotomous personality should explain himself and not duck behind the skirts of the unseen analyst. Nevertheless matters which are purely analytic of *this* problem for *this* decision maker, and which another analyst and decision maker on a similar problem would review from first principles, will I hope be judged by the reader as incidental, even if they consider that they would take a different view in the circumstances. The selection of and some aspects of the processing of attribute information might be of this type. When I wish to make my assumed role unambiguous I may use the first person acronyms ADEMI (As decision maker, I) or IMRANI (In my role of analyst, I).

I also had a clear view of the investment philosophy that I wished to adopt. This was not one of trend spotting but is essentially a Value investing approach of the type first popularly promoted by Benjamin Graham, notably in *The Intelligent Investor* (1973, 4th Edition), first published in 1949. Graham's position is that it is difficult to pick market turning points; "..if [the investor] places his emphasis on timing, in the sense of forecasting, he will end up as a speculator and with the speculator's financial results." (p95). One should instead look to the fundamental characteristics of the companies underlying the shares and should seek to obtain good value for money for the part of the company one purchases.

My extension of this is that a company can be thought of as a package of properties to which can be attached value. These can be financial or non-financial, tangible or intangible, subjective or objective, relate to the present, the past or expectations of the future. The issue then is to buy the maximum quantity of constituents of that value for the minimum outlay.

If one had a clear prior view of how those combine to form a money equivalent, the problem becomes relatively simple and can be tackled within the models of MPT. What would have been effectively measured is the Alpha of a share, this can then be combined with assessed systematic risk Beta (or vectors of Betas) to generate investment conclusions. Indeed, as discussed in Chapter 4, this appears to be the approach of "Quant" investment houses. But, as mentioned, assessment of Alpha is the difficult part and the decision efficacy, given Alpha, is of far less consequence. Moreover there may be some qualities that relate to value that are not causes of cash performance. However, the key issue is that whether others can make these translations or not, IMRANI could not.

In any case this is not the only approach, nor probably the dominant approach, adopted. Professional houses also depend on the flair of managers to qualitatively translate company data into views of whether company is a good "buy" or not. They translate the disparate information they have on a company, its attributes, into a view of value, seemingly without using an explicit intermediate Alpha. This is also the approach of the press. The *Investors Chronicle,* for example, draws on quantitative data and qualitative information to argue value conclusions concerning individual shares eventually summarised on a semantic scale "sell", "high enough", "fairly priced", "good value", and "buy". Attention is sometimes paid to specific indicators, Price/Earnings is often a starting point for discussion. Another popularised by investor Jim Slater is "PEG" which is attentive to expected earnings growth. Graham would commend a wide range of financial indicators of the type outlined in Cottle, Murray, and Block (1988), (a recent edition of a book Graham published in the thirties), whilst ensuring that accounts examined reflected what they purported to and were adjusted where necessary. I felt that I had not the interest, time, nor confidence, to adopt a predominantly qualitative approach; nor the desire to depend on the *recommendation* of others. A quantitavist by training and disposition, I sought a number based approach that did not require me to make a *prior* judgement of how factors translate to value, particularly in a situation in where I recognised my objectives as vague. However, as will be seen, I was and am prepared to use the *opinions* of others who have made such judgements.

I perceived the problem as one of buying a package of Attributes, in a similar way that a blender of animal feed may seek to buy formulation components, each having some but different amounts of nutrients which contribute to their value. But the blender's problem is simplified by constraints on each nutrient within the compound he is required to develop. Hallerbach and Spronk (1997) expressed the same idea as balancing attributes at the portfolio level in a way that best suits an investor's preferences. This is a departure from return-variance planar thinking. But the question becomes how to do this?

It was DEA that stimulated the approach I adopted. Treat shares in the same way as DMUs. Determine, what each share does best and judge it in those terms. It was immediately apparent that whilst one could generate efficient shares and what I now call CAFs, it was what I wanted that counted; efficiency needed to be judged in my terms not from the standpoint of some anthropomorphic share. (In this I had a different outlook from Smith (1990) who suggests DEA as a method of measuring the financial performance of companies, to resolve the difficulty of selection from alternative financial performance ratios). Nevertheless, each potential optimum would be revealed with a circumstance under which it would be optimum. It was also apparent that the potential optima identified were those without constraints on value, allowing complete latitude on the CAF. Even if I was too vague about my objectives immediately to articulate a CAF that reflected them, it should be possible to progressively reduce the latitude, guided by the results achieved, progressively eliminating potential optima until one remained and an order for all other shares obtained.

In the set of analyses I describe first, I first sought a single share. This analysis process employed what I have described as the Basic Technique. This parallels the whole process for some types of decision analysis. However, it is limited within the investment selection subject area of the core problem, as it excludes consideration of interdependent risk effects which, as has been discussed, are important to portfolio formation. Nevertheless the analysis can be used as a valid first step towards a full portfolio formation process, by generating valuation listings which can be used as one input to a subjective portfolio selection, or as a means of articulating valuations for non-interactive attributes, which can later be used to condense the number of attributes used in a full portfolio analysis. I will later describe how this was extended to generate portfolios, whilst allowing for interdependent risk.

The analysis which I describe first represents a complete and self-contained sequence leading to a single choice. As in all analysis and model-building, it depends on previous experience. In structuring the model I had learnt from problems revealed in previous experiments. In this series new issues became apparent and the means employed to resolve them as they arose are reported. This researcher has never experienced tidy decision development or modelling either as a manager or, in his early career, as an analyst. This is a consequence of seeking to approximate situations as bounded problems when they never are. Events here were not wart free and necessitated backtracking, but the description has been sanitised only to the extent that blind alleys having no bearing on subsequent runs, or repetitions due to errors, or after "losing the thread", have not been reported. In seeking to convey the *flavour* of real rather than stylised problems, the description is long. The issues discussed are mainly those of decision maker and analyst. Whilst the reader may find some particular practicalities of detailed interest, the narrative need not always be read in its full detail

I should add that, in my early work, I sought the Decision Maker's hidden value function and secured option reduction mainly by incorporating the Decision Maker's declared preferences between the Attribute combinations of specific pairs of companies. It is since (and because of) this early work that I have become less comfortable with my own ability to make, and the general soundness of, such judgements. In the 1998 work I relate first, I continued to use of this mechanic but also explored other methods to help home in on a single value function and a best share selection and share rank ordering.

8.3 Data for the Basic Approach

I purchased Company REFS which publishes detailed company financial statistics in a comparable format, news developments and company comments (latterly on CD). This was and is the main effective source of data for all my share analyses. However, I originally "cut and pasted" data from the CD into Excel for processing. Hemmington-Scott, originally the publishers of REFS, later kindly assisted by supplying much of the same data in a more easily processed form but nothing was used that was not calculable from the REFS Company Pages. I subscribed to Infotrade, which although also publishing some company data, made daily share price data available for downloading. I also regularly received *Investors Chronicle* for background and sometimes referred to newspaper internet archives when finalising trades.

I confined my interest to shares in the FTSE350 index which provided a large enough pool to work with. Although there would be interesting opportunities outside this, good selection within it should secure most of the benefits of a wider universe. Moreover, the data management problems of a larger pool would become difficult. I also consider myself risk averse and concentration on major shares appeared "safer". (I later excluded financial shares from the set of shares considered, for reasons mentioned below. Investment Trusts are constituents of the index but, as these are themselves share portfolios and not trading companies, these were also excluded). In the particular analyses I describe here, I also initially excluded one other company with an excessively low PE ratio which would have resulted in the spurious exclusion of more normal shares from efficient sets. Two further examples of companies having highly distorted data were identified during the course of the analysis. Their treatment will be described. Overall, some 317 shares were initially available for selection in the analysis.

8.4 My objectives as Decision Maker

ADEMI felt that the following Vague objectives were relevant to my share selection decision. I made prior assessment of the relative importance of these factors:

- High level of company earnings relative to a share price.
- Solidity and lack of uncertainty of those earnings
- Growth in historic earnings relative to share price
- Growth in future earnings relative to share price
- Financial security/conservatism of the company
- Good Dividend yield
- Minimal impact of adverse factors already known to the market, as evidenced by a company's share price performance over the preceding year relative to the market
- Good cash generation relative to earnings of the company

ADEMI perceived that as the past was a partial guide to the future, value can be attributed to historic statistics. But I also made vicarious use of other peoples' judgements by using brokers' Consensus Forecasts of the future level and growth of earnings.

8.5 The original 1998 analysis using the Basic Method

8.5.1 The Attribute variables used

The variables used were:

- Earnings per Share / Present Price per Share (EPS/PPS) for latest actual year (Yr 0) (E0)

- Cumulative EPS/ Present PPS for Yr-5 to Yr 0 (QE-5,..,QE-1) (QE-5 was directly used, QE-4 to QE-1 were used to calculate WE)
- A linearly weighted average "wedge" of the EPS from Yr-4 To Yr 0 with weights of 1,2,3,4 for each successive year (WE) see below
- Forecast EPS/ Present PPS for Yr +2 (E2)
- Growth in EPS/ Present PPS from Yr-5 to Yr 0, split into four variables, ie that amount constituting
  - ... growth from positive EPS positions within the period (T^{++})
  - ... declines from positive EPS positions within the period (T^{+-})
  - ... growth from negative EPS positions within the period (T^{-+})
  - ... declines from negative EPS positions within the period (T^{--})
- Growth in EPS/ Present PPS to Yr +2 (T2)
- Dividend per Share/ PPS for Yr 0 (D0)
- (Cash Flow-EPS)/PPS for Yr) (C0)
- A single measure representing Net-Gearing-Cash Richness (for ungeared Companies) and Negative Equity (where it existed) (GCR), defined:
  - ... Zero, if a company's assets are less than its borrowing (i.e. it has negative equity)
  - ... 100-100x (Borrowings net of Liquid Assets)/ (Borrowings net of Liquid Assets +Shareholders' Equity), for positive net borrowing
  - ... 100-100x (Liquid Assets net of Borrowings)/ (Liquid Assets net of Borrowings +Shareholders' Equity), for negative net borrowing.
- A binary "Is Financial?" variable signifying whether the Company is a Bank or Insurance Company for which some statistics are not available (IF)
- One year Relative Strength of the share, if it is negative, and 0 if it is positive (RSNO)

ADEMI felt that these attributes were generally reflective of my Vague Objectives. They were calculated for each share in the analysis set.

It will be noted that most variables above are normalised expressions of relative value found by dividing a "money flow" per share by price per share. They are not strictly dimensionless but are effectively so as money flows can be equated to momentary money value by discounting. The other variables are dimensionless measures to which similar value can be attached.

Amplification of the purpose or structure of some variable sets adopted in my analyst role is appropriate.

8.5.2 "Block" and "Wedge" weighting of historical Earnings per Share

ADEMI sought to include a weighted average of historic EPS to which could be attached value. As analyst this presented minor difficulties. I wanted to reflect a diminishing importance of less recent elements of the historic EPS vector. I wanted the diminution to be smooth, to be parsimonious in parameters, to be determined in the analysis not predetermined, and to be manageable within LP structure. I would have liked exponential weighting but it could not be readily approximated within a linear model.

In earlier analyses 6 historic EPS measures were used. These were the latest EPS, the cumulative EPS for the last two years, the cumulative for the last three years, for the last four, for the last five and for the last six. A weighted sum of such composites creates a weighted sum of the variables from which they are composed, but if the weighting coefficients of these variables is constrained to be greater than zero (a feature of the normal LP formulation), diminishing influence weighting of the primary variables is achieved.

The problem with the formulation is that it allows "unsmooth" weighting patterns for which justification in terms of a rational decision maker's value set would be difficult to justify. As it also implied a capability of fitting a six parameter model of just a single factor to a share, and failed the test of reasonable parsimony.

However, IMRANI felt an adequate reflection could be achieved with a two variable model. One of the variables, the "Block" was the cumulative EPS over six years. The other was a linear declining weighting, or triangular "Wedge" of relative influence in which the latest year had a relative weight of 5, and each of the 4 preceding years had a weight of 4, 3, 2, and 1 respectively. In practice this was achieved within the LPs by equally weighting four of the five cumulative variables already defined, and constraining them to the weight that the analysis assigns to the latest EPS (E0). A long-term influence could be reflected by the Block having a greater weight relative to the Wedge and vice versa.

In more recent analyses a different approach has been adopted.

8.5.3 Historic Earnings trend

In earlier explorations, high scoring shares included a significant number of shares where high increases in earnings had been achieved but from poor or negative earnings bases. ADEMI felt that this did not reflect the value I put on growth in different situations. I considered that:

- Growth from a loss-making position is less valuable than corresponding growth from positive earnings.
- The loss of perceived share value resulting from a positive earnings figure A declining to a lesser one B, is greater than the gain in share value that is attributable to an improvement in profit of the same amount, that is if earnings were to move from B to A.
- A decline in earnings from a loss-making position has more effect on perceived value than the corresponding decline from positive earnings.

To give effect to this, the Earnings per share movements over 5 years were split into four variables, separating those parts which represented decline within the period relative to the previous year, from those parts constituting growth, in combination with whether the growth or decline was from positive or negative earnings. The aggregate of the four trend variables always equalled the total trend over the whole period. This facilitated incorporation of both non-linear and irreversible or "hysteresis" effects, and also penalised unsteady earnings patterns relative to, otherwise equal, steady ones.

8.5.4 Present value versus past value ambiguity and the "Relative Strength" work-around

The purpose of the analysis is to calculate the *current* value of a share in terms of its attributes. These attributes may reflect recent events or past events, as in the inclusion of Historic EPS data. But these data also contribute to the *past* value of a share. It was noticed that in early experiments a number of shares were appearing to be efficient but had fallen significantly in price over the latest data year. Recent statistics of such shares were poor. In consequence, in seeking to show the shares in their Best Possible Light, the program over-emphasised historical attributes. It, in effect, synthesised a *past* valuation and, when this was compared with a *present* share price it scored such shares well. Such an ambiguity is clearly a dangerous one.

To avoid the issue, one approach might be to exclude all historic data but ADEMI felt that historical attributes did influence present valuation. Another might be to exclude from the consideration all shares which have fallen relative to the market. However, this would exclude some shares which were genuinely efficient against current considerations.

A different approach was adopted. One-Year Relative Strength (the percentage by which the share moved relative to the market average over the preceding year), was incorporated, but only for shares having negative Relative Strength (ie declining) shares only. The variable was set to zero for all shares with positive Relative Strength. This enabled the following approaches:

- Prevention of Negative Relative Strength shares from pushing out otherwise efficient shares of better Relative Strength from the efficient set.
- Inclusion of valuation penalties if and when ADEMI felt that it reflected valuation of the attribute's negative impact.
- The discounting of the historic components of the various EPS measures, for falling shares, to offset potential valuation bias.

8.5.5 Net Gearing-Cash Richness

The rationale for the net cash element of this expression is that it treats cash as having an opposite but similar magnitude of impact to borrowings, where net borrowings are smaller than equity. It also discounts the effect of large net cash assets. Later, the treatment of these factors was reviewed and changed.

8.5.6 The "Is Financial?" variable

The raw materials for the Gearing-Cash Richness measure is not available for Banks or Insurance Companies. In the analysis these companies were given gearing statistics as if they had negative equity. To compensate for this conservative treatment, financial houses were assigned a dummy variable to which the procedure could assign value. However, throughout the analysis, constraints were placed on the value that could be assigned to it. Later, financial companies were excluded from analysis on grounds of administrative simplicity, as Hemmington-Scott were not able to supply simply accessible data to me.

8.5.7 The Option Reduction runs

Run 1. Finding the initial Efficient Set. Initial Option Reduction

The LP was structured as described in Chapter 5. The additional constraints mentioned above were incorporated. One other set of pre-emptive constraints was imposed on the analysis. This was on the weightings, $w_S(T^{++})$, $w_S(T^{+-})$,$w_S(T^{-+})$, $w_S(T^{--})$, permitted for the historic trend attributes T^{++}, T^{-+}, T^{+-}, T^{--} for any share S. These were:

$$w_S(T^{+-}) \leq w_S(T^{++}) \leq w_S(T^{-+}) \leq w_S(T^{--})$$

Constraints were included to ensure that the valuation for any share was no greater than 1. (In these runs the Andersen-Petersen adaptation was *not* adopted, though I now usually prefer to exclude the investment being assessed from the Comparison Set). 13 independent variables corresponding to attributes were incorporated. An LP was individually run for each of the 317 shares in the analysis set, maximising the Maximal Comparative Advantage (MCA) for each, and finding the Comparative Advantage Function (CAF) and the Best Possible Light set of attribute weightings, subject to not violating the constraints discussed.

MCAs were found for every share. The contribution of each attribute to the aggregate MCA, and the efficient Peers and other shares scoring well under its CAF valuation, were recorded for each share. The individual attribute weights were also available from each LP, but were not routinely recorded by the summarising program.

The MCAs found are tabulated in Table 8.1. The number of efficient shares, that is having MCAs of 1, was 99.

Run 2. Starting the Second Stage Reduction to restrict Criterion Space. Further pre-emptive assumptions.

The imposition of further constraints restricting the latitude of the decision maker's value function, demotes some of the originally efficient set. But none outwith the set can become efficient. It is therefore permissible to restrict subsequent consideration to a reduced Comparison Set of efficient shares. As the macro controlled computerised analysis sequence for 317 shares with 317 share constraints, at the time, took over 8 hours to run on a 486DX4, it was worth reducing the considered set in this way. This was done for subsequent runs (in fact 103 shares were included in the modified Comparison Set). [Much faster software and hardware subsequently reduced this problem].

In Run 2, ADEMI took a less open view on some aspects of the value of forecast growth amount in EPS relative to the future level and on the relative potency of historic growth or declines classified in the attributes T^{++}, T^{+-}, T^{-+} and T^{--}, and this view was reflected in the tighter constraints:

$$2.w_S(T^{+-}) \leq w_S(T^{++}) \leq 0.5.w_S(T^{-+}) \leq 0.25.w_S(T^{--})$$

- The relationship between the value of future EPS trend and future EPS level depends on Discounted Cash Flow considerations. In principle the relative value of an increasing level of real earnings can be equated with a corresponding steady level of real earnings through Net Present Value calculations. But it poses the question, "How long might we expect a forecast trend to persist?". To get a simple handle on this, IMRANI undertook some simple statistical analyses directed to finding the extent to which growth in one year is related to growth in the following year. From this I assumed a rate of erosion of the forecast earnings trend in subsequent years and, using DCF, approximated the value equivalence of a forecast trend, relative to steady state earnings. ADEMI considered this represented the lower bound of the value of trend, for reasons related to the possible conservatism of the estimate, and because ASDEMI considered a strong projected trend as a measure of safety, not just an indicator of future earnings alone.

ADEMI considered that twice the lower bound constituted a reasonable upper limit.

$$0.155.w_S(E2) \leq w_S(T2) \leq 0.325.w_S(E2)$$

(sic. The inconsequential discrepancy from intention was spotted later)

When these constraints were added to the model and the LPs re-run, 64 shares were still found to have an MCA of 1, a reasonable reduction of the efficient set. These are shown in Table 8.2.

Table 8.1 Maximal Comparative Advantage (OI)* after Run 1

	MCA(OI)		MCA(OI)		MCA(OI)		MCA(OI)
Abbey National	1.000	Christies International	1.000	Legal & General	0.883	Schroders	0.801
Admiral	0.837	Close Brothers	0.813	Lex Service	1.000	Scottish & Newcastle	0.709
AEA Technology	0.620	CMG	0.871	Liberty International Holdings	0.665	Scottish Hydro-Electric	1.000
Aegis	0.436	Coats Viyella	1.000	LIMIT	1.000	Scottish Media	0.564
Aggregate Industries	1.000	Cobham	0.577	Lloyds TSB	0.881	Scottish Power	0.989
Aggreko	0.641	COLT Telecom	0.752	Logica	0.742	Sears	0.982
Airtours	0.975	Compass	0.392	London & Manchester	1.000	Securicor	0.435
Albright & Wilson	1.000	Cookson	0.927	London Clubs International	0.925	Sedgwick	1.000
Alliance & Leicester	1.000	Cordiant Communications	0.332	London Forfaiting Co	0.924	Select Appointments	0.935
Alliance Unichem	0.681	Courtaulds	0.742	London International	0.463	Sema	1.000
Allied Domecq	0.840	Croda International	0.731	Lonrho	1.000	Senior Engineering	0.733
AMVESCAP	0.437	CRT	0.881	LucasVarity	0.988	Serco	0.366
Anglian Water	1.000	Daily Mail & General Trust	0.468	M&G	0.773	Severn Trent	1.000
Arcadia	1.000	Danka Business Systems	1.000	Man (ED&F)	0.922	Shell Transport & Trading	0.710
Argos	1.000	Davis Service	1.000	Manchester United	1.000	Shire Pharmaceuticals	0.767
Arjo Wiggins Appleton	1.000	De La Rue	1.000	Marks & Spencer	0.679	Siebe	0.632
Arriva	1.000	Debenhams	1.000	Marley	1.000	SIG	1.000
Asda	0.734	Delta	1.000	Mayflower Corporation	0.822	Signet	1.000
Ashstead Group	0.906	Devro	0.853	McKechnie	1.000	Slough Estates	0.466
Associated British Foods	0.582	DFS Furniture	1.000	Medeva	1.000	Smith & Nephew	0.862
Associated British Ports Holdi	1.000	Dixons	0.650	Meggitt	0.679	Smith (David S) Holdings	0.988
Atkins (WS)	1.000	Electrocomponents	0.792	MEPC	0.527	Smith (WH)	0.824
Avis Europe	0.508	Elementis	0.943	Mersey Docks & Harbour Co	0.835	SmithKline Beecham	0.525
BAA	0.614	EMAP	0.642	Meyer International	0.856	Smiths Industries	0.719
Bank of Scotland	0.900	EMI	0.877	MFI Furniture	1.000	Somerfield	0.802
Barclays	1.000	English China Clays	0.923	Micro Focus	0.998	South West Water	1.000
Barratt Developments	1.000	Enterprise Oil	0.926	Millennium & Copthorne Hotel	0.822	Southern Electric	1.000
Bass	0.669	Express Dairies	1.000	Mirror Group	0.797	Spirax-Sarco Engineering	0.814
BAT Industries	0.963	Fairey	1.000	Misys	0.304	St Ives	0.924
BBA	0.684	FI Group	0.929	Monument Oil & Gas	1.000	St James's Place Capital	0.626
Beazer	1.000	First Leisure	0.701	More Group	0.644	Stagecoach Holdings	0.577
Bellway	1.000	FirstGroup	0.612	Morgan Crucible	0.971	Stakis	0.965
Berkeley Group	0.932	FKI	1.000	Morrison (WM) Supermarkets	0.854	Standard Chartered	1.000
BG	1.000	Flextech	0.596	National Express	0.954	Storehouse	1.000
BICC	1.000	Galen Holdings	0.780	National Grid	0.739	Sun Life & Provincial Holding	0.458
Billiton	1.000	Gallaher	1.000	National Power	0.875	Tarmac	0.803
Biocompatibles International	1.000	General Accident	1.000	National Westminster Bank	0.970	Tate & Lyle	0.789
Blue Circle Industries	1.000	General Cable	0.459	Newsquest	0.727	Taylor Woodrow	1.000
BOC	0.601	General Electric Co	1.000	Next	0.855	TBI	1.000
Body Shop International	0.791	GKN	1.000	NFC	0.864	Telewest Communications	1.000
Bodycote International	0.929	Glaxo Wellcome	0.429	Northern Foods	0.841	Tesco	0.701
Booker	1.000	Glynwed International	1.000	Northern Rock	1.000	Thames Water	1.000
Boots	0.822	Granada	0.660	Norwich Union	0.786	Thistle Hotels	0.927
Bowthorpe	0.864	Great Portland Estates	0.545	Nycomed Amersham	0.743	Thorn	1.000
BPB	0.863	Great Universal Stores	0.867	Ocean Group	0.868	TI	0.762
Bradford Property Trust	0.680	Greenalls	0.906	Orange	0.410	Tomkins	1.000
Britannic Assurance	1.000	Greene King	0.807	P&O	0.872	Travis Perkins	1.000
Britax International	1.000	Guardian Royal Exchange	1.000	Parity	0.965	Trinity International Holdings	0.737
British Aerospace	1.000	Halifax	0.760	Pearson	0.554	TT	0.976
British Airways	0.694	Halma	0.856	Peel Holdings	0.586	Unigate	1.000
British Biotech	0.998	Hammerson	0.506	Pentland Group	0.843	Unilever	0.871
British Energy	1.000	Hanson	1.000	Perpetual	0.954	United Assurance	1.000
British Land	0.355	Hardy Oil & Gas	0.414	Persimmon	0.968	United Biscuits (Holdings)	0.831
British Petroleum	0.657	Hays	0.676	Pilkington	0.967	United News & Media	0.908
British Sky Broadcasting	0.480	Hazlewood Foods	0.818	Pillar Property	0.484	United Utilities	1.000
British Steel	1.000	Hepworth	0.824	PizzaExpress	0.800	Vaux	0.903
British Telecommunications	0.899	Hewden Stuart	1.000	Powell Duffryn	1.000	Viridian	1.000
British Vita	0.949	Highland Distilleries	0.709	PowerGen	0.893	Vodaphone	0.417
British-Borneo Petroleum	1.000	Hillsdown Holdings	1.000	Premier Farnell	0.727	Wassall	1.000
Brixton Estate	0.584	House of Fraser	0.864	Premier Oil	0.822	Weir	0.979
Brown (N) Group	0.627	HSBC Holdings	0.906	Provident Financial	0.684	Wessex Water	1.000
Bryant Group	1.000	Hyder	1.000	Prudential Corporation	0.686	Wetherspoon (JD)	0.688
BTG	0.734	IMI	0.973	Racal Electronics	0.585	Whitbread	0.673
BTP	0.856	Imperial Chemical Industries	0.676	Railtrack	0.774	Willis Corroon	1.000
BTR	0.856	Imperial Tobacco	1.000	Rank	0.864	Wilson Bowden	0.961
Bunzl	0.865	Inchcape	1.000	Reckitt & Colman	0.590	Wilson Connolly Holdings	1.000
Burford Holdings	0.479	Independent Insurance	1.000	Redrow	0.958	Wimpey (George)	1.000
Burmah Castrol	0.845	Inspec	1.000	Reed International	0.502	Wolseley	0.786
Cable & Wireless	0.789	Ionica	1.000	Rentokil Initial	0.455	Wolverhampton & Dudley Bre	0.900
Cadbury Schweppes	0.662	Jarvis	0.914	Reuters	0.651	Woolwich	0.815
Cairn Energy	0.708	JJB Sports	1.000	Rexam	1.000	WPP	0.630
Caledonia Investments	0.607	Johnson Matthey	0.996	Rio Tinto	0.655	Yorkshire Water	1.000
Capita Group	1.000	Johnstone Press	0.729	RMC	0.763	Yule Catto & Co	0.852
Capital Radio	0.743	Kalon Group	0.914	Rolls-Royce	0.750	Zeneca	0.606
Caradon	0.839	Kingfisher	0.717	Royal & Sun Alliance Insuran	0.791		
Carlton Communications	0.885	Kwik-Fit Holdings	0.929	Royal Bank of Scotland	1.000		
Carpetright	1.000	Ladbroke	0.613	Rugby Group	0.883		
Cattles	0.591	Laird Group	1.000	Safeway	0.646		
Centrica	0.916	Land Securities	0.538	Sage	0.233		
Charter	0.840	Laporte	0.833	Sainsbury (J)	0.648		
Chelsfield	0.537	LASMO	0.511	Scapa Group	0.888		

* Basis of this set of runs was that each option under consideration was in Reference Set

Table 8.2- Shares in Efficient Set after Successive Preference Elicitation

	Run1	Run 2	Run 3	Run 4	Run 5	Run 6	Run 7	Run 8	Run 9	Run 10	Run 12	Run 13	Run 14	Run15
Abbey_National	1	1	1						1					
Aggregate_Industries	1	1	1	1					1	1				
Albright_&_Wilson	1													
Alliance_&_Leicester	1													
Anglian_Water	1	1	1											
Arcadia	1													
Argos	1	1	1	1			1			1	1	1		
Arjo_Wiggins_Appleton	1	1	1											
Arriva	1	1	1	1	1		1					1		
Associated_British_Ports_Holdings	1													
Atkins_(WS)	1	1	1	1						1		1		
Barclays	1								1	1				
Barratt_Developments	1	1	1											
Beazer	1	1	1											
Bellway	1	1	1											
BG	1													
BICC	1													
Billiton	1	1	1	1					1	1				
Biocompatibles_International	1	1	1	1										
Blue_Circle_Industries	1													
Booker	1	1	1											
Britannic_Assurance	1	1	1	1	1									
Britax_International	1													
British_Aerospace	1													
British_Energy	1	1	1	1	1									
British_Steel	1	1	1	1										
British-Borneo_Petroleum	1													
Bryant_Group	1	1	1											
Capita_Group	1	1	1	1										
Carpetright	1	1	1											
Christies_International	1								1	1		1		
Coats_Viyella	1	1	1											
Danka_Business_Systems	1	1	1											
Davis_Service	1	1	1	1	1		1	1	1	1	1	1		
De_La_Rue	1	1	1											
Debenhams	1	1	1											
Delta	1	1	1	1										
DFS_Furniture	1	1	1	1										
Express_Dairies	1	1	1											
Fairey	1													
FKI	1	1	1											
Gallaher	1	1	1											
General_Accident	1													
General_Electric_Co	1	1	1											
GKN	1													
Glynwed_International	1	1	1											
Guardian_Royal_Exchange	1													
Hanson	1	1	1											
Hewden_Stuart	1													
Hillsdown_Holdings	1	1												
Hyder	1	1	1											
Imperial_Tobacco	1	1	1											
Inchcape	1													
Independent_Insurance	1													
Inspec	1	1	1											
Ionica	1	1	1											
JJB_Sports	1													
Laird_Group	1	1	1											
Lex_Service	1	1	1	1		1			1	1	1	1	1	1
LIMIT	1								1	1				
London_&_Manchester	1													
Lonrho	1	1	1											
Manchester_United	1	1	1											
Marley	1													
McKechnie	1	1	1	1										
Medeva	1													
MFI_Furniture	1													
Monument_Oil_&_Gas	1													
Northern_Rock	1													
Powell_Duffryn	1	1	1											
Rexam	1	1												
Royal_Bank_of_Scotland	1													
Scottish_Hydro-Electric	1	1												
Sedgwick	1	1	1	1										
Sema	1	1	1	1										
Severn_Trent	1	1	1											
SIG	1													
Signet	1	1	1	1	1		1							
South_West_Water	1	1												
Southern_Electric	1	1												
Standard_Chartered	1													
Storehouse	1	1	1											
Taylor_Woodrow	1													
TBI	1	1	1	1						1	1			
Telewest_Communications	1													
Thames_Water	1	1	1											
Thorn	1	1	1	1	1		1			1	1	1		
Tomkins	1	1	1	1							1	1		
Travis_Perkins	1									1				
Unigate	1	1	1	1										
United_Assurance	1	1	1	1					1	1				
United_Utilities	1													
Viridian	1	1	1	1						1				
Wassall	1	1	1											
Wessex_Water	1	1	1											
Willis_Corroon	1	1	1	1	1	1			1	1	1	1	1	
Wilson_Connolly_Holdings	1													
Wimpey_(George)	1	1	1	1										
Yorkshire_Water	1	1												

Run 3. Allowing for some Larichev-type preferences

At the time of undertaking this analysis, I had not fully developed the Larichev Decomposition procedure described in Chapter 7. Nevertheless, in this part of the analysis, I used some of these ideas. I established a reference case constituted by the average of the attribute levels of the 317 shares under consideration. Then for a selection of attribute measures, I sought to find the mix of efficient shares that gave each attribute in turn its most favourable value, subject to no other attribute being permitted to take up a value inferior to a reference case. These best values for each attribute were efficient solutions and were also found using LP (requiring one run per attribute). ADEMI then ranked preferences amongst each variation from the reference case.

Table 8.3-Larichev Choice variations from reference case

Case	Attribute	Reference Level	Best Possible Value of Designated Attribute*	Difference of Best Value from Reference
1	QE-5	0.256	0.545	0.289
2	WE	0.729	1.479	0.750
3	E2	0.060	0.117	0.057
4	T^{++}	0.0343	0.0996	0.065
5	T2	0.0066	0.0463	0.039740
6	CO	-0.026	0.148	0.174
7	DO	0.021	0.063	0.042
8	GCR	70.0	157.3	87.300
9	RSNO	-3.10	0	3.100

*All other attributes at their reference level or better

Thus if, as decision maker, I prefer Case3 to Case2, it implies a preference for a level of 0.117 for attribute E2 and 0.729 for WE, to a level of 0.060 and 1.479 for the same variables, all other variables remaining at reference. Thus, I prefer an increment of 0.057 in attribute E2 to one of 0.750 in attribute WE. Accordingly:

$$0.057.w_S(E2) \geq 0.750.\ w_S(WE)$$

Unfortunately ADEMI remained vague regarding many of these comparisons and felt unable to express preferences between many of the above cases. I was nevertheless prepared to assert that I preferred:

Case 3 to Case 2 to Case 1

Case 2 to Case 4

and Case 6 to Case 7

The constraints corresponding to these were incorporated and the LPs were re-run.

The number of efficient shares reduced from 66 to 58 as a result. These are shown in Table 8.2.

In this instance, these preferences only helped to refine the search a small amount. However, this does not imply that the approach will always lack potency. Thus, if Case 5 was asserted as preferable to Case 3, a further reduction of 14 shares would have resulted.

Run 4. Incorporating Preference Bracketing. An over and under compensation methodology.

An alternative [1,1] methodology is Preference Bracketing (see Chapter 5). This asks the subject to propose relationship bounds which he or she is confident with, rather than presenting pre-determined choices that he or she may be unable to distinguish.

ADEMI sought to compare the sum of all the EPS components (relative to current share price) (E) to Gearing/Cash Richness (GCR), Cash Flow -Latest EPS (relative to share price) (CO), Relative Strength (non-positive only)(RSNO). In DM role I considered the following statements to be reasonable:

(7% E, 80% GCR)$\precsim$(5% E, 30% GCR$\precsim$(7% E, 40% GCR)

(4% E, -1.6% CO) $\precsim$(5% E, -2.6% CO)$\precsim$(4% E, +2.4% CO)

(7% E, -30% RSNO)$\precsim$(5% EPS, -5% RSNO)$\precsim$(7% EPS, -10% RSNO)

These implied :

10% ΔGCR$\precsim$2% ΔE$\precsim$50% ΔGCR

1% ΔCO$\precsim$1% ΔE$\precsim$5% ΔCO

5% ΔRSNO$\precsim$2% ΔE$\precsim$25% ΔRSNO

These were translated into LP constraints eg $w_{C0} \leq w_E \leq 5w_{C0}$ and the shares reassessed. The efficient set reduced to 26.

Run 5. Seeking weight balance

In this run I sought to avoid excessive dependence on certain variables which appeared to be getting undue prominence. ADEMI looked at the most favourable (usually largest) value of an attribute occurring amongst all 317 shares in the full analysis set. I specified that this level of an attribute could not make a bigger contribution to the MCA than a selected, though arbitrary limit, specific to the attribute. Upper limits for the corresponding weighting parameters were set accordingly. The following additional constraints were inserted in the LPs:

$$172.w_s(GCR) <=.5$$

$$0.17.w_s(E2) <=.7$$

$$1.13.w_s(T^{+-}) <= .15^*$$

$$0.11.w_s(DO) <=.25$$

The efficient set was reduced to 7.

Run 6. Preference between companies

At this stage it became possible to scrutinise the shares within the efficient set individually. It was observed that one, Britannic Assurance, was an unreasonable outlier. I excluded this company from the constraint structure so that it would not influence the valuation of any other share. This might allow shares to re-enter the efficient set including those not in the short-list of 103, pushed-out after Run 1. However, as no other shares had been constrained by Britannic, recalculation was redundant.

ADEMI felt that of the six remaining shares, two (Arriva and Thorn) had attributes which to me made them less preferable to two others (Davis Service and Willis-Corroon) in the analysis set. Constraints were added to achieve the implied reduction in valuation latitude.

The efficient set reduced to two, Lex Service and Willis-Corroon.

Run 7 Back-tracking to re-model

Both Lex and Willis-Corroon were, in the light of the data, acceptable shares with appealing sets of attributes. However they both had, what seemed to me as decision maker, a high weighting of the Gearing/Cash Richness parameters. In the case of Willis-Corroon in particular, over half the MCA was accounted for by this attribute. Willis-Corroon was very cash rich and was reasonably cheap. It had a generally solid earnings history but had declined in the past year and was not expected to fully recover quickly. It had come to the top of the pack by virtue of its cash.

ADEMI felt that this had been over-emphasised in the results of the analysis. However, I was keen to penalise shares with poor gearing. It seemed that in using a single variable to reflect Cash richness and Gearing I had denied myself an opportunity for a more distinctive treatment. In reality, I did not attribute equal value to movements in Gearing to those of Cash Richness. I decided to split the variable into two, as well as constraining the coefficient of the Gearing attribute to be larger than that for Cash Richness.

It was necessary to go back in the sequence of runs. As the change was a radical one the full set of 317 shares was used again. The share constraints subsequent to Run 5 were removed. The constraint $2.w_S(T^{+-}) \leq w_S(T^{++}) \leq 0.5.w_S(T^{-+}) \leq 0.25.w_S(T^{--})$ was also inadvertently reinstated to its Run 1 form,
$w_S(T^{+-}) \leq w_S(T^{++}) \leq w_S(T^{-+}) \leq w_S(^{T--})$.

The efficient set increased again this time to Argos, Arriva, Davis-Service, Signet and Thorn.

Run 8. Constraints reinstated

At this stage the company constraints developed for Run 6 were reinstated; the Cash Richness constraint was set so that at a Cash Richness of 72% the maximum contribution it could make to the MCA was 20%. However, the continued use of the over-relaxed form of the historic trend constraints, was not spotted.

The run resulted in a single efficient share Davis Service. Again whilst quite satisfied with Davis Service as a selection, I was not satisfied with what seemed an overweight given to one attribute, this time the Cash Flow-EPS difference.

Run 9. Contradictions

Some piece-meal experimentation was now undertaken with individual constraints being tried and individual shares being tested rather using a full formal run. It was apparent that the imposition of constraints designed to achieve a balance effect with which ADEMI was comfortable resulted in infeasible LPs. This implied internal contradictions between some of the constraints imposed. As a result of this I removed or relaxed some existing constraints that I was no longer confident in, whilst introducing or reinstating old ones. In the formal run 9, I:

- reinstated $2.w_S(T^{+-}) \leq w_S(T^{++}) \leq 0.5.w_S(T^{-+}) \leq 0.25.w_S(T^{--})$
- set 72 $CR \geq 0.1$
- set .26 T^{++}>= .1
- removed the Run 3 constraint where Case2 (WE at its extreme) was preferred to Case 4 (T^{++} at its extreme).

Run 10. Continuation

Attribute weight balance in the observed efficient set, still did not meet my perceptions of my valuation objectives. The difficulties were caused by the upper and lower bounds of attribute contributions and these constraints were thrown out (I now use this mechanic with circumspection). I introduced a relative upper bound on the T^{--} attribute which was taking up excessive values relative to T^{++}.

This was preparatory to more systematically imposing absolute "top-down" weighting restrictions after scrutiny of the BPL weightings, rather than by mixing floor and ceiling constraints. These used simultaneously had inhibited a clear view of what was going-on and caused infeasibility. (I now favour this approach).

16 efficient shares emerged.

Runs 11 to 14 Capping large weightings

Solutions now appeared that placed excessive weight on the Relative Strength parameter. This penalised falling price shares by more than ADEMI felt reasonable. The parameter for RLNO was capped with an upper-bound of 0.04.

Then Dividend D0 parameter also appeared too high in many BPL value functions. This was also capped.

An efficient set of 7 shares meeting all the constraints was found which were again individually checked. Another distorting share, TBI, was excluded. This resulted in another small temporary extension of the short-list

The Cash Richness parameter CR was allowing some shares to have an excessive MCA contribution from this factor. This was accordingly capped at a level which reduced it to what ADEMI considered reasonable.

The shortlist reduced to Lex Service and Willis-Corroon. The latter still depended on CR for a very high proportion of its RV. The CR cap was therefore further reduced to the minimum consistent with feasibility, that is to .005

Only one share emerged from the next run with a score of 1; Lex Service.

Run set 15. Fixing a single Comparative Advantage Function

There was still available latitude in the valuation within the preference constraints developed whilst imposing an MCA of 1 on Lex. To find a unique function Lex was constrained to equal 1. The DO parameter still seemed high so the LP was run to minimise this. Once this had been fixed at its minimum value, no more weight flexibility could be found. This fully determined a unique valuation function within the preferences expressed. Table 8.4 records the parameters found and Table 8.5 the scores of the top 100 shares using these weights.

Table 8.4- Unique Valuation Function Found after Run Set 15 with Attribute Contributions for Lex

	IF	G	CR	E-5	E-4	E-3	E-2	E-1	E0	E2	T^{++}	T^{+-}	T^{-+}	T^{--}	T2	D0	C0	RSNO
Attribute Values	0	0	0.473934	0.336595	0.301566	0.247554	0.192172	0.138356	0.077104	0.091585	0.04364	0	-0.00157	0	0.014481	0.034247	-0.009	0
Attribute Weights	0.4	0.008152	0.005	0	0	0	0	0	0.000296	4.071739	7.999508	0	60.35954	120.7191	1.893359	11.078	4.076185	0.04
Factor Contributions	0	0	0.00237	0	0	0	0	0	2.29E-05	0.372911	0.349098	0	-0.0945	0	0.027419	0.379384	-0.03669	0

Table 8.5 MCAs after Run Set 15

Top 100

Company	MCA
Lex Service	1.000
Arriva	0.900
Thorn	0.900
Davis Service	0.900
Willis Corroon	0.900
Tomkins	0.882
Christies International	0.812
Viridian	0.811
Aggregate Industries	0.793
Britax International	0.750
Sema	0.727
Atkins (WS)	0.722
GKN	0.701
Unigate	0.642
Kwik-Fit Holdings	0.623
United Assurance	0.603
Billiton	0.582
Storehouse	0.569
LIMIT	0.568
Perpetual	0.551
Barclays	0.522
McKechnie	0.519
Capita Group	0.515
General Electric Co	0.507
Argos	0.504
National Express	0.486
Associated British Ports Holdings	0.481
Bodycote International	0.475
Abbey National	0.475
Ocean Group	0.471
Railtrack	0.467
LucasVarity	0.447
Partty	0.446
Thistle Hotels	0.445
Somerfield	0.411
Select Appointments	0.410
Thames Water	0.407
Arcadia	0.395
St Ives	0.384
BPB	0.382
Cable & Wireless	0.381
Bowthorpe	0.380
Hewden Stuart	0.379
FI Group	0.378
IMI	0.374
Morrison (WM) Supermarkets	0.368
Alliance & Leicester	0.363
Boots	0.360
CMG	0.346
BTP	0.345
Admiral	0.340
P&O	0.334
Mersey Docks & Harbour Co	0.334
Vaux	0.330
General Accident	0.329
Millennium & Copthorne Hotels	0.328
Bank of Scotland	0.327
Stakis	0.327
Carlton Communications	0.326
Independent Insurance	0.326
Mayflower Corporation	0.324
Asda	0.322
M&G	0.321
Severn Trent	0.319
Scottish Hydro-Electric	0.317
Bunzl	0.318
Southern Electric	0.318
Royal Bank of Scotland	0.305
Unilever	0.297
British Telecommunications	0.296
Travis Perkins	0.295
Ashstead Group	0.295
Barratt Developments	0.284
Powell Duffryn	0.279
British Aerospace	0.275
Tesco	0.275
Burmah Castrol	0.274
Great Universal Stores	0.261
PizzaExpress	0.260
Logica	0.248
Galen Holdings	0.246
Sedgwick	0.238
FKI	0.237
CRT	0.235
Northern Rock	0.219
Legal & General	0.197
TBI	0.197
Lloyds TSB	0.195
Hays	0.185
Halma	0.182
Scottish Power	0.175
Anglian Water	0.156
South West Water	0.147
EMAP	0.144
Smiths Industries	0.135
Laporte	0.132
Racal Electronics	0.124
Close Brothers	0.121
Schroders	0.119
Trinity International Holdings	0.118

8.5.8 Conclusion following this example

Some aspects of this analysis were successful. The ability of the Dora-D analysis method to turn statements of preference into a valuation function and to reduce decision options in a transparent way, was well established. There was reasonably effective convergence on a valuation function that defined a unique solution, whilst being consistent with declared preferences.

The unique function actually derived was adequate, though with limitations. Lex and many of the other shares high in the final list are good shares. This analysis is based on a price of 511p in July 1998. I purchased them in September at 421p. This continued to fall during 1998 but rose during 1999 and on 11 May they were 667p).

However some aspects did not seem fully satisfactory. The emerging scale was too spread-out and did not appear to reflect relative value. This may be because of a high weighting given to measures which are not inherently proportionate to value, such as Cash Richness. However, there might have also been issues of configurality involved which were not reflected. I can see in my values a desire for some degree of conjunctive configurality feeling (as decision maker) that a share should score well across the range of attributes, rather than being exceptionally good in a few. There are some areas where I could be quite explicit. Whilst a high EPS/Price (the inverse of PE ratio more popularly used in investment analysis) is desirable, a very high ratio could be indicating risk. I consider this statistic to be coincave with value- a factor I did not accommodate in this analysis although I made this explicit later.

Arriva and Thorn stubbornly stayed high in the valuations despite my suspicion of them. I did not like Arriva because of its excessive Gearing but it is arguable that other favourable attributes adequately compensate. At the time of originally writing this up, I still did not rate Thorn which had a good history but a poor EPS trend forecast. An adequate weight is given in the unique function to the forecast level of EPS but not of the forecast trend. ADEMI relaxed the upper bound of the relationship between the forecast level and the forecast trend before Run 10 but the relaxation may not have been sufficient. Despite reducing the CR coefficient to the minimum consistent with earlier constraints, I still view it as high and I must therefore have been inconsistent or, there remained some structural mis-specification. Consideration was also given to the inclusion of other non-linearities. In particular I noted that ADEMI give high priority to the avoidance of negative forecast earnings trends but this was not reflected in this analysis structure. This implied a need to split the forecast trend variable into separate growth and decline variables, in similar manner to the approach I adopted for historic trend. I have subsequently adopted this approach.

However, I felt that the imperfection in the value function that emerged was primarily due to cognitive issues in the expression of my underlying preferences rather than their processing. I was happy with most of the techniques explored here. The historic trend pre-emptive assumptions worked well in that I still feel that they approximate my value system fairly. However, my "objective" calculation of the equivalence of the value of forecast trend versus forecast level may not have reflected the qualitative importance which ADEMI attaches to the earnings trend forecast, as a broader indicator of share value.

The limited use I made here of direct preferences between packages of company attribute preference, proved to have little reduction effect on this occasion. I had already established that it can be a mechanically potent elimination method within the option reduction process. The issue is not its mathematical efficacy but its psychological reliability. It is not a mechanic that I now commend. I do not find this form of [n,m] comparison easy, preferring fundamentally decomposed alternatives. Nevertheless, other decision makers might reliably use the mechanic in some situations, say, with less attributes.

After these analyses, I did not feel that my use of Larichev type comparisons before Run 3 was helpful as a means of expressing my value system. I had found it difficult to declare a clear preference between some choices. I felt that I could better express preference through the Preference Bracketing methodology used in Run 4. This is also a form of fundamentally decomposed comparison. With the passage of time and a greater understanding of my poor ability to articulate firm value (which I share with other humans), I now attribute problems to value vagueness not to technique impairment. One cannot have a clearer elicitation structure than Fundamentally Decomposed Preference. In these circumstances, we can then more reasonably say that "can't say" choices are broadly equal. Nevertheless, the analyst must not immediately rush to attribute equality of value to such cases.

The use of lower-bound absolute limits on parameter values was ineffective, serving to generate LP infeasibility. I now prefer to "squeeze" from the top by capping. It may have a place, when sources of infeasibility can be easily traced. Parameter Capping, after considering the attribute weightings and their contributions of particular efficient solutions, appeared a useful technique that could be introduced earlier in the analysis sequence. The advantage of the method is that it corrects observed undesirable weightings emerging from real cases. In our metaphor it invites the decision maker to object to a feature "proposed" by an option within its best possible light CAF. To force the value revealing statement, "I don't like this!", I now make much greater use of this mechanic and of Fundamentally Decomposed Preference.

The method used in Run 5 to generate upper bounds of the maximum value contribution of particular attributes, is methodologically flawed. It can cause attribute weights to be forbidden for some options whilst allowing them for others. In the early stages of analysis, it can be used to avoid LP bounding problems as well as numerical problems, arising from large cancelling contributions from attributes of opposite polarity.

8.6 Extending the Analysis for Portfolios

8.6.1 Dealing with risk

The method above essentially selects a single share. It also measures the value of other shares against the same criterion. One can argue that high ranking shares from the list should form part of a portfolio, and, indeed, this will tend be so. However, the methodology provides no guidance on the depth of the list from which shares should be drawn nor a means of specifying their proportions. From the beginning it was clear that its use on the portfolio problem would be a limited, approximate, though useful, aid to portfolio decision making.

The need for several shares, for diversity, arises from the need to ameliorate risk, and this is the element which must be incorporated in the analysis. This is a non-linear function of the proportions of the shares in a portfolio, however its detail is defined. The "Frontier Probing" technique described earlier was designed to address this issue and I applied it here.

As decision maker and analyst I had access to some relevant risk data. These included measured systematic risk Betas for all shares, provided quarterly by Hemmington-Scott. I also had daily share prices for shares quoted on the London Stock Exchange, for several years. However, I did not have access to a convenient cheap source of Beta vectors, for use in multiple index models, or the data to generate them easily. I rejected as impracticable, a "team of one" developing the parameters for such a model on a routine basis; I needed off-the-shelf statistics as far as possible. (As the data pre-processing required each Quarter is large, this still seems an appropriate position. However, the feasibility of developing the parameters of, say, a 3 or 4 Beta model from a Principal Components Analysis of the variance-covariance matrix of the prices of shares considered, could be investigated).

I also did not have calculated parameters concerning residual variance.

ADEMI was initially unclear about the exact nature of the risk to be mitigated. This was in, principle, of two types. First was the expected volatility of value movements of the portfolio being developed, essentially the statistical risk considered in MPT. Second was the risk of non-statistical catastrophic performance not reflected in the stochastic characteristic of share prices (seemingly not addressed by MPT). I wished to dilute, if I could not avoid, the problems of purchasing a Marconi, or Enron. The first could be incorporated by some form of Standard Deviation statistic. I also contemplated using Entropy to reflect portfolio variety and to represent a measure of insurance against the second type of risk. However, IMRANI concluded that this objective could also be adequately accommodated with a variance measure, particularly with the approximation I eventually used.

But the issue necessitated some re-scrutiny of objectives. Was I trying to minimise the volatility of my share portfolio or something else? I recognised I sought to invest in the stock exchange as a whole. I was seeking to outperform it in terms of the value characteristics I defined, but I was also seeking a portfolio which was otherwise similar to the market. If the short-term fluctuations were to echo the FTSE350, I would be satisfied. There was thus a dimension of index emulation in what I wanted. Initially I therefore tried a model in which I included both a representation of portfolio standard deviation and deviation from the market. I was intending to establish parameters for both of these attributes within the Dora-D methodology. The statistical calculation of the first of these was more complicated but, more importantly, the statistic was closely related to that of the second, and I came to see it as misconceived. Minimisation of random deviations from the market was a closer reflection of what I wanted- simply to incorporate and to apply a penalty to what was left over after Beta correlation had been taken out. I subsequently felt that this was more in keeping with those aspects of the Capital Assets Pricing Model (CAPM), which I later adopted. I will discuss the simple measure of variance used shortly.

IMRANI originally included Beta as a parameter of the model, as a linear attribute of a share to which value should be attached. This aspect was to be treated like all other attributes considered above. I considered that my valuation of the disbenefit of Beta would not be in line with the market's. Indeed, ADEMI felt that as I was risk averse I would put a high value on a low Beta.

This missed an important implication of the CAPM: It does not depend on my valuation of Beta. If an investor can define an optimum portfolio for high Beta (within the range where many shares have a higher Beta) and an optimum portfolio for a low Beta, the optimum portfolio for any intervening Beta will be a linear combination of these two. My wife and I were investing in a variety of other investments such as Building Societies, Government Stocks, ISAs, etc. These had low Betas and, indeed, were likely to be broadly efficient. We did not seek an overall portfolio with a Beta of greater than 1. It follows that if I could find an Optimal Equity Portfolio, with higher Beta than we needed, that portfolio combined with any proportion of cash-like assets, would also be pareto-optimal. A reasonable Beta to choose is Unity, the Beta of the market, reinforcing the idea of emulating the index. The problem then becomes to find an optimum portfolio subject to it having a Beta of less than one.

This approach has an additional justification. Measured Betas tend to regress to one (Elton and Gruber 1981 pp141-142). Assigning value to historic Beta on the basis that it was an approximation to future Beta would cause bias. However, a constraint approach, with a constraint on Beta equal to 1 (the value to which the Betas regressed) would be unbiased. In the absence of specific information on future Betas, a constraint of 1 on historic Beta implies a constraint of 1 on expected future values.

Moreover, in addition to principles there were practicalities. Ideally I would have measured the residual share price standard error, that is the standard error of its price after taking out the market effect, what in MPT is called non-systematic or diversifiable risk, eg:

$$\begin{array}{l} \varepsilon_{st} = p_{s,t} - p_{s,t-1} - \beta_s(p_{m,t} - p_{m,t-1}) \\ \quad \text{or} \\ \varepsilon_{st} = p_{s,t} - p_{s,0} - \beta_s(p_{m,t} - p_{m,0}) \\ \quad \text{Where } \varepsilon_{st} = \text{random variable representing residual variation} \\ \qquad\qquad \text{of share, } s, \text{ in period, } t. \\ \qquad p_{s,t} = \text{price of share relative to its average price} \\ \qquad p_{m,t} = \text{normalised "price" of the market} \\ \qquad\qquad \text{ie index value x (average share price/average index value)} \end{array} \tag{8.1}$$

I felt that it would constitute an unreasonable analysis burden to calculate these statistics on a regular census basis. I therefore sought a model of them. It seemed possible that the residual errors could themselves be related to Beta. If a share was sensitive to market fluctuations, might it not vary a lot about it? I analysed a sample of 21 shares and plotted residual error against Beta. There were some indications that this could be so but were insufficient to establish either a statistically significant or practically useful relationship. I therefore adopted a constant residual error assumption for all shares under consideration.

Whilst recognising the value of later returning to this issue, I decided as decision maker and analyst that the practical benefits would not repay the analytical outlay at this stage. In early portfolio runs I made use of the more complex (spurious) model and established that the Dora-D structure is able to accommodate more complex assumptions. The simplification can be defended on sensitivity grounds: amongst otherwise equal shares, optimum portfolio proportions would be in inverse proportion to the variance of residuals. However, the penalty for variations from the optimum proportions seems relatively low, as the partial differentials are zero.

8.6.2 Getting a "handle" on the valuation of risk

ADEMI had no intuitive notion of the value I attach to the avoidance of uncertainty, and even now I cannot readily attach a valuation function weight to it. I might be able to give an analyst a vague indication of an acceptable and, with greater confidence, provide a rough indication of the spread of shares I would like to see in a portfolio. IMRANI attempted to gain some insight of my negative utility of variability as a decision maker.

I recognised that I would be prepared to take some risk for an attractive gamble. I attempted to articulate this (in an investment situation) as being indifferent to paying £10,000 for a "gamble" with a 0.333 probability of £0 return and a 0.667 probability of £20,000. The expected value of such a gamble is £3333. However, the Standard Deviation of the return is £9428. The net value of the gamble is zero; therefore the negative utility is 35% of Standard Deviation. Originally I failed to clarify the reference frame of such a gamble but later perceived it as a gamble relative to the market. What expected gain would I wish to make relative to the market in order to justify a variation about it?

IMRANI regressed logarithms of fortnightly differences in share prices against corresponding statistics for the FTSE100 index and, found the residual sum of squares for each of 22 shares. Assuming residual independence, and averaging over the shares sampled, I converted this to an annual (and natural logarithmic) basis, finding a measure of standard deviation of the residuals of approximately 0.41 (loosely interpretable as a 41% of the mean share price). Another analysis found that the standard deviation of the natural logarithm of 12 month relative prices (relative strength +100), gave a comparable figure of .45. In principle this is exploitable but there remains the issue of how one converts an annual SD to value. What is the appropriate scaling to provide a capital value equivalent? One approach might be to assume uncertainty premiums are effectively loss of income and to discount annual increments in SD in a discounted cash flow analogue. On this basis, a scaling factor of the order of 3 might be appropriate. However, one could argue that one should assume that the share is held until such time as fundamentals realistically convert to price changes and take the variance as a capital movement. This is vaguer. The first if I did not find it convincing is more manageable. This would lead to an uncertainty loss of value of 3x.35x.41x Portfolio Value for a portfolio of one share. If the share value in MCA terms (excluding uncertainty issues) in a practical portfolio is 0.75 (this now seems too conservative), we could justify a model factor, c, of 0.32. The "value" for other portfolios would be given by:

$$v_u = -c.\sqrt{\sum f_i^2}$$

Where v_u = contribution to portfolio value of uncertainty
c = factor incorporating utility of uncertainty and residual variability(after market factors of a single share)
f_i = proportion of share i in the portfolio

(8.2)

(This equation would require little adjustment to incorporate a non constant assumption for standard deviations of residual uncertainty for individual shares).

Notwithstanding the above, I found it difficult to relate intuitively to the logic discussed for generating the c factor. I felt at greater ease judging c in terms of its implications on the characteristics of the portfolio. I found myself liking shareholding which were typically 7% or 8% of portfolio value, with individual shares having no more than double this representation, and rejecting as "too small to bother with" those of less than about 3%. A c around 0.5 seemed to generate portfolios with subjectively attractive characteristics and seems reasonable in the light of the considerations above. This was used as a starting value for the analysis discussed below, though *in this instance* the value used was increased as the portfolio generated seemed too concentrated.

8.6.3 New and modified variables and prior constraints

Over the few years since seeking to aid my decision making through the approach described, the model I have used and the methodology I have employed has often changed. This is unsurprising as decision analysis, though directed to finding preference, is, given its lability, bound to challenge and modify it. In some cases Decision Analysis serves to solidify preference. For example, in a difficult area, once having stated a preference to the precision a decision maker feels able, he or she may not feel it fruitful to revisit the area.

Moreover, the elicitation mechanics I have used have also altered, as I have explored ideas as analyst and researcher seeking to satisfy me better as decision maker, and as my theoretical ideas have developed in response to introspection, or with greater "understanding" of impairment of cognition. In general I have become steadier in the mechanics that suit me, I have "fixed", or defined close limits, for some elements of my preference function. Nevertheless, many elements of my preference function are determined anew each quarter, although the definition of most attribute variables is little changed from those specified earlier.

The following are the attribute variables used in recent runs:

- E0, QE-5,..,QE-1, E2, T^{++}, T^{+-}, T^{+}, T^{-}, D0, C0, RSNO as described above but with some differences of calculation, (see below).
- New variables NG and CR replacing GCR; PT2, NT2, replacing T2.
- New variables SRSS, SG, Beta, IR

The following attribute variable used before have been excluded without explicit replacement.

- WE

Variables E0 and E2 and the intermediate Earnings per Share data used to calculate

QE-5,..,QE-1were subject to modified calculation to reflect that ADEMI no longer sought to impute value to a Earnings per Share linearly with the raw statistics. I used the following transformation to generate a modified measure which ADEMI considered reflected linearity:

$$\boxed{\begin{array}{l} e_m = \dfrac{e.e_u}{\sqrt{e^2 + e_u{}^2}} \text{ if } e \geq 0, \; = e \text{ if } e < 0 \\ \text{Where } e = \text{ true earnings yield (=eps/pps)} \\ \qquad e_m = \text{ modified earnings yield} \\ \qquad e_u = \text{ upper limit of modified earnings yield} \end{array}} \tag{8.3}$$

The effect of this modification is an imposed asymptotic limit of e_u on the modified value but as e tends to zero e_m tends to e. I used a value .15 for e_m.

ADEMI eventually adopted fixed weight ratios for the trend variables, to reflect my more solid views. This were reflected as follows:

$$\boxed{\begin{array}{l} w^{+-} = 2.w^{++} \\ 3.w^{-+} = w^{++} \\ w^{--} = 2.w^{+-} \\ \qquad \text{Where } w^{\prime\prime} = \text{ weight of } T^{\prime\prime} \end{array}} \tag{8.4}$$

IMRANI abandoned the Wedge approach, in favour of constraints on the weights on the cumulative earnings yield variables (the reader will recall, to ensure a declining contribution of historic data). I simulated a "faster than linear" decline, by requiring the weight attached to the average of the components of a cumulative composite, to be less than the weight attached to the average of the cumulative composite of one year shorter duration (ie that the expected contribution from a cumulative variable, should not be greater than that for any of the defined cumulative variables over shorter periods). The weight constraints were:

$$\boxed{6.w_{QE-5} \leq 5.w_{QE-4} \leq 4.w_{QE-3} \leq 3.w_{QE-2} \leq 2.w_{QE-1} \leq w_{E0}} \tag{8.5}$$

However, the cumulative variables continued to be modified by the Relative Strength workaround explained before.

ADEMI judged the avoidance of Gearing to be more important than the corresponding quantum of surplus cash. The original single integrated measure was therefore replaced by two. Negative Gearing (NG) and Cash Richness (CR), ie:

- NG= - Net Borrowing/ (Shareholders Equity + Gross Borrowing)
  = -1, for negative equity
- CR= Net Cash/ (Shareholders Equity + Gross Borrowing)

Net Borrowings is Gross Borrowings less Cash, and Net Cash is Cash- Gross Borrowings. This constitutes a change of definition and of origin. The source data used was also slightly different. NG is specified as a negative number so that improvements correspond to additions to value.

ADEMI was also incompletely satisfied with the future earnings trend variable T2. If current earnings have taken a temporary dip even a relatively modest recovery may create the beguiling illusion of an attractive trend. I sought a more conservative treatment and therefore redefined the variable, taking as reference, not the latest situation, but the best of recent years, as follows:

$$\boxed{T_2 = E_2 - \max\{E_0, E_{-1}, E_{-2}, E_{-3}, E_{-4}, E_{-5}\}} \tag{8.6}$$

I also tried to reflect asymmetry of values of decline from those of increase; as had been done, ab initio, for historic trend. Thus, the variable T2 was redefined into two variables PT2 and NT2, such that:

$$\boxed{\begin{array}{l} T_2 = PT_2 + NT_2 \\ PT_2 = 0, \text{ if } T_2 < 0 \\ NT_2 = 0, \text{ if } T_2 \geq 0 \end{array}} \qquad (8.7)$$

The weight of NT2 was constrained to be greater than that for PT2.

The additional variables were as follows:

- SRSS = A measure of standard deviation of residuals for a portfolio after allowing for market related movements, based on an assumption of independence, normalised so that SRSS=1 for a portfolio consisting only of 1 share. It is defined by

$$\boxed{SRSS = \sqrt{\sum (f_i)^2},\ f_i = \text{proportion of share } i \text{ in the portfolio.}} \qquad (8.8)$$

  This is the only non-linear attribute incorporated requiring a B_{pl} type parameter (as in Chapter 60.

- SG = (£ Turnover per share Yr 0 -£ Turnover per share Yr -1)/ Present Price per Share (PPS).

- BetaD, is a simple modification of the normal expression of Beta. It is Beta as defined and calculated by Hemmington Scott, minus 1. It is a constant that relates the expected change in the share price to a change in the market price, assuming a model of the form:

$$p_i = \alpha_i + (\beta_i + 1)p_m \tag{8.9}$$

Where

p_i = price of share i relative to its previous price at reference moment

p_m = market index relative to index at reference moment

β_i = BetaD

$\beta_i < -1$ for shares prices negatively correlated with the market

$\beta_i = -1$ for shares prices uncorrelated with the market

$-1 < \beta_i < 0$ for shares with positive correlation but less sensitive than the market

$\beta_i = 0$ for shares with positive correlation and the same sensitivity as the market

$\beta_i > 0$ for shares with positive correlation and greater sensitivity than the market

- IR is an Inverse Rank indicator. Shares under consideration are ranked in inverse order of their market capitalisation, allowing the possibility of a company size value premium.

8.6.4 Portfolio dynamics and stability

No difficulty of principle arises if a decision maker seeks a long term portfolio without existing investments. However, in practice I seek to use a static model to solve a dynamic problem in which an existing portfolio is subject to periodic review. If the model is used to review my portfolio from first principles at each review (normally at quarterly intervals) I may end up with a completely different portfolio. Issues of cost may make this ill advised but other factors are also relevant. I chose the shares for my perception of their long-term value with the intention of having a stable portfolio. The quantified expression of my selection values are likely to be labile, yet I can qualitatively assert that I do not wish to have a volatile portfolio. Moreover, even within the context of relatively stable values, the selection of constituents may be sensitive to relatively small variation in attribute weights and magnitudes, and that some period to period variation in their measurement may not be significant in the longer term. As analyst, I sought a pragmatic work-around, that would satisfy me as decision maker.

More than one approach seemed possible, but the approach adopted was to restrict the permitted portfolio turnover rate. Within my quarterly review system, I nominated a retention rate of 70-80%. This would, notionally, allow me to replace a "good" portfolio turned "bad" over a year. It is, of course, still open to me to generate first principles portfolios and to review the model's suggestions and to cut deeper or more shallowly if I see fit. The following describes the structure of additionally imposed constraints which operationalises this:

$$
\begin{aligned}
& q_i \le f_i && \forall i \in \{1,\dots,n\} \\
& q_i \le r_i && \forall i \in \{1,\dots,n\} \\
& \sum_{i=1 \text{ to } n} q_i \ge K \sum_{i=1 \text{ to } n} r_i
\end{aligned}
$$

Where

f_i = proportion of share i in new portfolio

r_i = proportion of share i in old portfolio

q_i = amount by proportion of share i in old portfolio retained in new portfolio

K = proportion of old portfolio to be retained in the new portfolio

(8.10)

8.6.5 Analysis short-cuts

When I started this work, the speed of my computer and the software caused runs to take several hours. Now times are much reduced and are generally manageable without assisting the non-linear optimisation routine built into the software. However, at some stages of analysis, the non-linear model may take one or two minutes to reach an optimum. As 50 to 100 passes may be necessary to generate frontier constraints and to introduce preference constraints, the exercise can still be time-consuming. I employed a number of short-cuts to speed up the process.

I remind the reader that finding the CAF of the test portfolio is a linear problem and this part is very quick. Finding the efficient peer using that CAF is non-linear and potentially time consuming. This section of the computation cycle can be substantially accelerated by using the efficient peer portfolio used to generate a frontier constraint, as an initial solution in the identification of the next one. This works well, although IMRANI feels it prudent to check the results obtained after convergence by starting with a radical and non-feasible portfolio.

Another approach is a simple linearisation of the non-linear function within the model. The approach successfully used was to replace the attribute $\sqrt{\sum f_i^2}$, by

$$\frac{\sum f_i \cdot \tilde{f}_i}{\sqrt{\sum \tilde{f}_i^2}} \tag{8.11}$$

Where f_i are as previously defined

$\tilde{f}_i$ are the f_i for the most recently found efficient peer portfolio used to generate a frontier constraint

If the $(f_i - \tilde{f}_i)$ are small, then this constitutes a good approximation. This can be guaranteed by imposing constraints of the form:

$$\begin{aligned} &(f_i - \tilde{f}_i) \le \delta \\ &(\tilde{f}_i - f_i) \le \delta \\ &\delta \ge 0 \end{aligned} \tag{8.12}$$

Where $\delta =$ a small number

If δ is set at 0.01 the approximation is satisfactory, though there is often unimportant cycling between solutions close to the optimum. This precision is perfectly adequate for defining portfolios' proportions. Moreover, one can usually move from any portfolio to any other realistic one, in about 15 steps restricted in this way. This is typically much less than the number of cycles necessary to generate frontier constraints. It is open to another analyst to vary δ during the analysis or, as I successfully did, to establish frontier constraints using this quick approach but to define a precise solution using the non-linear optimiser.

One other approach might be used. There is in this approach to portfolio analysis a simple exploitable analogue to the more general Kuhn Tucker conditions of Quadratic Programming. Amongst those investments constituted within a portfolio, the marginal value with respect to f will be the same for all the investments within the portfolio. (If this were not so, an arbitrage gain can be secured by increasing the proportion of an investment with higher marginal value, at the expense of one with a lower one). The marginal gain for any investment is the partial differential of the objective function with respect to the proportion of that investment. If we generalise the formula for the cost of residual variance (from the unit standard deviation assumption used above) we can say:

$$v = \sum_{\text{all } j} w_j . \sum_{\text{all } i} f_i a_{ji} - c\sqrt{\sum_{\text{all } i} (f_i \sigma_i)^2}$$
$$= \sum_{\text{all } j} w_j . \sum_{\text{all } i} f_i a_{ji} - c.S$$
$$\therefore \frac{\partial v}{\partial f_i} = \sum_{\text{all } j} w_j . a_{ji} - {c.f_i \sigma_i^2}/{S}$$
$$= V_i - d.f_i \sigma_i^2 \tag{8.13}$$

Where σ_i = standard deviation of price residuals for investment i after allowing for market related movements based on an assumption of independence.

$S = \sqrt{\sum_{\text{all } i} (f_i \sigma_i)^2}$

V_i = linear elements of value exclusively associated with per unit of price

Other variables as previously defined

Accordingly, if one can establish the weights of the linear attributes and then rank all projects in the order of the calculated V_i, then there will be a cut-off point, say, V_C such that all investments with V_i greater than V_C will be in the portfolio and all others will be excluded. V_C represents the marginal value of all investments in the portfolio. We can say for all investments in the portfolio that:

$$V_C = V_i - d.f_i \sigma_i^2$$
$$\rightarrow f_i = \frac{V_i - V_C}{d.\sigma_i^2}$$
$$\because \sum_{\text{all } i} f_i = 1$$
$$d = \sum_{\text{all } i} \frac{(V_i - V_C)}{\sigma_i^2} \tag{8.14}$$

In summary the cut-off marginal value V_C determines the f_i and fully defines the portfolio. In the simplified assumptions ADEMI have used, the optimal fractions are proportional to the $(V_i - V_C)$. In principle this could be applied directly if weights were to be established outwith the Dora-D procedure. In the formulated problem, the pre-emptive constraint requiring BetaD to be no greater than zero, introduces a minor complication. Cut-off rates are also used in MPT (Elton and Gruber, p184) though the above derivations are not immediately related.

8.7 Using Frontier Probing to generate portfolios- A practical example.

To illustrate the use of the methodology to generate buy and sell decisions, I describe its use in the review conducted in early May 2002, based on price data for 29 April and financial data published by Hemmington-Scott and Company REFS (ie some four years later than the "single share" analysis described). As before, I have tried to relate it without over-tidying the analysis. However, some minor arithmetic errors were made in the live run in the specification of Fundamentally Decomposed Preferences. These were tested against the value function developed, and confirmed as having no significance. However, the analysis was re-synthesised without this imprecision. (I use the term re-synthesised rather than replicated. There can be sensitivity in the parameters of a particular frontier constraint selected at a particular moment to initial conditions and sensitivity of the detail of portfolio weights to small changes in value weights. Issues were therefore revealed in a different order). Also, as one moves towards a practical portfolio there is also an inevitable and desirable stage of "tinkering" and exploring variations conducted at a speed and style, such that detailed recording is impracticable without distorting the consideration process, and is unilluminating.

The analysis may be seen as unrealistic in one respect. I have tended to redevelop weights from first principles in each quarterly review (except as mentioned above). Had I not been developing methodology, it is likely that I would have settled on a set of weights, which I would have changed slowly from period to period. I would not expect other decision makers to re-synthesise their value functions once they had established a representation to their general satisfaction.

The share portfolio of my wife and myself consisted principally of shares within that part of the FTSE 350 for which data was conveniently available, selected on the basis of past model analyses. There was nevertheless some untidiness. Three financial sector shares were held, arising from purchases prior to their exclusion from explicit analysis or for other reasons. Two telecommunications shares were held arising from a series of acquisitions and divestments from a purchase made several years ago. Both of these have been marked for disposal, one, Vodaphone, is within the review group and, though a "sell" on the basis of the model, has been temporarily retained by my wife and myself on timing grounds. Two further technology shares (one purchased prior to the development of the model and one purchased more recently on speculative grounds) are retained. One of these, Sage, is in the review group and not favoured by the model. Our "stake" in this has been reduced but I still consider it a desirable holding. (I remark, en passant, that the attributes peculiar to companies whose development prospects outweigh their prospective shorter term contributions, have not at this stage been incorporated in the model. Ad hoc treatment appears appropriate in circumstances where, generally, we favour a conservative perspective).

At the time of this analysis we held three shares, purchased on the basis of the model, in companies that had dropped-out of the index for which data was routinely provided (in one case the share had been promoted again but data had not yet been re-included in the material provided by Hemmington-Scott). The practice I adopt in such cases, is to synthesise the value function without them and to assess them for *sale or retention* outside the main analysis, using a Ready-Reckoner Spreadsheet based on the model and using data of the same form. *Purchases* from outside the FTSE350 are not considered. Also available for disposal on an individual share or shares on the basis of the model, is a holding in an investment trust, Foreign and Colonial. Its disposal is a matter of judgement but I am usually disinclined to purchase in parcels of much less than 5% of portfolio value, preferring to keep this already diversified holding, rather than purchasing smaller parcels which might be suggested by the model.

The shares held within the review group prior to this analysis were as shown in Table 8.6.

Of these only Sage and Vodaphone were not originally selected within a Dora-D based procedure. The three shares originally selected by aid of the model, but now outside the routine review group are Ashstead Group, New Look and Mayflower Corporation.

Table 8.6 Main Portfolio before Analysis

Barratt Developments	12.40%
Bellway	7.34%
Brake Bros	9.13%
Dairy Crest	13.17%
Davis Service	8.06%
Enterprise Inns	5.28%
FKI	6.30%
Innovation Group	0.57%
Rexam Plc	7.49%
Rolls-Royce	4.89%
Sage	3.60%
Taylor Woodrow	10.84%
Travis Perkins	6.16%
Vodaphone	4.72%

8.7.1 General approach of May 2002 analysis

ASDEMI, based on experience to date over the project as a whole, decided to base the value self-elicitation on 4 principles:

- Retention of particular weight relativities, established by experience of previous analyses (eg those connecting historic EPS trend variables to each other, keeping open their collective connection to other variables). These have been discussed above.
- Selection of an appropriate Reference Portfolio.
- Ranking of Fundamentally Decomposed Choices derived in a "portfolio" variation of Larichev Decomposition.
- Attribute Weight Capping.

Within the reference group there were three shares which, ab initio, I excluded from consideration within frontier defining portfolios, because of extreme magnitudes of one or more statistics. These were Energis, Eurotunnel and Marconi.

8.7.2 The Reference Portfolio

The Reference Portfolio can be an important influence on the value function used to determine an optimum portfolio. As discussed, the methodology seeks to find the Efficient Peer of the Reference Portfolio, subject to whatever additional constraints, reflecting a decision maker's preferences, that might also be added-in. As also discussed it might be reasonable in some circumstances to accept such an efficient peer without additional elicitation. On the other hand if expressed preferences fully circumscribe available latitude, the Reference Portfolio has no *ultimate* influence.

Bases for possible Reference Portfolios might include:

- A good qualitative guess or "pet" portfolio.
- A portfolio derived by other methods (eg lexicographically)
- An existing or previous portfolio with satisfactory characteristics.
- An "index" or average market portfolio.

In this case, ASDEMI was in some sense trying to emulate the market whilst wanting the characteristics of financially well-managed, reliable companies, and a simple criterion. The Reference Portfolio therefore chosen was the top 15 shares in the Reference group by Market Capitalisation in equal proportions. The shares were Anglo American, Astra Zeneca, BAE Systems, BG Group, BP, BAT, British Sky Broadcasting, BT, Diageo, GlaxoSmithKline, Rio Tinto, Shell Transport and Trading, Tesco, Unilever, Vodaphone. The attributes associated with this portfolio for May were as in Table 8.7.

Table 8.7 Reference Portfolio Attribute Values

SRSS	NG	CR	QE-5	QE-4	QE-3	QE-2	QE-1	EPS0	EPS+2		
0.258	-0.317	0.001	23.96	20.24	16.38	12.73	8.774	4.531	5.526		
T^{++}	T^{+-}	T^{+}	T^{-}	PT2	NT2	DO	CO	RSNO	BetaD	SG	IR
2.335	0	-1.627	-0.112	0.578	-0.436	2.306	2.71	-4.939	-0.494	0.045	284.0

8.7.3 Generating Fundamentally Decomposed Choices for ranking, based on the Larichev Method

In the methodology described in Chapter 7, a Best Dominated Choice (BDC) is derived from the set of all extreme-point efficient options. Maximal Efficient Choices (MECs) are then generated by finding the highest incidence of each attribute, from amongst the permitted efficient options, and combining each of those attribute magnitudes into a package with other attributes drawn from the BDC. Every choice so generated thus differs from every other choice only in two attributes. They are thus weakly efficient [1,1] choices.

The methodology of generating BDCs was originally conceived for situations where all options were explicit. Only recently have I appreciated that, at least, an approximate, "Best Dominated Choice" for the Portfolio extension of Dora-D could also be simply generated. Instead I used, and use here, the Reference Portfolio in the same way as the BDC. It should be noted that this Portfolio though generally inefficient will not be dominated by all, or even most, efficient options, though the efficient [1,1] choices generated from it will, indeed, dominate it. However, if our Reference Portfolio is sensibly chosen, and is not itself close to efficiency, we can reasonably expect our optimal solution to lie in the positive orthant (oriented by value) of the attribute space with the Reference Portfolio at its origin. Notwithstanding, this expectation is not essential. The selection of Reference Portfolio does not prevent the value function eventually developed, pointing to a solution which does not dominate the Reference Portfolio.

To generate the MECs I used the NLP facilities of What's Best to find the portfolios which, in turn, maximised each attribute (or minimised them in the case of unfavourable attributes) subject to no attribute having an inferior value to those in the Reference Portfolio. The MEC corresponding to a particular attribute was a vector with the attribute at its most favourable value and with all other elements set to be no less than Reference Portfolio levels. The most favourable values of each attribute were as in Table 8.8.

Table 8.8 Most Favourable Attribute Values

SRSS	NG	CR	QE-5	QE-4	QE-3	QE-2	QE-1	EPS0	EPS+2		
0.064	0	0.881	N/A	N/A	N/A	N/A	N/A	9.098	9.849		
T^{++}	T^{+-}	T^{+}	T^{-}	PT2	NT2	D0	C0	RSNO	BetaD	SG	IR
N/A	N/A	N/A	N/A	3.032	0	4.954	14.54	0	N/A	1.054	N/A

Instead of attribute values for individual QE-n, a composite attribute referred to as adjH EPS was used instead. This was the aggregate over all the accumulation periods, including EPS0, of the average EPS over the period. This was to enable a preference constraint to be imposed reflecting aggregate EPS weight, whatever period it related to. The maximum value of this composite was 47.47 compared with 25.30 for the Reference Portfolio. Similarly as the *relative* weights between T^{++},T^{+-},T^{-+},T^{--} were pre-determined, an adjusted T attribute, expressed in equivalent terms to T^{++} was used. This composite attribute, adjT , was equal to $T^{++}+0.5T^{+-}+3T^{-+}+6T^{-}=5.532$.

As BetaD was to be constrained to be less than or equal to 0 and no valuation view was to be taken on its weight, no minimum value of this attribute was sought. In this series of analyses this constraint was never critical. Although a minimum SRSS value was found this statistic was not used. As IR for the reference portfolio was already the maximum that could be obtained for an equal weight portfolio of 15 shares, this was not used to generate a [1,1] choice corresponding to the attribute. The alternative treatment used is mentioned below.

In the next stage I specified [1,1] choices for which preferences would be declared. These are the differences between the attribute magnitude in the Reference Portfolio and the value of that attribute in its MEC; that is the differences in the values of Table 8.7 from those of Table 8.8. Relevant differences are shown in Table 8.9. The value for PT2 is as used, including a small transcription error. The magnitude above for IR is based on different and arbitrary principles. I found the *minimum* IR subject to other attributes remaining above their reference portfolio levels. At this stage I omitted consideration of adjH EPS.

Table 8.9 Differences between Most Favourable Value of Attributes and their Values in the Reference Portfolio

NG	CR	AdjH EPS	EPS0	EPS+2	Adj T	PT2	NT2	D0	C0	RSNO	SG
0.317	0.880	22.18	4.567	4.323	8.750	2.545	0.436	2.648	11.83	4.939	1.009
IR (see text)											
260											

I then ranked my initial preferences for the other differences. These were:

$$(\Delta C0=11.83) \succ (\Delta \mathrm{Adj\ T}=8.75) \succ (\Delta EPS+2=11.83) \succ (\Delta NG=0.317) \succ (\Delta CR=0.88) \succ (\Delta SG=1.009) \succ (\Delta PT2=2.545) \succ (\Delta EPS0=4.568) \succ (\Delta D0=2.648) \succ (\Delta NT2=0.436) \succ (\Delta RSNO=4.933) \succ (\Delta IR=260)$$

Where $(\Delta S=n) \succ$ means that a change in Attribute S of n is strictly preferred to ...

This translates to LP constraints as follows:

$$11.83W(C0) \geq 8.75W(T) \geq 11.83W(\Delta EPS+2) \geq 0.317W(NG) \geq 0.88W(CR) \geq 1.009W(SG) \geq 2.545W(PT2) \geq 4.568W(EPS0) \geq 2.648W(D0) \geq 0.436W(NT2) \geq 4.933W(RSNO) \geq 260W(IR)$$

Where $W(S)$= Weight of Attribute S in value function.

In addition to these, I imposed that the weight of NT2 must be at least twice that of PT2. Also, whilst I had notionally made comparisons between EPS0 and EPS2, I recognised that I was using EPS0 as a proxy for all historic EPS measures and I therefore represented the relativity found ostensibly for EPSO, as a constraint on the weight of AdjH EPS.

8.7.4 Initiating Frontier Probing

In setting up the LP/NLP cycles, I imposed some technical constraints. First to prevent unbounded solutions and to speed the early iteration process, I imposed a limit of 10 on the valuation of any portfolio. I also limited the valuation contribution of any individual attribute to 1.0. I set-up the first cycles of the LP/NLP routine to make use of the linear approximation already discussed. This requires that a portfolio found at the end of any cycle, is a close relative of the initial solution for each cycle. I imposed that the proportion of any share in a portfolio at the end of a cycle, would not differ by more than ± 1 percentage point. I also required the first such initial solution to be the Reference Portfolio. The reader is reminded that the Frontier Probing Cycles follow the sequence:

1. Specify constraints, including:
   - Constraints on the latitude of the value function. Initially here, those derived from the ranking of FDCs.
   - Constraints arising from the requirement that no *explicit* portfolio may have a value of more than 1.
   - Technical constraints and any other constraints imposed on solutions.
2. Find the CAF of the Reference Portfolio by maximising the value of that portfolio, subject to those constraints.
3. Find the portfolio having the highest value, using the CAF found, subject to technical constraints- and its value.
4. If the portfolio found has a valuation materially above 1, the CAF violates the requirement that no CAF is allowed that causes *any* portfolio to be valued at more than 1. Accordingly, generate a constraint to ensure, in subsequent cycles, that portfolio shall always be valued at no greater than 1. Add this to the constraints used in the prior cycle.
5. Start the next cycle.

Within the Excel-What's Best-VBA Macro approach adopted, the cycles are controlled semi-automatically. Initial constraints and evolving preference constraints are manually specified but the generated constraints are added-in automatically. I ran the routine through 10 cycles.

At this point the "value" of the portfolio found was 1.0 (though still marginally above zero). The actual portfolio was of no consequence but the CAF found was examined. ADEMI felt this indicated an excessive contribution to value of attribute CO, which was at its artificial upper bound of 1.0. The technical objections to constraining *contribution* other than as a crude bounding constraint has been discussed, I moved to capping the weight of CO and limited this to 0.2.

I ran the routine through to cycle 12. The value of the portfolio then found was 1.47; I therefore ran it through to cycle 15 without modification. The value of the portfolio then produced was 1.03. This would have been too high at the terminal stages of analysis but was low enough to review weights.

The contribution of EPS2 seemed excessive and I capped the attribute weight at 0.05. At this stage I switched to precise optimisation and ran the analysis through to cycle 19. The contribution of CO was still high at 50.5% of value and the weight was capped at 0.02.

After cycle 20 objective function was 1.03. The shape, but not the content, of the portfolio was examined. I felt that it was insufficiently diversified. I accordingly sought to penalise concentration somewhat more severely by increasing the weight of SRSS from 0.5 to 1.0. Cycle 21 was then run.

The measured portfolio value was 1.03. DO now seemed somewhat high, its weight was capped at 0.05.

In cycle 22 the objective dropped to 0.814. This normally implies mutual incompatibility between value indicating constraints- in this case either over-zealous capping or interaction between the caps and the Larichev ranking constraints. Now CO seemed too light and the attribute weight limit was raised to 0.025. I ran cycle 23.

The objective function still indicated problems and I decided to demote NG and CR in the ranking to below SG and PT1. This was shown to be to little effect in cycle 24.

I decided to break the link between EPS2 and SG in the FDC rankings. SG seemed low and the constraint was critical. I then ran to cycle 26.

This resulted in a portfolio value of 1.00 but unfortunately, without the Larichev ranking constraint, SG became excessive and its attribute weight was capped at 0.3. This pushed the objective function down to below 1 again, to 0.914 in run 27

I then allowed historic EPS (which in the set of runs was only influencing EPS0) to come up by relaxing its coefficient in the Larichev constraints to 4.0. In run 28 this had the effect of restoring optimal portfolio value to 1.00.

The weight of NG now seemed too high notwithstanding its earlier demotion. I limited its weight to 0.3 in cycle 29 which again over-suppressed valuation of the optimal portfolio to 0.861.

The constraint on CR in the FDC rankings suppresses the coefficient of CR relative to NG. A relative weighting of NG being at least twice that of CR is quite sufficient and this effect is achieved by dropping the weight of CR in the Rankings to 0.632. But this did not allow significant movement in the value of the objective, 0.870.

I then relaxed the NG cap from 0.3 to 0.5 and in cycle 31 the objective went into the residual latitude area with a value of 1.004.

The general characteristics of the weights and contributions in the optimal portfolio at this point, seemed generally satisfactory. T^{++} seemed somewhat light and I therefore experimented with fixing its weight at 0.03. Cycle 32 demonstrated that this did not use up residual latitude (Objective = 1.003)

I therefore did the same with NT1 forcing its weight to 0.15 in run 33. The optimum portfolio value was then 1.008, sufficiently close to unity not to require the interposition of further frontier constraints. I was then generally content with the attribute weights and the relative contribution of the attributes to the value of the optimal portfolio found at that point. These then constituted the weights of the attributes in the CAF of the Reference Portfolio under the preferences declared, and consequently my value function; they are shown in Table 8.4. The optimal portfolio corresponding to this and the attribute values for this portfolio are:

Table 8.4. Coefficient of Attributes in ADEMI's Value Function after Frontier Probing

SRSS	NG	CR	QE-5	QE-4	QE-3	QE-2	QE-1	EPS0	EPS+2		

1.0	0.454	0.227	0	0	0	0	0	.0358	.05		
T^{++}	T^{+-}	T^{+}	T^{-}	PT2	NT2	DO	CO	RSNO	BetaD	SG	IR
0.03	0.015	0.09	0.18	0.075	0.15	0.05	0.025	0.013	0	0.3	.00025

Table 8.5. Optimal Portfolio corresponding to Value Function

AMEC PLC	0.65%
Anglo American PLC	3.47%
ARRIVA PLC	10.26%
Barratt Developments PLC	8.03%
Carpetright PLC	5.33%
Collins Stewart Holdings PLC	10.79%
Dairy Crest Group PLC	4.92%
Davis Service Group (The) PLC	1.49%
DFS Furniture Company PLC	15.46%
G K N PLC	5.10%
ICAP PLC	0.94%
Inchcape PLC	1.95%
McCarthy & Stone PLC	1.45%
Next PLC	0.53%
Persimmon PLC	15.73%
Rexam PLC	0.94%
Taylor Woodrow PLC	6.15%
W H Smith PLC	3.46%
Wimpey (George) PLC	3.35%

Table 8.6 Attribute values for Optimal Portfolio after Frontier Probing

SRSS	NG	CR	QE-5	QE-4	QE-3	QE-2	QE-1	EPS0	EPS+2		
0.306	-0.180	0.454	35.35	30.79	25.55	20.15	14.32	7.69	8.49		
T^{++}	T^{+-}	T^{+}	T^{-}	PT2	NT2	DO	CO	RSNO	BetaD	SG	IR
3.70	0	-0.57	0	0.87	-0.11	3.32	7.58	-0.028	-0.366	0.320	129.7

8.7.5 Generating dealing decisions

It will be noted that of the 19 shares generated in the analysis thus far, 5 already form part of the existing portfolio. The initial suggested proportions of these shares was 21.5%.

The issue of portfolio stability has been discussed. Generally my wife and I are prepared to turnover 20-30% of an existing portfolio, selling the worst and buying the best. However, this can be increased, if shares suggested seem attractive, or reduced, if not. An additional factor here is the size of parcels suggested, we generally prefer not to purchase in parcels much below 4%. We are also averse, since a price fall in a large holding, to large parcels, say, over 12% of aggregate portfolio. (I recognise a dichotomous attitude in myself, and believe I am perhaps relaxed with "flatter" portfolios than the statistical approach leads to). It is also open to us to convert Foreign and Colonial, if an attractive proposition is suggested.

These aspects are questions of judgement which have been incompletely modelled. However, optimisation within turnover constraints can be readily handled.

The next stage therefore was to generate an optimal portfolio, in this instance such that 70% of existing holdings are retained. When this was done the shares in Table 8.7 were suggested.

Table 8.7 Suggested Portfolio with70%

Existing Portfolio Retention

ARRIVA PLC	4.75%
Barratt Developments PLC	12.40%
Bellway PLC	4.88%
Brake Bros PLC	2.29%
Collins Stewart Holdings PLC	5.28%
Dairy Crest Group PLC	13.17%
Davis Service Group (The) PLC	8.06%
DFS Furniture Company PLC	9.85%
Enterprise Inns PLC	0.68%
Persimmon PLC	10.12%
Rexam PLC	7.49%
Rolls-Royce PLC	4.89%
Taylor Woodrow PLC	10.84%
Travis Perkins PLC	5.29%

However, within the Comparison Set, as mentioned, there are two shares, Sage and Vodaphone, to be retained at least temporarily. When these are forced in the suggested portfolio became, as per Table 8.8.

Table 8.8. Suggested Portfolio with Vodaphone and Sage forced in.

ARRIVA PLC	4.80%
Barratt Developments PLC	12.40%
Bellway PLC	2.78%
Brake Bros PLC	0.25%
Carpetright PLC	0.08%
Collins Stewart Holdings PLC	5.31%
Dairy Crest Group PLC	11.79%
Davis Service Group (The) PLC	8.06%
DFS Furniture Company PLC	9.77%
Persimmon PLC	10.04%
Rexam PLC	7.49%
Rolls-Royce PLC	4.89%
Sage Group (The) PLC	3.60%
Taylor Woodrow PLC	10.84%
Travis Perkins PLC	3.18%
Vodafone Group PLC	4.72%

There were no Increase suggestions; Retain (or Substantially Retain) suggestions were Barratt, Dairy Crest , Davis, Rexam, Rolls Royce, and Taylor Woodrow; Reduce suggestions were Bellway, and Travis Perkins; Sell suggestions (including shares where very small retentions were indicated) were Brake Bros, Enterprise Inns, FKI and Innovation Group; Suggested significant Buys were Arriva, Collins Stewart, DFS and Persimmon.

All these were reviewed, particularly using the REFS detailed company fact sheets, which include Brokers' recommendations. My wife and I rejected Arriva (which had previously been in and out of our portfolio on the basis of the model) on the grounds of uncertainty related to public transport policy. Persimmon seemed attractive but there was a diversity issue; Barratt, Bellway, Taylor Woodrow, and Travis Perkins were in related markets. However, as some of these would probably be sold or reduced this was not critical. Before finalising others, the model suggestions were re-analysed constraining Arriva out, new purchases were subject to an additional arbitrary limit of 10% of portfolio value. The revised recommendations are in Table 8.9.

Table 8.9 Suggested Portfolio with Arriva excluded.

Barratt Developments PLC	12.40%
Bellway PLC	4.00%
Carpetright PLC	1.62%
Collins Stewart Holdings PLC	6.98%
Dairy Crest Group PLC	13.17%
Davis Service Group (The) PLC	8.06%
DFS Furniture Company PLC	10.00%
G K N PLC	1.40%
Persimmon PLC	10.00%
Rexam PLC	7.49%
Rolls-Royce PLC	4.89%
Taylor Woodrow PLC	10.84%
Travis Perkins PLC	4.41%
Vodafone Group PLC	4.72%

In the original analysis Collins Stewart was suggested as 7.4% and GKN was higher than Carpetright, at 2.0% at this point. It was decided to sell Brake Bros., Enterprise Inns, and FKI and to sell approximately half the holdings in Bellway and in Travis Perkins. It was further decided to reinvest all proceeds with miscellaneous cash balances in the relevant accounts principally in Collins Stewart, DFS, and Persimmon in approximately equal proportions. A smaller purchase of approx 1/3 the quantum of the other 3 was also considered, for which Carpetright and GKN were candidates. GKN was decided upon. Of the unconstrained portfolio value, measured as 1.008, a 0.102 drop arises from the requirement of 70% retention of the existing portfolio. There is a further 0.064 from constraining Arriva out and Sage and Vodaphone constraints. The purchase quantum roundings (embracing differences in the recommendations from the reconstructed model) account for a value deviation of 0.0034.

Generally there is considerable insensitivity of value, to imprecision in proportions of shares in a portfolio provided, the issue is the quantum not the fact of inclusion. Thus a 3 percentage point increase from optimum in the holding of Collins Stewart, causes an attrition of measured value of only 0.002.

8.8 Conclusions relating to the portfolio procedure

I am broadly happy with the application of the portfolio procedure to my investment problem based on the two mechanics used. This is despite, in this review, generating mutually incompatible sets of preferences at a point in the analysis. Further thought is necessary to ensure, as analyst and decision maker, that the FDCs that are developed are the most useful ones, in the sense of allowing the easiest judgements. As analyst and decision maker I recognised an unhelpful introspective pressure to force a preference rather than settling for "don't know" and then, if necessary, seeking to exploit "close to indifference". Generally where there was a mental conflict I felt "happier" with Capping, using this to avoid extremely unbalanced contribution effects. However, I recognise that this could be a self deception and believe that comparison of FDCs is a more direct principle.

I am increasingly conscious of wanting more conjunctive valuation (two symptoms of this may be my apparent dislike of parametrically polarised models, feeling happier when all factors have a moderate representation in the valuation; and my desire for "flatter" portfolios). More recently I have started to explore its incorporation in the portfolio model. I would also like to find ways to avoid onerous pre-processing of data and I have started to explore whether rank data, used cardinally, in association with the general configural model, may allow simplification without loss of power. It would appear to diminish the problems of spurious data in the tails of distributions and could avoid some of the complicated transformations I have hitherto felt constrained to adopt.

I am also exploring partitioning into groups of attributes (not necessarily disjoint), as an aid to achieving easier weight evaluation; notably dividing attributes into safety factors and performance factors.

Chapter 9 Testing the approach using simulation

9.1 Introduction

In this chapter, I review the performance of the methodology. I consider only issues of the mechanical efficacy of Dora-D to convert statements of preference, which simulate a variety of elicitation devices, into consistent and, for the expressed values, optimum decisions. Accordingly, I ignore issues of the psychological reliability of the elicitation device. Indeed, I assume that the simulated decision maker is totally reliable and consistent in the weights he or she attaches to attributes and, in binary preference situations, he or she can declare an accurate strong preference however slight the value advantage. The main purpose of the simulations was to:

(a) Pragmatically demonstrate that Dora-D will progressively reduce the potential optima and find an optimum.

(b) Indicate the relative speed of convergence for the mechanics used.

Most of the simulations test variations in elicitation mechanics for the discrete decision linear model (ie the basic model), though a configural discrete model and a portfolio model are also demonstrated.

I start by outlining the data used. In order to facilitate comparison between mechanics, ten standard sets of data are predominantly used. These serve to represent problems of moderate size and complexity.

An encapsulation of the multiple attribute decision analysis is to find the weights that attach to attributes. In the simulations I use a concept of revealing "Hidden Weights". Both the concept and values used are explained to the reader. The simulated decision maker is assumed to express preferences exactly and consistently, in accordance with these values, but the simulated decision maker is deemed unaware of the weights, which are only revealed to a simulated analyst through the expression of preference, within the analysis process being considered.

Two criteria of analysis performance are then discussed. One, the Information View, focuses on the number of options that remain potentially optimal. The other, termed the Performance View, concentrates on the average value, assessed using the "hidden weights" of the options, that at a particular stage remain potentially optimal, given the preferences declared.

The methodology used is briefly discussed which is centred on the Basic Methodology using the Andersen-Petersen variant. An issue arises here and the use of a different option elimination criterion in these simulations, from that commended for practical situations, is explained.

I then relate the various experiments undertaken. The first is by way of context setting. A decision by analyst and decision maker of how many attributes to embrace in an analysis, is itself an expression of value that influences both the number and identity of potential optima, just as expressions of preference do. At extremes, it either completely determines the optimum, or fails to eliminate any options, without the need for further analysis. Taking the Information view only, I examine how the number of potential optima is influenced by the number of attributes considered for inclusion in an analysis, and seek to establish relationships.

Simulations 2 to 6 consider the impact of reductions of value function latitude, secured by expressions of preference between options and different mechanics for identifying options for comparison. I address the extent to which reduction is achieved by Dora-D, if such expressions are reliable. Different methods of identifying options for comparison or prioritisation are examined.

Simulation 7 assesses the reduction achieved by the ranking of attribute weights.

I go on to consider the mechanical efficacy of Weight Capping, before examining two approaches to [1,1] decomposition.

Simulations 11 to 13 examine the effects of mis-specifying a non-linear value mechanism as a linear one. Two true underlying value structures are investigated. In one of these no major problems emerge. However, LP infeasibilities were caused in the other. Alternative methods of proceeding are investigated, one appearing to be more effective.

I then move to a situation in which non-linearity is considered possible, by assuming, *within the simulated analysis,* that the decision maker's values can be represented by the Modified Minkowski Metric. However, we still test this concept on a mis-specified model, by making the simulated decision maker's "actual" value function correspond to a multiplicative model. A model in which we simultaneously seek both the arithmetic weight and power parameters is considered first. This is unsuccessful. In Simulation 15 we consider the Conservative Fixed Parameter methodology discussed in Chapter 7, finding more encouraging results. Finally, within this group, the relationship between the configural parameter and the number of efficient options at Initial Option Reduction is assessed, demonstrating relatively low variation.

9.2 The data

All data are artificial. Ten sets of data were used for most of the analyses. Each of these was a 5x50 matrix simulating 50 potential decisions in a selection situation characterised by 5 attributes, elements of the matrix are the magnitudes of those attributes for each option. The values for all attributes and for all sets of data were generated in an identical manner. They were found using the random number generator of Microsoft Excel 97, from a seed of 2813 in a single pass. This generated a 50 x 50 matrix consisting of normally distributed random variables of unit mean and unit SD. This was divided into the ten sets. These parameters provide for data sets with predominantly positive numbers but with a number of negative numbers. This set of data is designated as 5x50data_2810.

The reader should note, nevertheless, that any positive multiplicative scaling of any variable constitutes a strategically identical problem. Any translation of origin also generates a problem that is comparable, in the sense that it generates identical efficient options in the initial option reduction, and corresponding expressions of preference will generate the same optimum answer and the same ranking. However, the attribute weights will be different, and final MCAs, although still represented on an interval scale, will also be different. The selection of attributes with identical parameters therefore implies no loss of generality with respect to the parameters. The selection of a normal distribution is arbitrary though, with an additive value function, is consistent with a normal distribution in consequential values.

Value is assumed to increase with increases in the magnitudes of all attributes.

9.3 The "Hidden Weights"

It is assumed that the decision maker acts consistently with respect to hidden weights, known to the researcher and declared to the reader, but only revealed to the simulated analyst through the expression and incorporation of preference, in the analysis process. In most analyses, exactly equal weights are used as the hidden valuation function (but see below). It is considered that this assumption is neither simpler nor more demanding than any other assumption. As the value function is an additive composite of components that, for this exercise, are normally distributed, the distribution of the option values will also be normally distributed, whatever the weights that underlie it, and can always be reduced to a distribution with standard mean and standard deviation. The identification of the extreme point of that distribution, the optimum option, would accordingly seem a priori to be no easier nor more difficult, whatever the weights that constitute it. However, this assumption was checked by comparing the effectiveness of initial stage reduction for an "extreme alternative", where the hidden valuation function has zero weight for all but one of the attributes.

One partial elicitation methodology, the ordering of weights, requires different treatment. To deal with this case, I effectively assume that there are infinitesimal differences from the evenly weighted case, such that first attribute is deemed to have infinitesimally higher weight than the second, the second than the third, the third than the fourth etc.

9.4 Criteria

9.4.1 Information View

I here take two complementary views of the efficacy of selection. One can be termed the Information View, the other the Performance View.

In the Information View the decision maker has a number of options. In the absence of values, criteria and methodology, we can say that each choice is equally likely. The task of the decision analysis is, by means of the expression of preference allied with a method of exploiting it, progressively to polarise the probabilities so that at the conclusion one option can be assigned a probability of unity of it being optimum, with all others having a probability of zero. If we had 64 choices at the commencement we would have a situation with an entropy of 6 bits; that is 6 bits of relevant information are necessary to convert options into a decision, a determined "system" with zero entropy. A statement by a decision maker that the value of the decision is determinable from a single attribute, married with knowledge of the magnitude of that attribute for each option, is sufficient to provide the information necessary to identify the optimum. Indeed, such a declaration provides the very much greater amount of information (in fact $\sum_{i=2 \text{ to } 64} \log_2 i$ bits) necessary to generate a complete ranking. Precisely 6 bits of information, sufficient to solve the optimum identification problem, would be provided by the statement "Option 8 is the option having the highest value for the relevant attribute". Alternatively, 2 bits of information would be provided, if an unordered listing of all the options in the top quartile were to be given.

If a decision maker were to assert that there were, say, 200 relevant attributes and that the value of any option were to be fixed by an undetermined linear combination of them, he or she would have provided virtually no useful information. But if he or she were to make the same assertion in respect of 2, 3, 5, or 10, some degree of exploitable information would already have been provided, *if* a suitable analytic tool were also available to evaluate it. (Of course, if the information enabled the decision maker to improve the probability of selecting the optimum *without* an analytic tool, some gain would be made; but he or she needs to double the probability to make a one bit gain. This is an issue of calculation not of judgement of value, and a person should not expect to outperform a machine on this task. Indeed he or she might seriously under-perform it, and could reach a totally erroneous conclusion (eg by excluding the optimum)).

It might in principle be possible to make an analytic calculation of the likely information gain, from the provision of such statements given a method of processing it. However, such a calculation appears an extremely complex one dependent on an assumption of a prior distribution, and involves the mathematics of statistical extremes. It is not attempted here.

Nevertheless, Dora-D can itself be used to estimate the exploitable information gain, by simulation. If remaining potential optimum options are equally likely, the information gain from the provision of attribute data, if *associated with* the means of converting that data into information, will be given by:

$$\boxed{\begin{array}{l} \log_2 n - \log_2 n_e \\ \quad \text{Where } n = \text{ number of options} \\ \qquad\quad n_e = \text{number of efficient options} \\ \qquad\qquad \text{(ie potential optima)} \end{array}} \tag{9.1}$$

Similarly, in subsequent stages, the information gain associated with a particular mechanic can be measured in terms of the reduction of the number potentially optimal options that remain.

9.4.2 Performance View

The Information View only provides a measure of the extent to which an optimum has been identified; that is the degree to which uncertainty in the specification of the best option has been reduced. It provides no indication of the amount by which an identified optimum is superior to an alternative candidate. If a non-optimal option is infinitesimally different in value from that of the optimum, the missing information is of no consequence, but this may not be discernible. An alternative view point that highlights this issue, is to examine the expected true value of remaining potentially optimal options and how this is improved by the provision of preference indicating information.

We can define the Performance Gain associated with the provision of any particular package of information, as:

$$\boxed{\begin{array}{rcl} \multicolumn{3}{l}{G = \dfrac{v_i - v_{av}}{v_{opt} - v_{av}}} \\ \text{Where } G & = & \text{performance gain} \\ v_i & = & \text{average true value of all options remaining} \\ & & \text{potentially optimal after imposition of prior} \\ & & \text{and preference constraints at any stage } i \\ & & \text{in the analysis} \\ v_{opt} & = & \text{true value of the optimum option} \\ v_{av} & = & \text{average true value of original options} \end{array}} \tag{9.2}$$

I use both Information Gain and Performance Gain, when testing elicitation mechanics, to gauge the impact of preference expressions within the analysis structures.

9.5 Methodology and methodological issues

In the simulations the Basic Methodology was used in the manner described in Chapter 5, except where indicated. The test simulations were started before I had a clear understanding of the relative merits of including the option under assessment within the Comparison Set, or using the Andersen-Petersen (AP) variation of excluding it. I now favour excluding it for reasons mentioned in Chapter 5 and have usually excluded it here.

Much of this is immaterial to these analyses. However, it should be noted that the instances of a measured MCA of exactly one, will be treated differently. I have argued elsewhere that such an option should be retained until it can be excluded on other grounds, as it represents a potential tie. However, I exclude them as potential optima in these analyses, where in most examined situations all attributes have finite weights. The reason for this is that a strict preference between pairs of real options are usually represented in these analyses by an $x \geq y$ value constraint rather than, more properly but inconveniently, a strong one using the form $x \geq y+\varepsilon$. Instances where MCAs of *exactly* 1 occur, within these simulations under AP conditions, will usually be where such a strict preference constraint has come into effect, implying a genuinely dominated, and hence non-optimal, option. This in no way invalidates the earlier suggestion for *practical* cases. Little is lost by retaining an option a little longer, particularly when it is overtly flagged, and its proper status can be readily checked.

9.6 The simulations

9.6.1 Simulation 1. Contextual examination of inherent extractable information in declared attribute data.

In this set of analyses I examine the information, relevant to the selection of an optimal decision, inherent in the specification by a decision maker of the number of attributes that he or she considers relevant. As has already been discussed, the selection of a single determined attribute will fully define the optimum solution. On the other hand, a failure to commit to any, or to allow a large number to be of potential relevance, provides no, or negligible, information. Accordingly, simply to allow that some attributes may be relevant, is itself an intermediate expression of value by the decision maker that can be exploited. The fewer attributes that are allowed, the more the latitude of the optimum is circumscribed. This analysis attempts a quantification of the impact of such specification and provides a context in which the efficacy of other expressions of preference can be judged.

In the analysis here I take only an Information View, judging the information gain by the difference between the system variety or entropy when all options are equally likely, and the entropy when only potentially optimal options are included. The number of potentially optimal (ie efficient) options are assessed by determining the MCA for each option in the option set, reviewed under AP conditions. The following 18 situations were examined:

(a) Situations with 10, 20, 30, and 40 options, each with 3, 5 and 7 sets of attributes designated as relevant.

(b) Situations with 50 options, with 2, 3, 4, 5, 7 and 10 sets of attributes designated as relevant.

The 5x50data_2810 was supplemented by a further set 5x50data_0422 generated in like manner to the first. This was used to provide additional attribute data for those situations involving 7 and 10 attributes. 10 cases were evaluated in each situation, that is a total of 180 cases. Consistent subsets of the 50 options cases were used to generate the cases for 10, 20, 30, and 40 options; in fact the first 10, 20, 30 or 40 options of each set. Similarly consistent subsets of the 5 attribute cases were used to generate the data for lesser numbers of attributes. The attribute data for the 10 attribute cases consisted of the data for the 5 attribute cases, plus the supplementary data. The data for the additional 2 attributes in the 7 attribute cases, was that for the 5 attribute cases, in addition to a consistent subset of 2 attributes from the supplementary data.

In addition to the above there were a set of single attribute situations for which an invariable single potential optimum could be assumed without calculation.

Each option, in each case, for every situation was evaluated using the Dora-D basic model, involving the execution of approximately 6000 LP runs. For each case, the number of efficient options was recorded. The potential optimum Entropy was calculated as:

$$\boxed{\begin{aligned} &E_i = \log_2 n_{e,i} \\ &\text{Where } E_i = \text{Entropy in respect of uncertainty of} \\ &\qquad\qquad \text{optimum option, for case } i \\ &\qquad\quad n_{e,i} = \text{number of efficient options for case } i \end{aligned}} \tag{9.3}$$

The mean Entropy was calculated from the ten cases representative of each of the situations. These results are shown in Table 9.1.

Table 9.1-Entropy in respect of uncertainty of optimum option related to total options and number of declared attributes

No. attributes	1	2	3	4	5	7	10	Starting
No. Options								Entropy
10	0.00		1.87		2.74		3.28	3.32
20	0.00		2.20		3.22		4.10	4.32
30	0.00		2.20		3.56		4.57	4.91
40	0.00		2.38		3.74		4.86	5.32
50	0.00	1.43	2.38	3.01	3.70	4.40	5.06	5.64

These data may be represented in alternative form, either as Information Gain achieved (Table 9.2), or as Proportion of Information required to determine the solution (Table 9.3).

Table 9.2- Information gain achieved by declaration of attributes related to total options

No. attributes	1	2	3	4	5	7	10	Starting
No. Options								Entropy
10	3.32		1.45		0.58		0.05	3.32
20	4.32		2.12		1.10		0.22	4.32
30	4.91		2.71		1.35		0.34	4.91
40	5.32		2.95		1.59		0.46	5.32
50	5.64	4.21	3.26	2.64	1.94	1.25	0.59	5.64

Table 9.3- Proportion of information required to fully determine optimum provided by declaration of attributes, related to total options

No. attributes	1	2	3	4	5	7	10	Starting
No. Options								Entropy
10	1.000		0.436		0.176		0.014	3.32
20	1.000		0.490		0.255		0.052	4.32
30	1.000		0.552		0.274		0.069	4.91
40	1.000		0.554		0.298		0.086	5.32
50	1.000	0.746	0.578	0.467	0.344	0.221	0.104	5.64

The following generalised observations can be made:

(a) A substantial reduction in the original difficulty of identification of optima is achieved by the specification of a limited number of relevant attributes; typically the entropy is of the order of 2-3 bits (corresponding optima shortlist 4-8) for a 3 attribute concentration, and 3-4 bits (corresponding optima shortlist 8-16) for 5 attributes. More than half the information necessary to identify a single optimum is provided, if 3 attributes are designated as being relevant, relative to a situation in which no opinion is expressed, for problems with a medium to high number of original options.

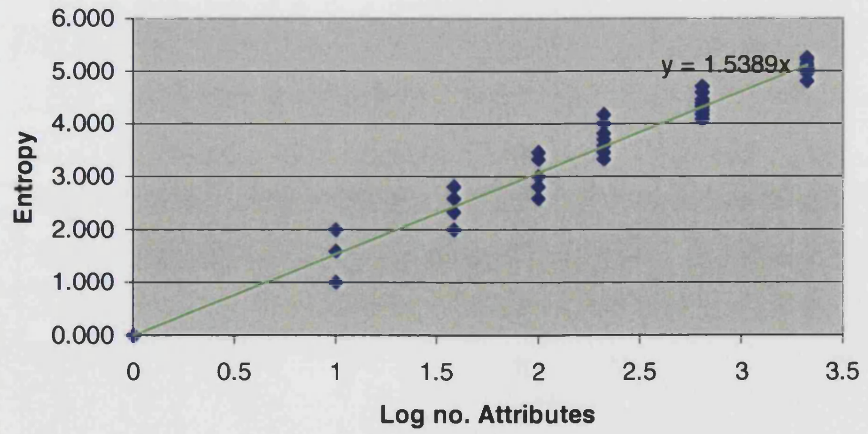
(b) Diminishing returns are achieved as the number of attributes declared to be relevant increases. Thus, for example, the option reduction achieved with 10 relevant attributes, is not materially better than the original problem when no opinion as to relevant factors is given.

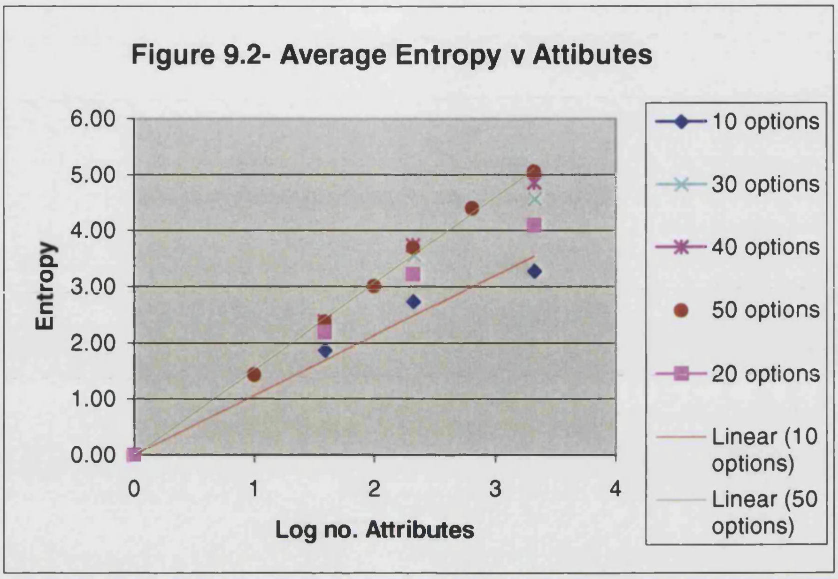
(c) Entropies for any specified number of attributes are naturally lower with lower numbers of starting available options, though the fall off is not as marked as I expected. The information gain is higher in both absolute and proportional terms, with higher numbers of initial options.

In order to attempt to formulate an empirical model of the relationship between information provision and number of attributes and total options, the logarithm (to base 2, which will be implicit throughout this Chapter) of the number of efficient options, was plotted against the logarithm of the number of declared attributes for all cases for the 50 option situations. This plot is shown in Figure 9.1. This plot has 10 points per attribute value, many coincident; coincident points are plotted as one.

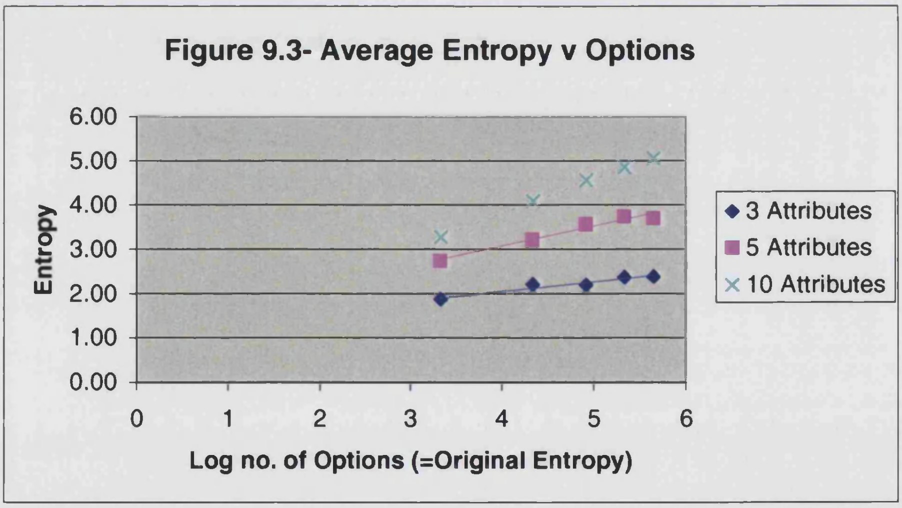
The graph strongly suggests a linear relationship through the origin. This is supported by a plot of *Average* Entropy versus the Logarithm of the number of attributes for the alternative number of options cases examined; as shown in Figure 9.2.

Figure 9.1- Entropy versus Attributes for 50 Option Cases





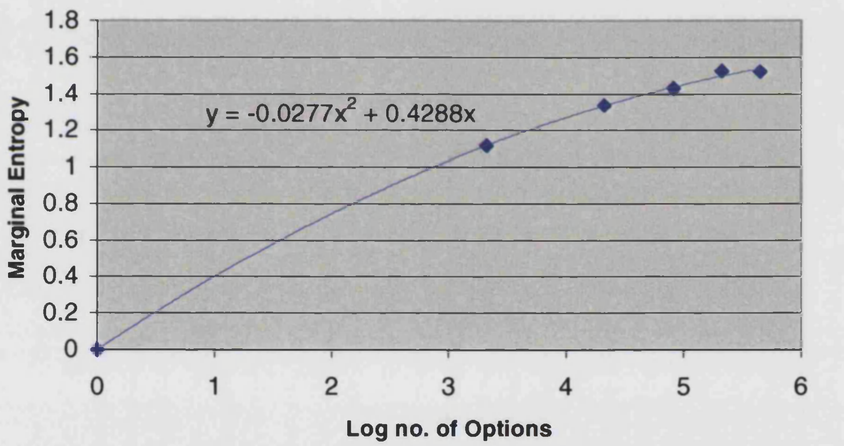
Similarly the Average Entropy of situations varying by the original number of options but fixed with respect to the number of attributes declared, was also plotted for alternative attribute numbers. These plots are graphed in Figure 9.3.



The plots suggest an adequate linear approximation over the data range, though the standing of this empirical relationship is of lower order than that I suggest for the relationship of Entropy to Attributes.

In order to develop a generalised relationship it is perhaps convenient to depend on the proportionality of Entropy to the log of the number of declared attributes. If one divides the log of the number of efficient options, by the log of the number of declared attributes and averages over all cases with a specific number of starting options, one achieves a measure of Marginal Entropy (with respect to log number of attributes) which can be related to the number of options. Figure 9.4 plots these points with a fitted polynomial.

Figure 9.4- Marginal Entropy v No. of Options



Thus an integrated rule of thumb for calculating Entropy is:

$$E_{in} = \mathrm{A}(n).(0.43\mathrm{P}(i) - 0.028\mathrm{P}(i)^2)$$

Where E_{in} = Estimated Entropy in respect of uncertainty of optimum option, for situation with i original options and n declared relevant attributes (9.4)

$$\mathrm{A}(n) = \log_2 n$$

$$\mathrm{P}(i) = \log_2 i$$

The use of this rule of thumb to *estimate* the statistics of Table 9.3 (which was based on the simulated data) is shown in Table 9.4. A comparison of the two tables shows that relative errors are highest for the higher numbers of attributes and distortions are exaggerated for the lower numbers of original attributes.

Table 9.4-Estimated proportion of information required, to fully determine optimum secured by declaration of attributes, related to total options

No. attributes	1	2	3	4	5	7	10	Starting
No. Options								Entropy
10	1.000		0.466		0.218		-0.119	3.32
20	1.000		0.510		0.283		-0.026	4.32
30	1.000		0.536		0.321		0.028	4.91
40	1.000		0.555		0.348		0.067	5.32
50	1.000	0.728	0.569	0.456	0.369	0.236	0.097	5.64

9.6.2 Simulation 2. Focused comparison of preference between most overvalued option with another.

One method of using Dora-D for single option decision identification, is for a decision maker to highlight anomalies in valuations suggested by MCAs and then specify preferences which circumscribe valuation latitude by "forbidding" the "out of order" relationships highlighted. I make no comment in this section on the cognitive reliability of this mechanic, considering only its analytic power. The approach may be given practical effect by identifying an option which appears overvalued and ranked above another decision which the decision maker considers is of greater value; developing a constraint which ensures that the first option cannot have an assigned MCA greater than the second; rerunning the evaluation;

identifying a further anomaly to be resolved in a similar manner; and continuing until the decision maker is satisfied with the ranking obtained.

This set of simulations, which was performed using the ten decision situations of 50 options with 5 declared attributes represented in 5x50data_2810, attempts to assess the optimum seeking efficacy of this process. In practice such anomalies would be judged subjectively. However, for the purpose of simulation, it is necessary to define a non-arbitrary criterion for selection of each "anomalous" pair. I assume here that the simulated decision maker selects:

(a) as most overvalued option (A); the option having the highest ratio of MCA to true value.

(b) as comparison option (B); the option, from within the group of options having higher true values than the most overvalued option, that option having the lowest calculated MCA.

Having done so, a constraint on value, $v(A) \leq v(B)$, is imposed, and the MCAs recalculated by rerunning the set of LPs for all options. The procedure is repeated until the decision maker is satisfied that there is no anomalous ranking.

For Data Set 1 the following results were obtained.

Table 9.5- Results simulation 2: most overvalued option

Data set 1

Preference	No of Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	10	3.322	0.731	2.322	0.411	0.527
37≤49	1	9	3.170	0.752	2.474	0.438	0.564
26≤15	2	6	2.585	0.807	3.059	0.542	0.661
19≤14	3	4	2.000	0.870	3.644	0.646	0.772
46≤26	4	3	1.585	0.891	4.059	0.719	0.808
21≤38	5	3	1.585	0.891	4.059	0.719	0.808
39≤32	6	1	0.000	1.000	5.644	1.000	1.000

In Table 9.5, Preference A≤B signifies that Option A is deemed by the simulated decision maker to have a value that is no higher than that of Option B. NTV is "Normalised True Value", that is the average "true" value of the options described, calculated using the decision maker's hidden valuation function, relative to the value of the true optimum. The remaining single efficient option after the declaration of these 6 preferences is the true optimum, for this data set, option Z. These results can also be represented graphically as in Figure 9.5:

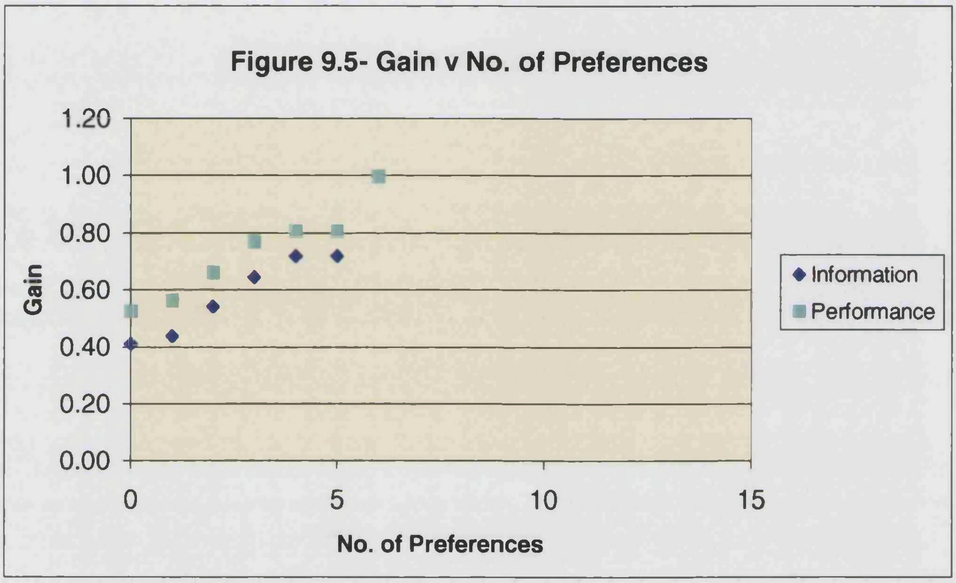


Figure 9.5- Gain v No. of Preferences

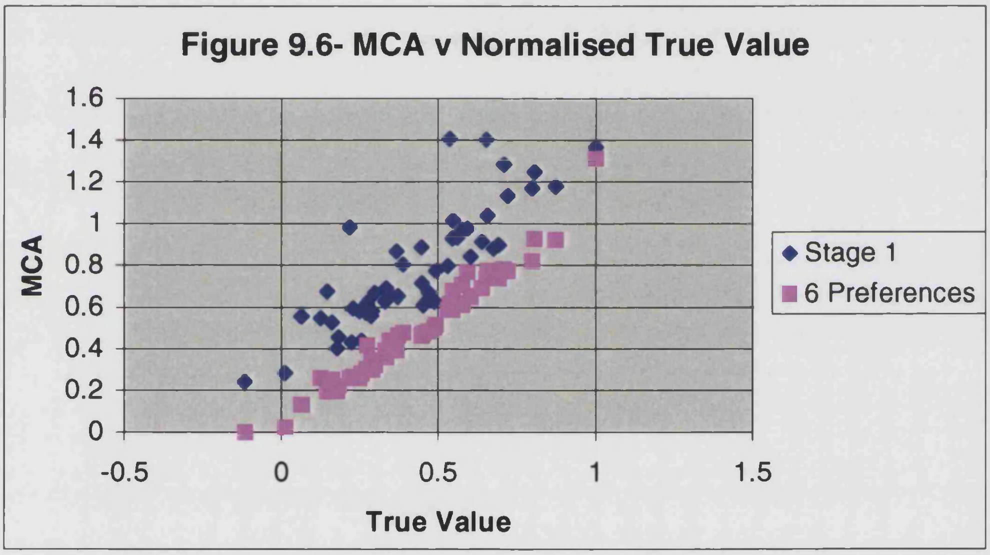
In undertaking these simulations for the other 9 data sets the following circumstances arose. In Set 2 there was a reduction from 12 efficient options to 3, after the specification of 5 preference pairs. However it took a further 5 preferences to determine the optimum. Similarly for Set 8, there was a reduction to 3 efficient options after the specification of 5 preferences, but it took a further 3 preferences to reduce the optimum candidates to 3, and a further 4 to distinguish the optimum. In both these cases the true values of the two additional options which were persistent in the efficient sets, were close in value to the optimum. The average of the NTV for the three persistent options was 0.970 for set 2 and 0.989 for set 8 suggesting that they had not been discriminated easily because they were close in value.

In a number of cases weakly efficient options (MCA=1) were found (eliminated as potential optima in this analysis). These appeared to arise when 1 efficient option was compared in a binary preference to another efficient option.

In three instances (Sets 6, 8,10) the optimum option was at some stage evaluated as the most overvalued option. In those instances the next most overvalued option was used as one of the members of the anomalous pair. In one instance (Set 8) an option was determined as most over-valued but already had the lowest MCA; the next most overvalued option was again used in the situation.

In all cases the hidden true optimum was correctly identified.

In all instances, expression of preference causes MCAs to migrate to lesser values (necessarily, MCAs can only reduce as value latitude diminishes). They move closer to a clear linear relation with the true value, with reducing dispersion. Figure 9.6 illustrates this for Set 1.



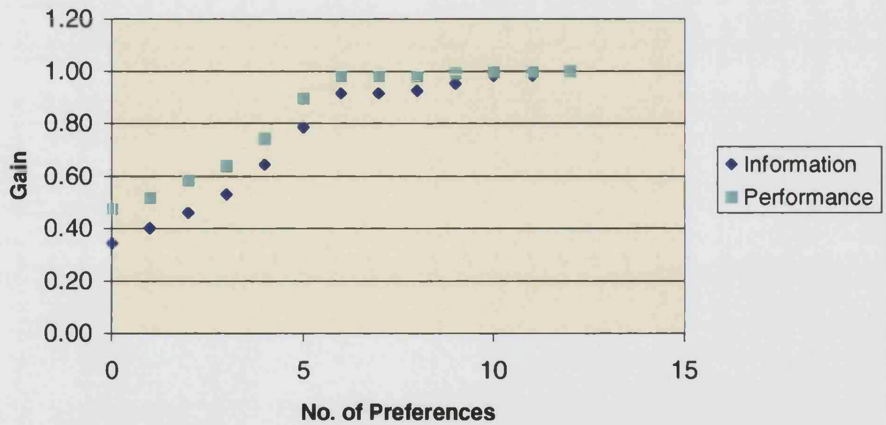
It should be noted that the information provided by six preferences is, in this instance, sufficient to resolve the optimum. However, it is *not* sufficient to provide an accurate full ordering of all options. Nor should one be surprised as the combinatorial information inherent in a complete order is of a far greater order than mere specification of one out of a few. The specification of further options would continue to improve the line.

The summary of results for all ten sets of data areas is shown in Table 9.6 and Figure 9.7 as follows:

Table 9.6- Results simulation 2: Most overvalued option
Average over 10 Sets

Preference	No of Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
	1	10.600	3.383	0.767	2.261	0.401	0.518
	2	8.600	3.048	0.800	2.596	0.460	0.583
	3	6.700	2.651	0.828	2.993	0.530	0.641
	4	4.300	2.011	0.877	3.633	0.644	0.744
	5	2.600	1.208	0.950	4.436	0.786	0.897
	6	1.600	0.475	0.991	5.168	0.916	0.981
	7	1.600	0.475	0.991	5.168	0.916	0.981
	8	1.500	0.417	0.990	5.227	0.926	0.980
	9	1.300	0.258	0.996	5.385	0.954	0.992
	10	1.100	0.100	0.999	5.544	0.982	0.998
	11	1.100	0.100	0.999	5.544	0.982	0.998
	12	1	0	1	5.644	1	1

Figure 9.7- Gain v No. of Preferences



9.6.3 Simulation 3. Focused comparison of preference between most overvalued efficient option with another option.

In the previous simulations it was assumed that a decision maker can subjectively spot the most "out of order" option and can partially correct valuations, by putting the test option below the worst evaluated superior case. In this simulation the perspective is altered so that the decision maker specifically considers only the most over-valued option amongst *efficient* options, that is those with an MCA greater than 1, but otherwise assesses them in like manner to simulation 2.

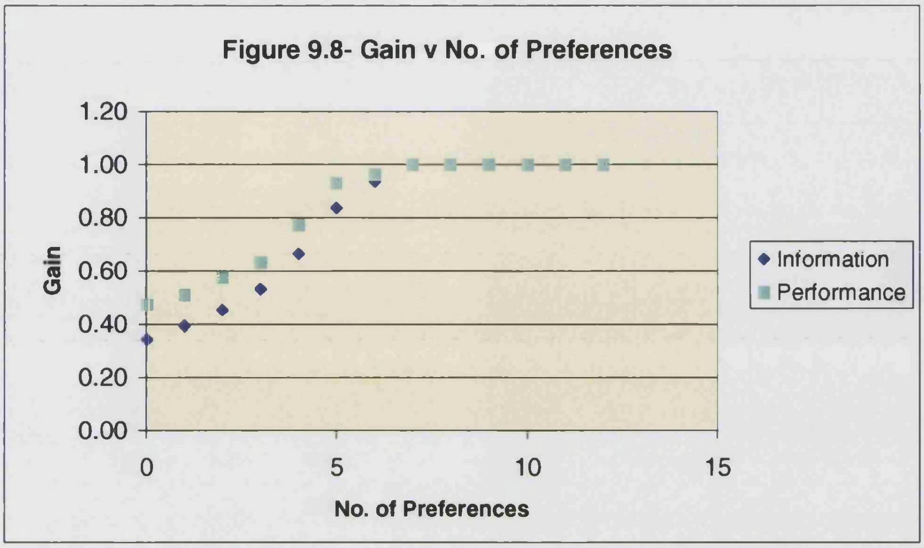
The average results obtained are summarised in Table 9.7 and Figure 9.8.

Table 9.7- Results simulation 3: Most over valued efficient option

Average over 10 Sets

Preference	No of Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
	1	10.900	3.415	0.764	2.229	0.395	0.513
	2	8.700	3.077	0.895	2.567	0.455	0.575
	3	6.500	2.644	0.822	2.999	0.531	0.634
	4	3.900	1.898	0.893	3.745	0.664	0.773
	5	2.100	0.917	0.969	4.727	0.838	0.932
	6	1.400	0.358	0.985	5.285	0.936	0.964
	7	1.000	0.000	1.000	5.644	1.000	1.000

The reader will note that there is little difference in the achievement relative to simulation 2, for lower numbers of preferences, but a small, not necessarily material, gain around four/five preferences.



9.6.4 Simulation 4. Comparison of preference between randomly generated pairs of options

In Simulations 2 and 3 the decision maker is assumed to make judged selections, the selection of the pair for scrutiny, as well as the expression of preference between the options within the pair, within the simulated process. In this set of analyses, it is assumed that the decision maker expresses preferences within pairs, but these pairs are randomly generated (once again using the facilities of Microsoft Excel), unguided by the Dora-D process.

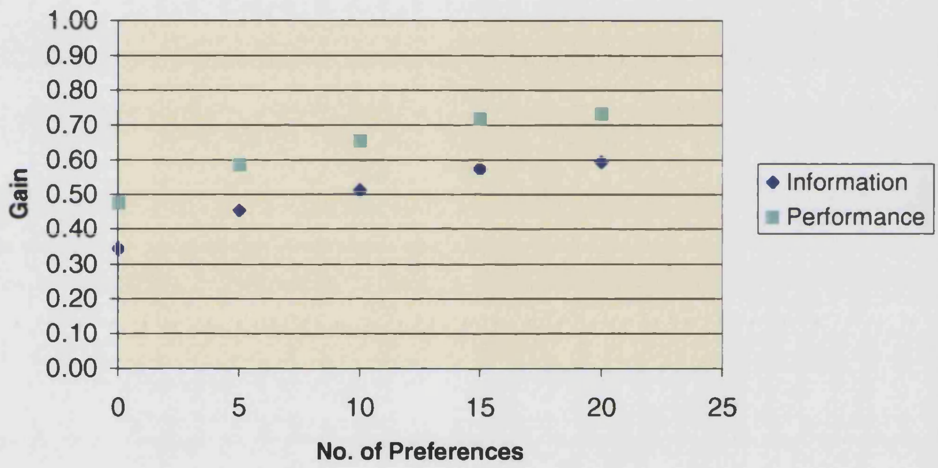
For manageability of analysis the constraints are introduced in blocks of five but the analyses are in other respects of identical form to Simulations 3 and 4; that is the analyses were performed on each of the same ten sets of 50 option 5 attribute data with 50 LPs being executed on each pass to determine the MCA under AP conditions of each option. Those strongly efficient options with MCA(AP) greater than 1 remain potential optima. A summary of the results obtained are shown in Table 9.8 and Figure 9.9.

Table 9.8- Results simulation 4; randomly generated pairs

Average over 10 Sets

Preference	No of Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
	5	8.700	3.085	0.801	2.559	0.453	0.587
	10	7.000	2.753	0.833	2.890	0.512	0.654
	15	5.700	2.407	0.864	3.237	0.573	0.718
	20	5.300	2.296	0.871	3.348	0.593	0.732

Figure 9.9- Gain v No. of Preferences



Whilst information is brought to the process in a way which reduces the number of potential optima, the unfocused nature of the mechanic provides far less exploitable information than the focused procedures of Simulations 3 and 4. The gain achieved by the expression of 15 random preferences is little better than that achieved using 3 focused ones. The information gain from 15 unfocused preferences is also roughly equivalent in information effect to the decision maker deciding to reduce the attributes considered relevant from 5 to 3.

9.6.5 Simulation 5. Progressive partial order development amongst efficient options.

In this set of analyses we return to the theme of making focused comparisons but we allow prior Dora-D passes to "select" the options over which the decision maker is asked to make preference judgements.

The procedure is that after Initial Option Reduction, the simulated decision maker is asked to express a preference between the options having the two highest MCA(AP), which it reliably does on the basis of its hidden valuation function. I again suspend judgement regarding the ability of a decision maker to reliably assert such a preference, but note that in some sense we can look on these as the most contrasted amongst the potential optima, as each has a substantial advantage against the other when evaluated using its own CAF.

A value constraint based on the preference is imposed for the second pass and another option is identified with the highest MCA(AP) amongst the remaining efficient options. The decision maker is "asked" to place this option in correct value rank with respect to the other two, forming a partial order of three options. The procedure continues in like manner, creating rankings of 3, 4, 5 etc until only a single efficient option remains.

The reader will note that the placing of a new option within an existing partial order, will usually be equivalent to the expression of more than one binary preference. Thus, if we say, when seeking to add a fifth option, that the fifth may with even chance slot anywhere within the ordinal scale defined by the other 4, there is 100% chance of 1+ comparison being necessary, an 80% chance of 2+, a 60% chance of 3+, a 40% of 4; or an average of 2.8. More generally we can say that the average number of preferences that need to be expressed is $n\left(\frac{1}{2} + \frac{1}{(n+1)}\right)$, where n is the number of options in the existing partial order. When assessing the effective number of preferences expressed in the creation of a partial order, allowance must also be made for the number of binary preferences used to generate the order, prior to the insertion of the last option. For ease of comparison with the binary preference mechanics explored, I will use this conversion to binary preference equivalence in the tables and graphs that follow. The rule worked well in comparison with the number of true preferences expressed, when averaged over the data sets of data used.

Using Set 1 data the results obtained were as follows:

Table 9.9- Results simulation 5; progressive partial order development Data set 1

Preference	Equivalent Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	10	3.322	0.731	2.322	0.411	0.527
19>37	1.00	9	3.170	0.752	2.474	0.438	0.564
16>19>37	2.67	8	3.000	0.764	2.644	0.468	0.585
16>42>19>37	4.92	6	2.585	0.791	3.059	0.542	0.633
16>32>42>19>37	7.72	5	2.322	0.788	3.322	0.589	0.627
16>32>29>42>19>37	11.05	3	1.585	0.807	4.059	0.719	0.661
16>47>32>29>42>19>37	14.91	1	0.000	1.000	5.644	1.000	1.000

Table 9.10- Results simulation 5; progressive partial order development Average over 10 sets

Preference	Equivalent Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
	1.00	11.700	3.522	0.751	2.122	0.376	0.486
	2.67	9.500	3.204	0.771	2.440	0.432	0.522
	4.92	6.700	2.657	0.791	2.987	0.529	0.564
	7.72	4.600	2.121	0.794	3.523	0.624	0.573
	11.05	2.600	1.309	0.875	4.334	0.768	0.738
	14.91	1.200	0.200	0.983	5.444	0.965	0.959

Figure 9.10- Gain v Equivalent Preferences
Set 1

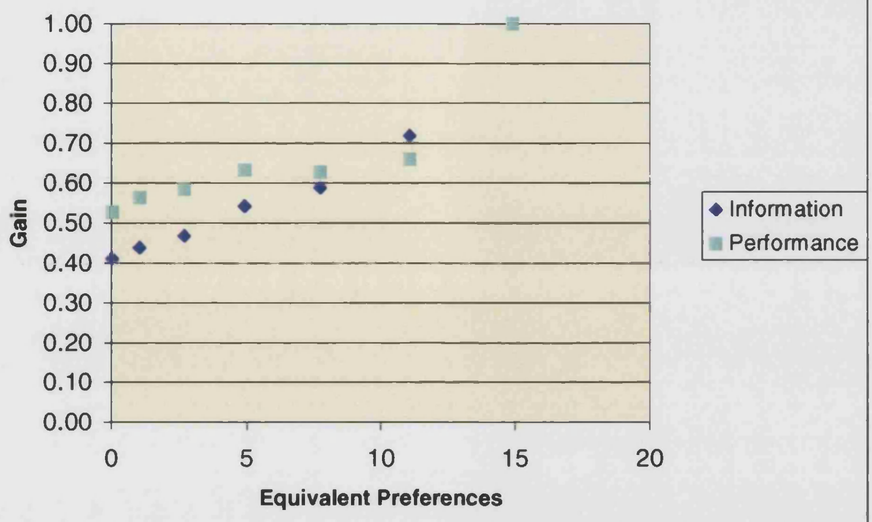
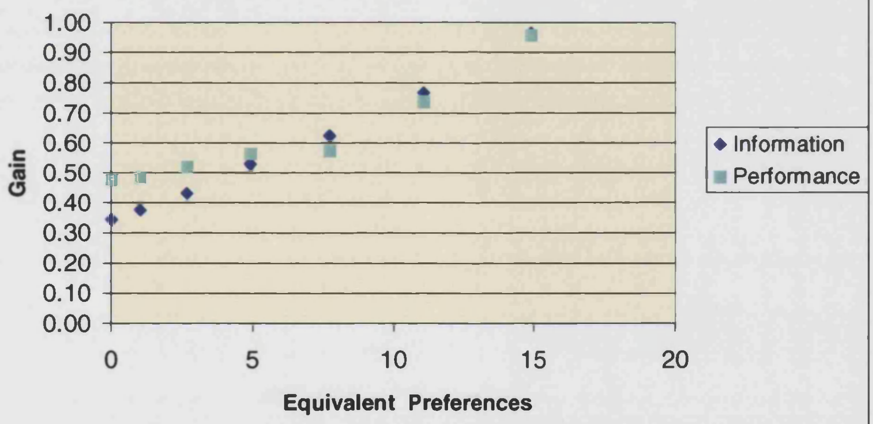


Figure 9.11- Gain v Equivalent Preferences



The mechanic appears more powerful than random pair generation but still considerably less so than the focused paired comparison, for example. However, it should be recognised that the procedure of Simulation 3 itself involves a complex judgement process which, if decomposed, can be construed as a partial order formation. It might be expected to be more discriminatory, here we merely seek to rank efficient options behind other efficient options (ie highly scoring options), there we rank them behind poorly scoring options, a more discriminatory principle.

Nevertheless, the procedure serves progressively to isolate the optimum and does so more economically than universal comparisons, ab initio. The situation is less clear-cut if one takes as a starting point the efficient options that remain after stage one reduction. One might note here that an efficient content-independent search, where a decision maker is cognitively competent to make consistent choices, requires one less comparison than the number of original options existing, in order to determine an optimum. In this simulation, the subsequent average pace of reduction in the number of potential optima, seems little different from straightforward elimination of the reduced set remaining. The factors here are unclear, but it could be useful to explore whether additional efficacy is related the number of potential optima remaining, compared to the number of attributes considered relevant.

9.6.6 Simulation 6. Focused comparison of preference between the efficient pair of options having the highest MCA(AP) statistics.

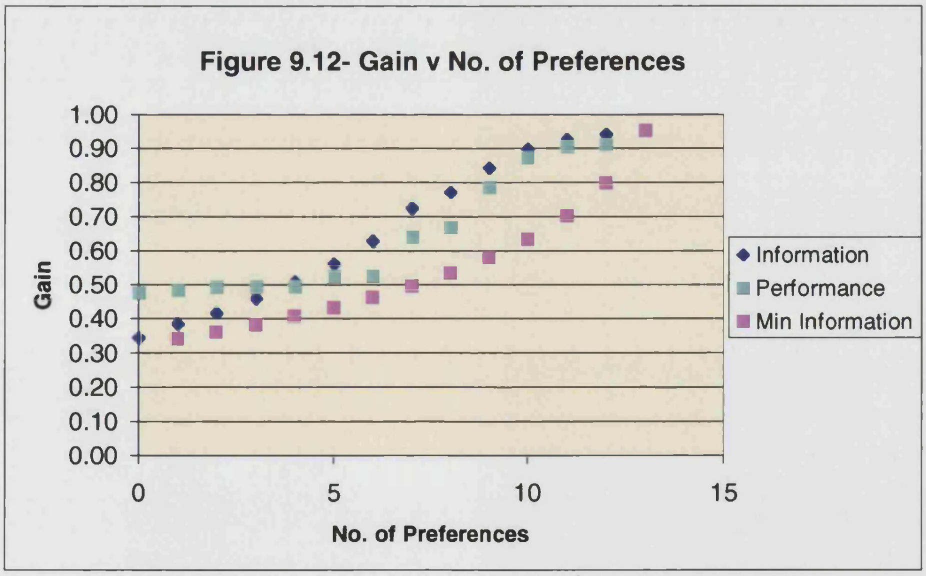
The mechanic of Simulation 5 had the merit of securing good information gain relative to the number of Dora-D reduction passes made. However, there was doubt as to the reduction effectiveness of the approach if the construction of partial orders is considered in terms of the number of paired comparisons to which it is equivalent. Reference was made to the fact that it did not appear to be superior to an unstructured succession of paired comparisons of efficient options, which (because each such comparison makes a previously efficient option, inefficient) will generate an optimum, after as many comparisons as there are efficient options following Initial Option Reduction, less one. In part this may be caused by the characteristic that to place an efficient option within an existing partial order, involves a comparison with one efficient option, but the other options within the order are no longer potentially optimal.

In this series of simulations we return to strict comparisons of individual pairs, recalculating the efficient set and MCAs (under AP conditions) with a Dora-D pass, after specification by the simulated decision maker of the preferred option within each considered pair. The pair tested on each occasion are the two options having the highest MCAs amongst efficient options remaining; retaining the principle adopted in Simulation 6. I have already indicated that these options can be considered to be the “most different” insofar as the penalty for choosing incorrectly when the other is the optimum is, on the information available, the largest for these two options. There are other grounds. They are also, prima facie, the two most likely options; each such comparison secures the maximum reduction in a measure of "possibility" even if this cannot be equated to a probabilistic measure. Finally, it secures the maximum guaranteed aggregate reduction in MCA amongst the potential optima, this will be at least equal to the second highest MCA as the MCA of the least preferred of the pair will reduce to no greater than 1 on the next Dora-D pass. MCAs of other hitherto efficient options may also be reduced (possibly to below unity); though not the MCA of the preferred option of the pair. There is a guarantee the mechanic will be at least as effective as an efficient content-independent search, as we know that in making any comparison between efficient options we must secure a reduction in the number of potential optima by at least one.

The results are summarised in Table 9.11 and Figure 9.12.

Table 9.11- Results simulation 6; Comparison between pairs of efficient options having highest MCA(AP)s. Average over 10 sets

Preference	No of Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
	1	11.300	3.469	0.751	2.175	0.385	0.484
	2	10.100	3.297	0.755	2.347	0.416	0.492
	3	8.700	3.052	0.755	2.592	0.459	0.493
	4	7.300	2.778	0.754	2.865	0.508	0.494
	5	6.100	2.481	0.766	3.163	0.560	0.523
	6	5.000	2.101	0.767	3.543	0.628	0.526
	7	3.900	1.562	0.822	4.082	0.723	0.641
	8	3.200	1.292	0.834	4.351	0.771	0.668
	9	2.500	0.891	0.896	4.753	0.842	0.786
	10	2.000	0.581	0.940	5.063	0.897	0.872
	11	1.700	0.417	0.953	5.227	0.926	0.906
	12	1.500	0.332	0.956	5.312	0.941	0.912



In Figure 9.12 the magenta points designate the information gain for an efficient content-independent elimination.

It will be noted that performance gain is limited over the first few preference introductions, implying that early eliminations are not much more likely to be of options of lower than average true value. The information gain relative to the minimum information line is useful, but it is not a radical gain. Indeed in two cases out of the ten, the mechanic did no better than content-independent elimination and in two further cases just one comparison was saved. Some methodological issues are at stake. The measure of both performance gain and information gain is dependent on an assumption of equal likelihood amongst the efficient options that remain. It is apparent that MCA(AP) is related to likelihood in the sense that options with a higher MCA have bigger domains of variation in the value function under which the option are optimal. The optimum entered the pair for comparison, on average, by the third comparison. There is over the ten data sets, an average of 13.2 initially efficient options. Therefore, on equal likelihood assumptions, we might expect the optimum to enter the comparison pair on average by the sixth comparison.

There is one further ground for considering the approach is a useful one. I argue in this thesis that values are labile; that a decision maker, quite independently of measurement or elicitation methodology issues, cannot stably and with precision retain weights within his or her value system. *Amongst efficient multiple attribute* options, a decision maker may not be able to discriminate value beyond a limited, say a 3 bit, capability. This implies there is a level at which the potential for value improvement of a particular option will be spurious. The level of the MCA in excess of 1 is a measure of the maximum value advantage that a particular option can achieve relative to the best of the rest. It follows that we can set a value of MCA at which do not consider the matter worth pursuing, not because the value difference wouldn't be important if a particular value function was the "true" function, but because it can never be said with sufficient precision that it was. One might set an MCA(AP) as a threshold not of indifference but of discrimination, say, of 1.1. If one stops the search at this level, an average of 6.1 comparisons would have been made over the test set. In reality, the true optimum of the simulated decision maker (which differs from real decision makers in being able to retain its value's perfectly) would have been found by this time in every case. If the threshold were to be 1.15, the number of comparisons drops to an average of 4.3. The optimum would have been correctly identified in 9 out of 10 of the test cases and in the tenth case the value of the selected option would have been within ½% of the value for the optimum.

An area for possible future research is to obtain a greater understanding of how MCAs can be related to likelihood of optimality and, accordingly, get a fuller insight into the information and performance gain of the approach.

9.6.7 Simulation 7. Ranking of Attributes.

In this series, the simulated decision maker places attributes in order of potency. The effect on the reduction in the number of potential optima and the performance gain is evaluated. It will be recalled that the hidden weights are equal in value, that is equal in potency, and this remains the basis for identifying the true optimum and the value of all options. However, notwithstanding, it is assumed that the "decision maker" will declare weak preferences. The "decision maker" when “asked” ranks the attributes in the order 1, 2, 3, 4, 5.

For each set of the ten, the reduction of potential optima and their average Normalised True Value is found for five conditions. These are (1) No preference expressed; (2) weight of attribute 1 is greater than or equal to 2; (3) weight of attribute 1 is greater than or equal to attribute 2, which is greater than or equal to 3; (4) weight of attribute 1 is greater than or equal to 2, which is greater than or equal to 3, which is greater than or equal to 4; (5) weight of attribute 1 is greater than or equal to 2, which is greater than or equal to 3, which is greater than or equal to 4, which is greater than or equal to 5.

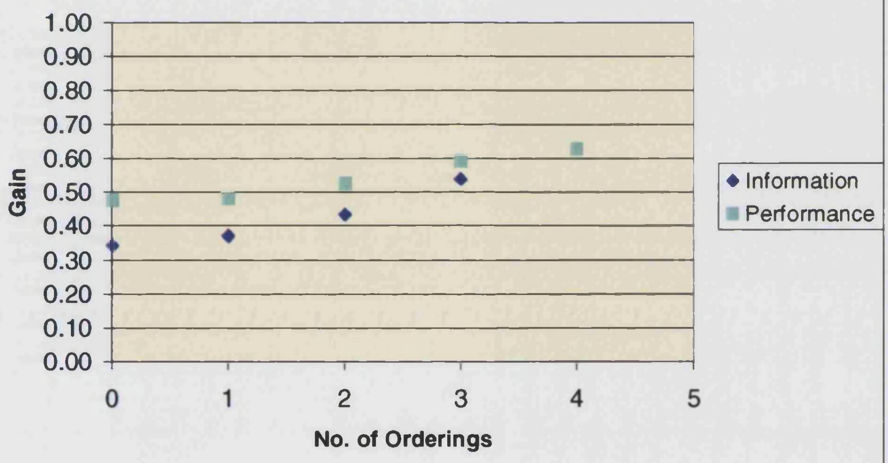
The gains obtained over the ten sets are summarised in Table 9.12 and Figure 9.13.

Table 9.12- Results simulation 7; Ranking of attribute weights by potency

Average over 10 sets

Preference	No of Attribute Orderings	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
w1>w2	1	11.900	3.556	0.748	2.088	0.370	0.481
w1>w2>w3	2	9.300	3.193	0.771	2.451	0.434	0.528
w1>w2>w3 >w4	3	6.300	2.609	0.800	3.034	0.538	0.591
w1>w2>w3 >w4>w5	4	4.400	2.089	0.819	3.555	0.630	0.630

Figure 9.13- Gain v Attibute Orderings



The effect of these orderings can be compared with the information gain implicit in reducing the number of relevant attributes. With the test data, stating two ordinal preferences is rather less effective than reducing the number of attributes by 1; and specifying 4 orderings seems more effective than reducing variables by 2.

Placing all variables in order provides a similar additional gain in information to that obtained in the initial option reduction. It appears an analytically potent mechanic, particularly as ordering of attribute weights seems a psychologically more reliable expression than the comparison of options varying simultaneously in multiple dimensions.

9.6.8 Simulation 8- Capping of Attribute Weights

As discussed previously, whatever the ability of this or any other technique to eliminate options on the basis of preference between *real* options, these will tend to require expression by a decision maker of [m,n] preferences. This, I have argued and assume, is generally not a cognitively easy task, though it may be within the competence of some decision makers in some situations.

A decision maker may, however, "know what he/she doesn't like", in the light of the implications of a Dora-D pass. Dora-D "suggests" CAFs and indicates for any options CAF the contribution to value of particular attributes. Decision makers may, from such evidence, be able to assert that particular attributes for some feasible CAFs are "over-weighted". This is a concept that I feel comfortable with as a decision maker, and, whatever the psychological justification, is well established as a pragmatic procedure within the OR paradigm, where a decision maker's satisfaction with the implications of a model, contributes either to its validation or indicates flaws that should be addressed. I suggest that a decision maker may reasonably express upper bounds of weights of some attributes, and can progressively reduce value function latitude through re-examining the implications of possibilities that persist, after successive Dora-D reductions.

This set of simulations explores the extent to which potential optima are reduced, and measures the information and performance gains by a decision maker "capping" attribute weights. It is assumed that the simulated decision maker caps weights for all attributes at, respectively, factors of 2.00, 1.50, 1.20, 1.10, 1.05, 1.02 of the hidden true value of the weights within "his" value system. It should be noted that in practice a decision maker or analyst cannot make an actual determination of his/her capping level. He or she merely states a level that they are "confident" represents an upper limit. However, he or she is able to see the impact of such judgements on the options that can remain efficient in the light of such judgements and their corresponding CAFs.

I also examine for the test cases, the effect of the decision maker capping only attribute1; attributes 1 and 2; attributes 1 to 3; and attributes 1 to 4 at the 1.2 level. Tables 9.13-9.15 show how the declaration of weight upper limits reduces potential optima, and the extent of the Information and Performance Gains over the test sets:

Table 9.13- Number of remaining efficient options/ potential optima Average of ten sets.

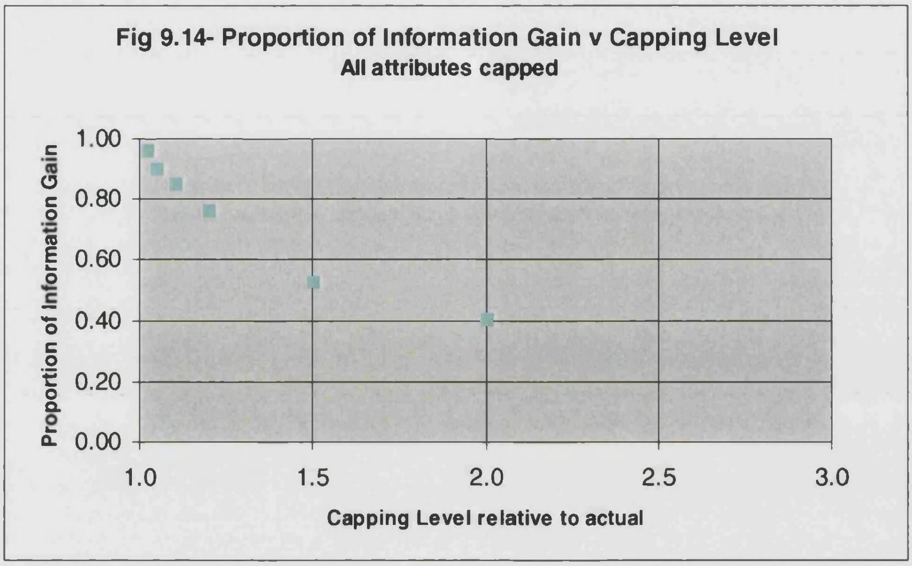
	Capping Level						
Attributes Capped	1.02	1.05	1.1	1.2	1.5	2	None
1				11.30			13.20
1,2				10.30			13.20
1,2,3				7.80			13.20
1,2,3,4				5.70			13.20
All	1.20	1.60	2.00	2.90	6.80	10.40	13.20

Table 9.14- Proportion of information required to fully determine optimum provided by declaration of Capping limits. Average of ten sets.

	Capping Level						
Attributes Capped	1.02	1.05	1.1	1.2	1.5	2	None
1				0.383			0.344
1,2				0.405			0.344
1,2,3				0.484			0.344
1,2,3,4				0.575			0.344
All	0.965	0.901	0.848	0.764	0.528	0.406	0.344

Table 9.15- Performance Gain provided by declaration of Capping limits. Average of ten sets.

	Capping Level						
Attributes Capped	1.02	1.05	1.1	1.2	1.5	2	None
1				0.515			0.476
1,2				0.534			0.476
1,2,3				0.586			0.476
1,2,3,4				0.641			0.476
All	0.999	0.972	0.954	0.903	0.682	0.561	0.476



In broad terms, capping all attributes at the 1.5 level provides half the information necessary to determine the optimum, and a similar impact is achieved by capping 4 attributes at the 1.2 level. If a decision maker can assess capping limits for all attributes which are 1.2 actual attribute weights, then the decision maker is reducing the scale of the problem to a similar degree as if he or she had asserted that 2, rather than 5, attributes were relevant.

9.6.9 Simulation 9- Supplemented Larichev Fundamentally Decomposed Preference

I have suggested that comparison of [1,1] choices should be more psychologically reliable than choices of Higher Order. In the next two simulation sets I consider preferences between such fundamentally decomposed choices generated from the options. In the first of these I use Larichev Decomposition. Using the method I have described in Chapter 7, r [1,1] choices (where r is the number of attributes) are constructed from the efficient options generated on each Dora-D pass. These r choices, five in this set of simulations, are ranked by preference to the decision maker. The constraints so generated are used to develop another set of efficient options and another set of [1,1] choices until no further reduction is possible.

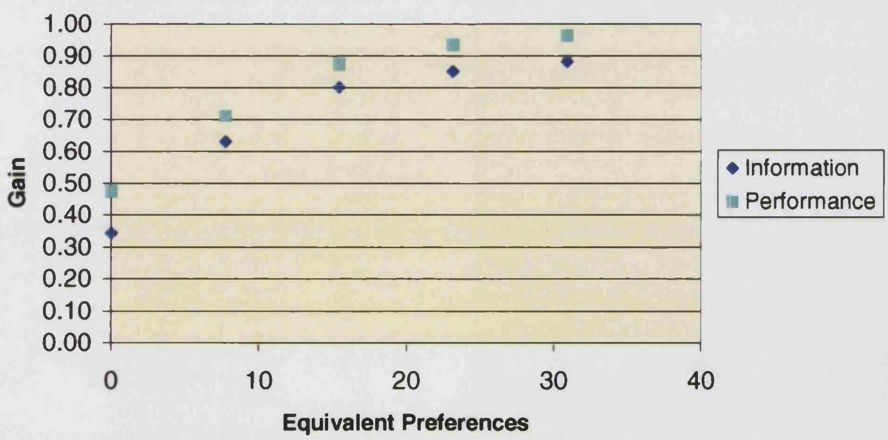
As will be commented, Larichev decomposition does not always reduce efficient options by as much as can be eventually achieved (it is surmised that the method will always reduce efficient options to no more than the number of attributes). Franklin decomposition can sometimes achieve further reductions, and so, in these simulations, switches are made to Franklin if this situation is encountered. Larichev is equivalent to Franklin for 2 remaining options. Accordingly, a Larichev reduction to 2 options cannot be improved by this extension.

As in Simulation 5, the ranking effectively involves several binary choices, an average of 7.72 per pass for the attributes of the test data. To facilitate comparison, equivalent binary preferences are tabulated in the analyses. The results averaged over the ten test data sets are shown in Table 9.16 and Figure 9.14.

Table 9.16- Results simulation 9: Larichev Fundamentally Decomposed Preference. Average over 10 sets

No of passes	Equivalent Binary Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
1	7.72	4.500	2.089	0.861	3.555	0.630	0.712
2	15.44	2.500	1.123	0.940	4.521	0.801	0.874
3	23.16	2.100	0.832	0.970	4.812	0.853	0.937
4	30.88	1.700	0.658	0.984	4.985	0.883	0.967

Figure 9.14- Gain v Equivalent Preferences



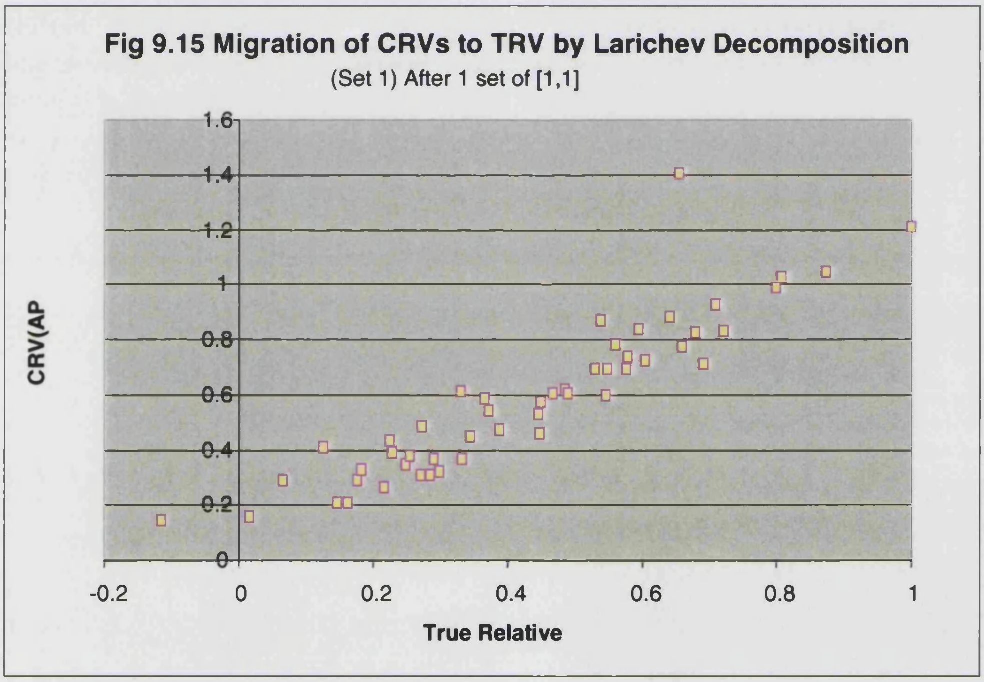
Only in 4 cases did potential optima reduce to 1 by successive unmodified application of the method. In two further cases they reduced to 2. In the remaining four cases further reductions were possible by supplementing the technique by

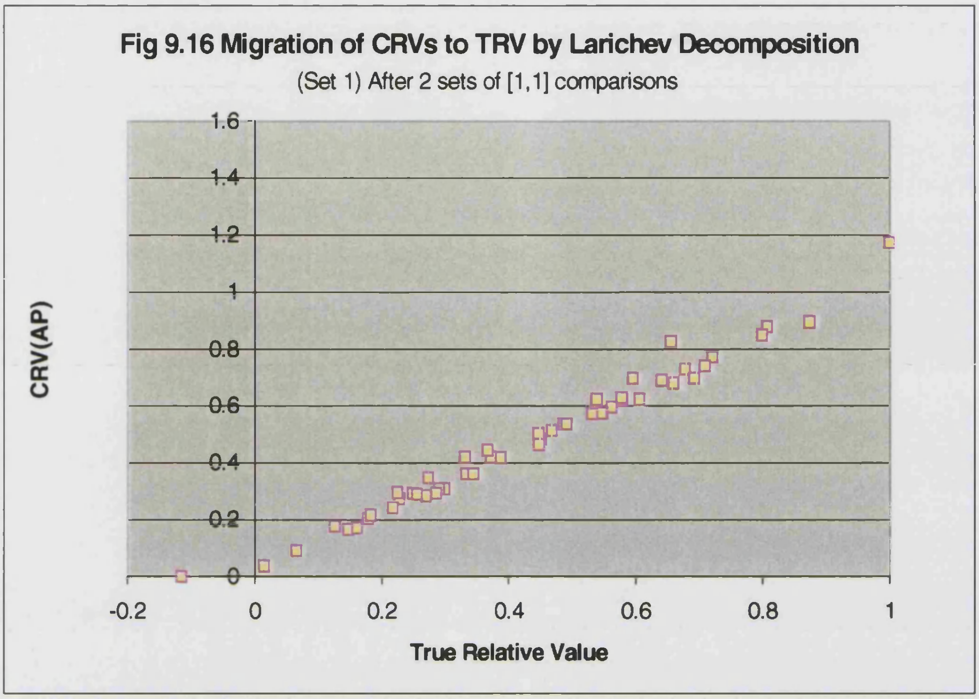
Franklin Decomposition; these were to two options in three cases, and to 3 options for case 8, where the true values of the final candidates were close.

Information Gain per pass was good and, per equivalent binary preference, was comparable to the methodologies explored in Simulations 4 to 7, many of which depend on cognitively more difficult elicitation. It is inferior in discrimination to the methods of Simulations 2 and 3 but it will be recalled that these involve slotting an overvalued option below another option selected for efficacy. This judgement brings extra information to the problem. Performance gain relative to information gain is marginally superior in a number of instances.

The method bears a similarity to comparison of attribute weights and the results obtained are similar to the comparable test of Simulation 7.

The migration of CRVs to true values using this method is illustrated in Figures 9.16 and 9.17.



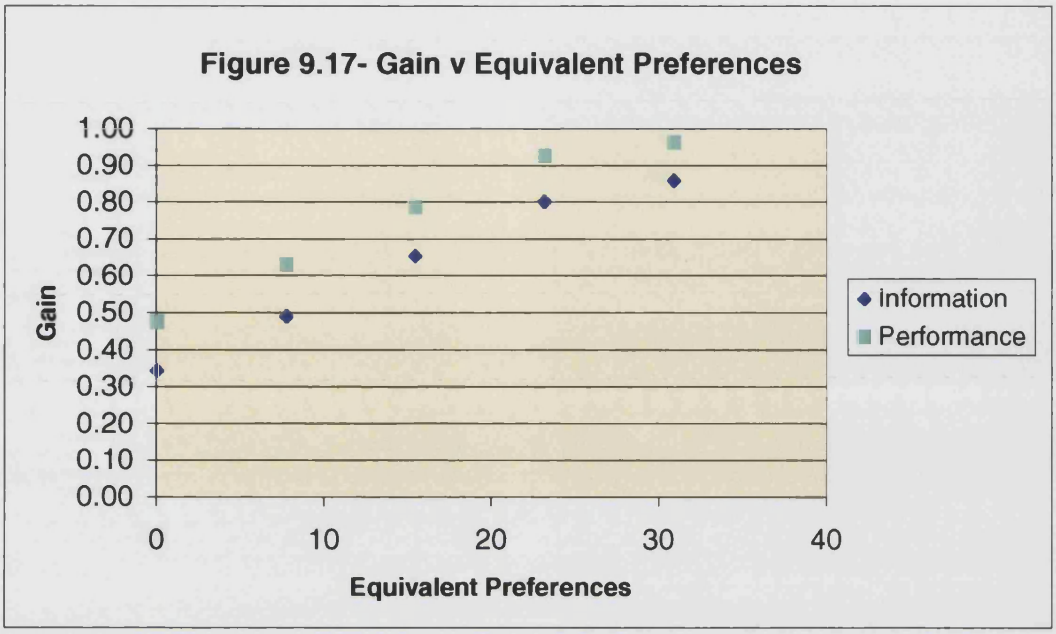


9.6.10 Simulation 10- [1,1] Franklin Fundamentally Decomposed Preference

In this series, the performance of [1,1] Franklin Decomposition is explored, *ab initio*. Pairs of efficient options are decomposed and attribute differences ranked, and converted into value constraints for *subsequent* Dora-D passes. The procedure is repeated until potential optima reduce to one, or decompositions of all pairs of remaining potential optima have been explicitly examined. The results averaged over the ten sets are summarised in Table 9.17 and Figure 9.17.

Table 9.17- Results simulation 10: Franklin Fundamentally Decomposed Preference. Average over 10 sets

No of passes	Equivalent Binary Preferences	No of Efficient Options	Entropy	NTV Efficient Set	Information Gain	Proportion Information Gain	Performance Gain
None	0	13.200	3.703	0.746	1.941	0.344	0.476
1	7.72	7.600	2.878	0.823	2.766	0.490	0.632
2	15.44	4.300	1.952	0.898	3.692	0.654	0.787
3	23.16	2.400	1.117	0.966	4.527	0.802	0.927
4	30.88	1.900	0.800	0.982	4.844	0.858	0.962



Generally, pure [1,1] Franklin Decomposition is inferior to supplemented Larichev Decomposition (that is, with switches to Franklin when the pure procedure "sticks"). The difference seems marked in the early passes, though after four passes both methods have secured good reduction. Franklin secured reduction to the optimum in 4 cases and to two options in 4 further cases. For case 8 it did not reduce below 4. This was the same case that supplemented Larichev could not reduce below 3.

The difference between the Larichev and the Franklin methods is, although they are both based on decompositions of efficient options, the virtual choices derived in the Franklin approach are not necessarily themselves efficient. The Larichev choices, though dominated, are weakly efficient. We can be sure that if the choices were strongly efficient, each binary comparison would secure at least a one option reduction. I surmise that the slightly weaker Larichev condition is still highly discriminatory and provides powerful elimination, at least to the point where remaining options can simultaneously sustain an MCA of 1 (ie are on an efficient facet).

9.6.11 Simulation 11. The effect of model mis-specification from making false linear assumptions. Non-linear additive values.

I have suggested, with specific qualifications, that the existence of a mental, multiple variable, valuation system, involving non-linearities of which a decision maker is unconscious, is a philosophically suspect basis for well-founded decision making.

However, it is possible that this is not so, or that a decision maker or analyst, who in other respects believe the techniques described here may be useful to them, does not believe it to be so. Therefore, to test the implication of making this form of false assumption, I consider here situations in which the hidden value function is non-linear in the attribute measures and the decision maker expresses preferences between choices on the basis of this function. However, the analyst models the situation as linear in the attributes.

The emphasis here is not on the speed of convergence and reduction efficiency but on the degree to which one can be misled. Before starting this analysis, I sought to predetermine an analysis mechanism from the several that were available. I sought to test correct elimination by Dora-D, distinguishing this from the effectiveness of any tie-breaking mechanism. I therefore decided to examine the effectiveness on two bases:

(a) The generation of a short-list of efficient options by successively feeding-back Larichev choices (and then [1,1] Franklin choices if and when Larichev "sticks") into Dora-D passes until no further reduction is possible. Then allowing the fictional decision maker to pick the best from the short-list that remains.

(b) Proceeding as in (a) until Dora-D secures no further reduction and then applying a tie-break procedure.

The tie-break procedure chosen, in prospect, was to find the "linear" value function which maximised the minimum value of the remaining options and to use this to determine the optimum. This overlooked that, whilst this particular tie-breaker can be a useful eliminator where several options remain, it cannot distinguish between options on an efficient facet (ie options for which there exists a CAF such that the

CRV of the options is simultaneously 1). On the first appearance of this situation, a supplementary criterion was adopted. This was to take the average of the normalised "linear" CAFs for the remaining potential "optima" and to select the option which was maximised with that function. In practice all remaining options after the Larichev/ Franklin reductions were of this "on facet" type.

In Simulation 11 hidden true value was defined to be additive and:

- linear with the square of Variable 1 of the data set for positive values- zero otherwise.
- linear with the Variable 2 to the power 1.5 for positive values- zero otherwise.
- linear with Variable 3.
- linear with Variable 4 to the power of 2/3 for positive values- zero otherwise.
- linear with the square root of Variable 1 of the data set for positive values- zero otherwise.

Thus the fictitious "analyst" continued to assume the linear Dora-D model with the standard test data, but the "decision maker's" preference declarations were determined by applying the above hidden non-linear value function to the same data. The following results were obtained:

Table 9.18- Simulation 11: Effect of linear mis-specification of non-linear additive model

Data Set	No. of Options after Larichev/ Franklin	Options ID	NTV of Options before Tie-break	True Rank of Options	Best Option Criterion (a)	NTV of Option	Best Option Criterion (b)	NTV of Option
1	1	19	1	1	19	1	19	1
2	2	12	1	1	12	1	12	1
		50	0.727	2				
3	1	25	1	1	25	1	25	1
4	2	17	1	1	17	1	17	1
		43	0.690	4				
5	2	16	1	1	16	1	16	1
		31	0.834	2				
6	1	33	1	1	33	1	33	1
7	1	39	0.971	3	39	0.971	39	.971
8	2	1	0.946	2	45	1	45	1
		45	1	1				
9	1	27	1	1	27	1	27	1
10	2	7	1	1	7	1	50	.743
		50	.743	3				
Av. Ten Sets	1.5		0.947	1.5		0.997		0.971

In nine of the ten test cases the normal Dora-D reductions included the true optimum within the final set. In the other case the third ranked option had a true value close to that of the optimum. In one case the tie-break failed to find the best option available selecting the third ranked option (out of 50) although this had a materially lower NTV. In no case was a preference expressed within the linear model which led to infeasibility.

In further tests it was established that in the ten data sets there were only 8 options out of the 134 options that were initially efficient within the non-linear model, that were not also efficient within the linear one. Thus the optimum is rarely excluded from the set for reduction, and, where this does occur, the likelihood is that there is a good solution within the set that is considered. Also, because decision makers need not necessarily resort to an automatic tie-breaker but can continue with other forms of elicitation, the risk of practically misleading conclusions seems small.

9.6.12 Simulation 12. The effect of model mis-specification from making false linear assumptions. Underlying multiplicative model. Retained constraints.

In this simulation it is assumed that the decision maker now determines his preferences according to an underlying multiplicative model, but that in other respects the simulation is similar to Simulation 11. Specifically, the fictitious "decision maker" values options in proportion to the geometric means of the attribute, subject to the minimum value of any attribute being taken as being 0.5, if its actual value is less than this.

In this set there were several instances of the simulated decision maker's rankings generating sets of inconsistent constraints. Where this occurred, all existing constraints used in previous passes were retained. Within the set of new constraints relating to the last stated rankings of preferences, the procedure adopted was to accept the binary preferences between adjacent higher ranking choices before the lower ranking ones. The last constraint which then resulted in infeasibility was permanently excluded and the remaining constraints allowed unless these too "caused" infeasibility. No attempt was made to find the most plausible violating constraint or to replace the constraint with others (eg to reflect preferences between "non-adjacent" choices). It is not suggested that this is the best procedure and another is used in Simulation 13.

It is only necessary to run one LP within the pass to establish LP infeasibility and adjustment is a quick process with the criteria mentioned.

The results obtained are shown in Table 9.19.

Table 9.19- Simulation 12: Effect of linear mis-specification of multiplicative model

Data Set	No. of Options after Larichev/ Franklin	Options ID	NTV of Options before Tie-break	True Rank of Options	Best Option Criterion (a)	NTV of Option	Best Option Criterion (b)	NTV of Option
1	1	47	.802	2	47	.802	2	.802
2	2	5	.636	13	12	.885	12	.885
		12	.885	2				
3	1	24	.949	2	24	.949	24	.949
4	2	11	.661	12	43	1	43	1
		43	1	1				
5	1	16	.882	6	16	.882	16	.882
6	1	30	1	1	30	1	30	1
7	2	19	1	1	19	1	29	.921
		29	.921	2				
8	2	13	1	1	13	1	13	1
		45	.995	2				
9	1	27	.826	6	27	.826	27	.826
10	2	6	.801	3	7	1	6	.801
		7	1	1				
Av. Ten Sets	1.5		0.891	3.6		0.934		0.907

The results obtained were visibly inferior to those obtained for the additive non-linear true model. In two cases the selected option was outside the "actual" top 5 in both the selection from short-list and automatic tie-break methods. The tie-break selected the inferior of the available options in 2 out of the 5 cases when more than one option remained.

9.6.13 Simulation 13. The effect of model mis-specification from making false linear assumptions. Underlying multiplicative model. Successive constraint elimination.

In this set we consider identical preference valuation assumptions to those considered above but the evaluation procedure is different. A short justification of this approach is appropriate.

Where there is a true linear relationship, the inclusion of value limiting constraints for dominated options could be redundant, but does no harm. However, if the true value relationship is non-linear, retention of such constraints, used to identify now excluded options, may serve to restrict the ability of Dora-D to find the best linear approximation with respect to the remaining candidates.

By the same token, whilst retention of previous linear constraints has use in a linear model, perhaps improving the precision of the value function, they can be a handicap in a non-linear situation where non-linear preferences can give rise to apparent inconsistencies if the models "expectation" is of linear valuation. Moreover, much of the information contained in prior constraints is already reflected in the options retained or rejected; if constraints have resulted in reduction they have largely "done their work" .

Accordingly in the simulation procedure investigated here the Larichev system is applied as before. But each time a reduction is secured the previously declared preference constraints are discarded. Moreover, the Comparison Set is reduced to the potential optima. That is, no constraint is imposed requiring the valuation of already eliminated options to have a value of less than 1.

Prior constraints are retained only:

(a) When no reduction is secured on a pass, and

(b) When there is no infeasibility

Thus, where Larichev fails to reduce, Franklin is followed, retaining prior preference constraints. If a further reduction is achieved Larichev is reinstated. Tie-breaks are determined by the magnitude of the CRVs for the uneliminated options, subject to already eliminated options not being permitted to inhibit respective valuations.

In one case the introduction of a single set of constraints reflecting a Franklin ranking was incompatible with a linear model resulting in infeasibility. In this case the Simulation 12 approach was used to eliminate one of the constraints.

The results obtained are shown in Table 9.20.

Table 9.20- Simulation 13: Effect of linear mis-specification of multiplicative model. Successive constraint elimination.

Data Set	No. of Options before Tie-break	Options	NTV of Options before Tie-break	True Rank of Options	Best Option Criterion (a)	NTV of Option	Best Option Criterion (b)	NTV of Option
1	1	47	.802	2	47	.802	2	.802
2	1	12	.885	2	12	.885	12	.885
3	2	14	1	1	14	1	14	1
		24	.949	2				
4	1	43	1	1	43	1	43	1
5	2	26	.944	3	31	.995	26	.944
		31	.995	2				
6	1	30	1	1	30	1	30	1
7	2	19	1	1	19	1	29	.921
		29	.921	2				
8	2	13	1	1	13	1	13	1
		45	.995	2				
9	1	49	1	1	49	1	49	1
10	2	6	.801	3	7	1	6	.801
		7	1	1				
Av. Ten Sets	1.5		0.949	1.6		0.968		0.935

These results are superior to those obtained in Simulation 12. No option inferior to rank 3 was shortlisted and, whilst the optimum was excluded in 3 cases, in no case was the rank of the best option in the shortlist below 2. Where the shortlist was extended relative to Simulation 12, it was to include a superior option and, where it was reduced, it was to exclude an inferior one. The best shortlisted option for each case was the same as, or superior to, that available in Simulation 12 and the automatic tie-break produced the same, or a superior, decision in each case.

The automatic tie-break selected the *inferior* available choice in 3 out of the 5 cases when its use was invoked.

9.6.14 Simulation 14. Configural Dora-D: simultaneous optimisation.

In this set of simulations the perspective is changed. It is assumed that a configural possibility is accepted and that the Configural Dora-D model, discussed in Chapter 7, is assumed. However the hidden value function of the "decision maker" is the multiplicative model used in simulation 13, not the modified Minkowski metric that is assumed in the analysis routine. What's Best was used in non-linear mode to find the modified Minkowski power parameter modifying the linear variables, simultaneously with finding the BPL weights. This set of simulations tests the extent to which the true optimum can be identified.

The 5x50data_2810 data was used. However, positive values are necessary to use the model and it is assumed that the 0.5 minimum attribute valuation which underlay the decision maker's hidden model previously, is recognised by the analyst, and the analysis routine, in this simulation set.

The preference indicating mechanic used is the Larichev/Franklin decomposition methodology. Criterion (a) which was the main test criterion, is used again here. An automatic tie-break method is used and this is, arbitrarily, as per Criterion (b) of the previous simulations. However it should be noted that, in this model, the "CRV" statistic does not have a comparable, or clearly interpretable, meaning. In practice it would be preferable to continue reduction using other approaches. Prior constraints were retained only:

(a) When no reduction had been secured on their introduction, and
(b) When there was no infeasibility
(c) In the terminal stages of analysis, prior to tie-break, when prior constraints were reintroduced (back to the last Larichev)

As previously, where a single set of Larichev or Franklin preferences produced infeasibility, individual constraints were eliminated preferring the elimination of preferences for lower ranking choices to higher ranking ones. Once an option returned a CRV of less than one, it was eliminated and permanently excluded from the Comparison Set.

As the Minkowski parameter tends to zero, *all* CRVs tend to one. To prevent this unhelpful condition the parameter was restricted to a lower bound of 0.2. During the analysis of Data Set 2 it became apparent that the model in conjunction with the What's Best software was not well-behaved in a variety of ways.

Behaviour was improved by the following:

- Setting the starting modified Minkowski parameter at 1.5.
- Placing an upper bound on this parameter of 3.
- Limiting maximum CRV to 2.
- Placing a lower bound on every weight of 0.001.
- Placing a lower bound on the magnitude of the most preferred attribute after its Minkowski transformation, of 0.001.

Whilst this assisted, there were further instances of failure to find a CRV greater than 1 when it was known one existed. Accordingly there may have been false eliminations.

The results obtained are shown in Table 9.21.

Table 9.21- Simulation 14: Effect of simultaneous configural optimisation with multiplicative valuation.

Data Set	No. of Options before Tie-break	Options	NTV of Options before Tie-break	True Rank of Options	Best Option Criterion (a)	NTV of Option	Best Option Criterion (b)	NTV of Option
1	2	16	1	1	16	1	16	1
		39	.704	4				
2	3	12	.885	2	22	1	12	.885
		22	1	1				
		23	.718	8				
3	1	33	.908	4	33	.908	33	.908
4	2	11	.661	12	43	1	43	1
		43	1	1				
5	1	31	.995	2	31	.995	31	.995
6	3	30	1	1	30	1	30	1
		36	.809	4				
		37	.826	3				
7	1	29	.921	2	29	.921	29	.921
8	3	1	.991	3	45	.995	45	.995
		30	.577	22				
		45	.995	2				
9	1	49	1	1	49	1	49	1
10	1	7	1	1	7	1	7	1
Av. Ten Sets	1.8		0.911	3.4		0.982		0.970

The performance of this implementation of the configural model, albeit applied to a structurally different configural situation, is inferior to that obtained on the assumption that valuation is linear using the constraint elimination approach.

The method is complicated, computationally slow, unreliable and unnecessary. It has no advantages relative to the Conservative Fixed Parameter formulation considered in Simulation 15.

9.6.15 Simulation 15. Configural Dora-D: Conservative Fixed Parameter formulation.

The methodology for Simulation 14 was predicated on the implicit assumption that there was an upper and lower bound of the modified Minkowski parameter, for which a potentially optimal option remained potentially optimal. The assumption is a false one. As has been discussed in Chapter 7, following that simulation, it was established that, whilst for most options (but not all) there may be an upper bound on the parameter for an option to remain efficient, an option which is efficient for a particular Modified Minkowski parameter is efficient for *all* smaller parameters. In other words, if an option is efficient under certain configural conditions, it will remain optimal under less disjunctive (or more conjunctive) configurations. The implication is that those options which are not efficient for the realistic minimum of the parameter will not be efficient at higher levels. A conservative shortlist of potentially optimal options can thus be established using the lowest reasonable Modified Minkowski parameter. No genuine additional potential optima are to be found by allowing the parameter to float.

Reduction can be achieved using preference indicating mechanics just as with the linear model, though mechanics involving FDC are likely to be more reliable as the meaning of the transformed attribute variables may otherwise distort elicitation.

Accordingly in this simulation the same Franklin/Larichev approach was again adopted. The same methodology was used for introducing constraints. However, in this instance the Modified Minkowski parameter was fixed at 0.2.

The results obtained are as shown in Table 9.22.

Table 9.22- Simulation 15: Effect of Conservative Fixed Parameter configural optimisation with multiplicative valuation.

Data Set	No. of Options before Tie-break	Options	NTV of Options before Tie-break	True Rank of Options	Best Option Criterion (a)	NTV of Option	Best Option Criterion (b)	NTV of Option
1	1	16	1	1	16	1	16	1
2	1	22	1	1	22	1	22	1
3	2	10	.757	8	24	.949	24	.949
		24	.949	2				
4	1	43	1	1	43	1	43	1
5	2	1	.898	5	31	.995	31	.995
		31	.995	2				
6	2	30	1	1	30	1	30	1
		36	.809	4				
7	3	19	1	1	19	1	29	.921
		22	.862	3				
		29	.921	2				
8	2	1	.991	3	13	1	1	.991
		13	1	1				
9	2	29	.912	4	49	1	49	1
		49	1	1				
10	3	6	.801	3	7	1	6	.801
		7	1	1				
		13	.750	10				
Av. Ten Sets	1.9		0.943	2.5		0.994		0.966

Although, in principle, though sometimes not in practice, the Larichev cycles of the analysis should have produced identical results to previously, the Franklin cycles need not. This is because the CRVs found would be calculated on different principles, with no promise that they would give identical rankings.

Generally, the results obtained by this approach were superior to those of the previous method, particularly in its ability to retain the first or second rank solutions in the potentially optimal set prior to tie-break. It is doubtful that the differences would be statistically significant or material in decision making terms. The superior validity of this method rests on a more methodological foundation. More interesting is the relative performance of this model to that of ignoring configurality altogether and assuming a linear model. There is a superficial advantage in the Configural model but it is by no means marked and one would have to perform many more simulations to form a view of its statistical significance.

In two out of ten of these, the true optimum was not within the reduced set before tie-break. In neither case did the best of the options left, lie outside the range of realistic value discrimination, but the optima's exclusion is nevertheless curious. This modest failure seems at variance with the apparent success, already discussed, of the Modified Minkowski metric in approximating (in strategic equivalence terms, with some precision) a bi-variable cross product term. A fuller investigation into the performance of the approximation would be instructive.

Philosophic, rather than the measured performance considerations illustrated in these simulations, govern appropriate treatment for real problems. If there is no reason to suppose configural valuation, the adoption of a pure linear model is reasonable until it is counter-indicated (perhaps by the expression of preferences which are inconsistent with linear behaviour but would be possible within a conjunctive model). Equally, if configurality is suspected, the general configural model might be assumed reverting to the linear model on grounds of parsimony, if this is equally consistent with all declared preferences at the conclusion. If configurality is explicit it should be modelled in structure and parameterisation to reflect the decision maker's declared intent as closely as possible.

If a specific configural structure is suspected, the use of the generalised configural model would not be the best approach. If, for example, specific interaction terms can be identified, it may be more appropriate to use the Supplementary Variables or Multiple Parallel Models methodologies. Both these call for specific prior suggestions regarding the form of configurality. These methods have not yet been tested for relative superiority. But one can assert that if the value model does not mis-specify the structure of the decision maker's valuations, then the optimum *cannot* be excluded before a tie-break stage. This, of course, also applies if the "real" model is Minkowski.

9.6.16 Simulation 16. Configural Dora-D: Relationship of number of efficient options to configurality.

In this set of analyses, the number of efficient options occurring for different levels of the Modified Minkowski parameter in the General Configural Model, is tested. The data used was as employed in Simulations 12-15. Parameters of 0.2, 0.5, 1.0, 1.5, and 2 were tested. The test embraced Initial Option Reduction only, ie it excluded preference constraints.

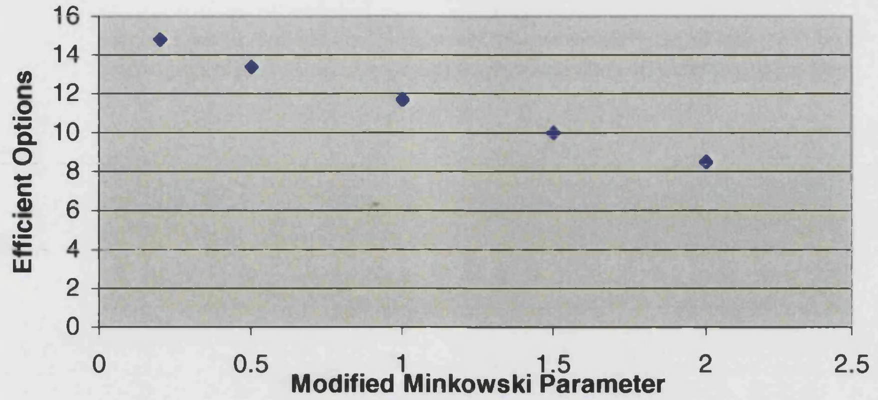
The results obtained are summarised in Table 9.23 and Figure 9.18.

The simulations confirmed my prior observation that all efficient options for higher parameters are within the set of options for lower parameters; for example the 8 efficient options for data set 2, parameter 2.0 are included within the 14 efficient options for parameter 0.2.

Table 9.23 - Variation of number of Efficient Options with configurality parameter.

No. of Efficient Options		Modified Minkowski Parameter				
		0.2	0.5	1.0	1.5	2.0
Data Set	1	11	10	10	10	10
	2	14	12	10	8	8
	3	15	14	13	13	8
	4	14	12	11	10	7
	5	16	14	13	11	10
	6	15	14	10	8	7
	7	14	14	13	10	9
	8	13	11	10	9	7
	9	17	15	14	9	8
	10	19	18	13	12	11
	Average	14.8	13.4	11.7	10.0	8.5

Figure 9.18- Relationship of no. of efficient options to configurality



The results illustrate remarkable insensitivity. There is a power of ten diference in impact between a parameter of 0.2 and 2.0; they are quite extreme parameters. Thus, a relative contribution to value of 0.8 with a configurality of 0.2, becomes a relative contribution of 0.1 at configurality of 2.0. Yet this reduces efficient options by an average of 6.3- rather less than 50%

Taking an information view, Initial Option Reduction provides approximately 1.8 bits of the 5.6 necessary to resolve the optimum but configurality contributes a maximum of 0.8, even for this extreme range.

9.6.17 Simulation 17. Testing Frontier Probing

In this set of analyses, I consider the portfolio extension. I examine the ability of Portfolio Dora-D to find an Efficient Peer of a Reference Portfolio, by finding and inserting explicit frontier constraints when implicit constraints are violated. This procedure was outlined in Chapter 6.

The analyses are again based on the data set 5x50data_2810. In this instance each of the 50 rows is considered to be a sub-option or potential portfolio component. Attributes 2 to 5 are considered to be additive and independent attributes and correspond directly to variables 2-5 in the data sets. Variable1 of the portfolio component variables is used to generate a single interdependent portfolio level and portfolio descriptive attribute. This attribute is defined as follows:

$$B_p = \sqrt{\sum_{\text{all } h} f_{ph}^2 b_h} \tag{9.5}$$

Where f_{ph} = proportion of potential portfolio component h in portfolio p

b_h = parameter governing dependence of Portfolio attribute B_h on component h.

In the simulations b_h was assumed to be the square of each element of variable 1 of each data set. Thus B_p may be thought of as representing the standard deviation of an unknown random variable associated with the portfolio, where the absolute value of b_h is the standard deviation of mutually independent random variables for the components. This variable was assumed to be negatively related to value, whilst all other variables were assumed to be positively related.

Only Initial Stage Reduction was considered (ie no value constraining preferences were introduced). Up to 40 Virtual Frontier Constraints were made specific in each simulation, introducing 1 constraint per cycle in the manner described in Chapter 6, based on finding the most violating portfolio. The MCA of each such portfolio was found but recorded only for every fourth cycle. The analyses here were based on Reference Portfolios, which for each data set were defined as being the first 25 potential components in equal proportions. The nominal MCA for the Reference Portfolio was also calculated. The reader should note that this statistic is always overestimated (and the nominal MCA divided by the MCA of the most violating portfolio always underestimates the statistic), until the MCA for the most violating portfolio is close to 1.

For numerical reasons, I imposed that the MCA of any portfolio and the contribution to MCA of Attributes 2 to 4 should be less than or equal to 10 at every stage.

Experiments were also conducted using Data Set 1 in which, respectively, 5 and 50 equal components test portfolios were used, instead of the 25 standard. The results obtained are not tabulated here.

The results obtained for the 25 component reference portfolios are illustrated in Table 9.24. In all ten cases, there was suitable convergence on a Peer Portfolio, with adequate solutions (violation of less than 5%) generally being obtained in under 16 cycles. Very nearly optimum solutions (violations of less than 0.05%) were typically obtained in under 24 cycles, by when the MCA of the Reference Portfolio was invariably accurately defined. Although not observable in this data, cycle by cycle results, when these were individually recorded, confirm that the measured violation of the most violating portfolio, did not invariably diminish on each cycle, although converging towards 1. For this number of attributes and potential components, an adequate Peer of the Reference Portfolio was typically developed after the identification and insertion of 16 violating constraints, and a good estimate of the MCA of the Reference after the insertion of 20

It will be seen that the number of components in each Efficient Peer Portfolio varied. Whilst sometimes single component portfolios were generated, all efficient Peers had less elements than the Reference Portfolio, with this data and portfolio attribute definition.

Table 9.24- Illustrating convergence of MCA for last violating option and valuation of reference portfolio.

	Data Set	1		2		3		4		5		6		7		8		9		10		Total	
	MCA of	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio	Last Violating	Ref Portfolio
Cycle	4	3.47	1.058	4.51	1.280	7.95	2.80	5.60	1.54	3.19	1.12	2.93	0.978	1.88	0.989	9.22	4.55	2.09	0.791	3.17	0.919	4.40	1.6025
	8	1.64	0.531	1.48	0.722	1.15	0.561	1.39	0.808	1.76	0.772	1.17	0.553	1.65	0.778	1.31	0.677	1.17	0.566	1.054	0.567	1.3774	0.6535
	12	1.071	0.440	1.089	0.568	1.028	0.537	1.044	0.616	1.17	0.656	1.005	0.512	1.017	0.722	1.013	0.581	1.065	0.520	1	0.557	1.0502	0.5709
	16	1.019	0.428	1.026	0.546	1.007	0.531	1.015	0.605	1.037	0.636	1	0.511	1.003	0.720	1.003	0.577	1.019	0.504			1.0129	0.5615
	20	1.009	0.425	1.005	0.538	1.004	0.529	1.006	0.604	1.006	0.631			1.001	0.719	1.001	0.576	1.010	0.503			1.0042	0.5593
	24	1.002	0.424	1.002	0.538	1.001	0.529	1.002	0.603	1.002	0.628			1.000	0.718	1.001	0.576	1.003	0.502			1.0013	0.5586
	28	1.002	0.424	1	0.538	1.000	0.529	1.001	0.602	1.002	0.628			1.000	0.718	1.000	0.576	1.001	0.502			1.00061	0.5585
	32	1.000	0.424			1.000	0.529	1.000	0.602	1.001	0.627					1.000	0.576	1.001	0.502			1.0002	0.5584
	36	1.000	0.424						0.602	1.000	0.627							1.000	0.501			1	0.5583
	40																	1.000	0.501			1	0.5583
Components in Efficient Peer Portfolio		8		1		5		7		8		1		8		6		7		1			

9.7 Summary and Conclusions

All the preference elicitation mechanics investigated for the linear Basic Model performed satisfactorily, in terms of achieving reliable convergence towards a single true optimum, or an adequate short-list containing it, at reasonable speed. Calculated MCAs progressively migrated to a straight line when plotted against true value, implying a progressive development of correct linear orders. Except with tautological elicitation (eg "How do you rank the remaining two options"), one could not necessarily reduce beyond "on a facet" options, but it appears that one can usually reduce to at least this extent.

Generally the expected cognitive reliability of a method would appear a more critical issue for choosing between mechanics than its technical efficiency within the Dora-D framework. Unsurprisingly, focused comparisons appeared markedly superior to random ones. The procedure for slotting an option into an existing ranking seemed somewhat less efficient, per equivalent preference, than generating a pair of focused and then re-computing, but the concept is far more efficient per computational pass. There seemed to be a very small gain from looking for the most over-valued efficient option, compared with seeking the most over-valued option. However, again the argument should not be resolved by technical efficiency considerations. It seems sensible to concentrate one's scrutiny on options that could be optimal, rather than to determine optimality on the basis of definitely non-optimal choices. It also means examining a smaller set.

It is clear that the simple ranking of attribute weights constitutes a potent mechanic. This is recommended for early use, whenever practicable.

The simulations here demonstrate the performance and information gain that can be achieved by capping, but the efficacy of the method is dependent entirely on the competence of decision maker (or possibly his or her confidence in the particular situation) to make the necessary judgements. The expressed terms of the Capping simulations do not fully stylise real situations. In the simulation a "true weight" and optimum exist and it is a question of the degree to which a corresponding optimum is revealed by an upper limit. In real elicitation a "true limit" is revealed by a capping statement, devoid of a "Capping Level" label (it is not an estimate of the limit of error of a mean estimate expressed with a particular degree of confidence) and the issue is how good this is at revealing optima, compared with eliciting "true weights" by other means. The simulations confirms that reduction-useful information is provided by such statements. I suspect the rest is personal.

It is clear that, of the decomposition mechanics explored, Supplemented Larichev (simulation set 9) is superior to [1,1] Franklin ab initio, and this is recommended of these two approaches. It is also clear that Supplemented Larichev is superior to the two, roughly comparable, "whole efficient option" mechanics explored in the simulation sets 5 and 6 on a *per pass* basis. On an *equivalent binary preference* basis over the limited intermediate range where they overlap, 9 and 6 appear of roughly comparable power. However, these are not being compared on a strict like for like basis. Were Larichev vectors to be compared as binary preferences (rather than ordered within a group), with a reselection of vectors made after each comparison (as in simulation 6), an improvement in *per equivalent binary preference* of Larichev should be expected. (It is proposed to examine this in follow-up work). In any event, the psychological reliability of [1,1] comparisons over [m,n] puts the issue of overall superiority of Larichev to "whole efficient option" comparisons, beyond doubt.

Established non-optimal options retained in the Comparison Set are redundant in a strictly linear model. However, the simulations suggest that if non-linearities exist, or if a model is misspecified, their inclusion can contribute to distorting results. Accordingly if there is any question that this may be the situation, options should be removed from the Comparison Set once they are established as non-optimal. This is anyway justifiable on the grounds of "independence from irrelevant alternatives" alone. It also seems that it is helpful to remove preference constraints once they have served their purpose in eliminating one or more options, in the same type of situations. With these precautions, and assuming there is no cavalier disregard of suspected major non-linearities, a linear mis-specification of a non-linear model may not necessarily be damaging and a "good" non-optimal option may be suggested even if the optimum is missed.

In cases of configurality of non-specific or unknown form, the Modified Minkowski Model is useful. The Conservative Fixed Parameter approach to finding the optimum within Dora-D is recommended. This guarantees (if genuinely of Minkowski form) the inclusion of the optimum in the efficient set following Initial Option Reduction.

The expression of Larichev preferences, which can, and should, still be presented to the decision maker in un-transformed form, will then secure reduction to a set, which again, is guaranteed to include the optimum. If the parameter is too conservative, more potential optima than necessary may remain. Accordingly, if no other reduction is possible, further elimination may be achieved by increasing the Minkowski parameter.

The insensitivity of the Modified Minkowski Model to parameter variations suggests that if the extent of configurality is unknown, but thought to be modest, it may do no great harm to ignore it all together. This would particularly commend itself if disjunctive circumstances apply.

The Frontier Probing technique works effectively and computationally manageably in finding efficient peers, MCAs, and CAFs for test portfolios.

It is believed that the Information measure, based on equal likelihood of efficient options used in these analyses, is conservative. If a reliable relationship between the MCA(AP) and probability of being optimum can be found, as seems possible, early reductions of all methods would be measured as providing greater information.

Chapter 10 Round-up

10.1 General observations

Others might have been tempted to entitle the ultimate Chapter, "Conclusions". I can make no such definite claims. Rather, I feel like a painter who has sketched out an outline with some parts of his work being supported by drawings, parts of the canvas having crude outlines and some parts still empty. In all areas, refinement is necessary.

I am also conscious that I have worked vulnerably at the interfaces of many areas where I have lacked specific training and long experience. I have not pretended to emulate the depth of knowledge and understanding of those working in the mainstreams of those areas. I have borrowed bits of Empirical Psychology, Evolutionary Psychology, Decision Theory, Data Envelopment Analysis, Mathematics, Mathematical Programming, Statistics, Modern Portfolio Theory etc. In using them I have abstracted their simplest and most accessible ideas. Yet in doing so, I think I have generated potentially important insights, and have remained true to the Operational Research tradition: or at least to the traditions of its founding fathers.

To me, the two most potentially potent ideas that come from Data Envelopment Analysis are the concepts of inductive analysis using "Best Possible Light", and the closely related notion of normalising weights or parameters of descriptive function, not by their sums, but by their aggregate contributions. Both these lie at the core of Dora-D and I hope that I have illustrated their use in a wide-ranging collection of problem structures. It may be that other areas of OR analysis could exploit them, as they seem, notwithstanding their simplicity, to be powerful concepts. It seems a pity that DEA seems to have de-emphasised the BPL aspect by relegating the *multiplier form* to secondary status, preferring to give prominence to the *envelopment form*, which now, though not originally, is usually designated the Primal. I am not aware of anyone else reflecting that the "upper bounding" used in DEA, and here with Dora-D, is a form of weight normalisation but this seems to be what it does; and far more revealingly than delimiting weights by ensuring that they add to one, to which no interpretative meaning can be attached. This parallel is illustrated within the thesis by formulating and solving MOLPs, without using traditional weight normalisation, within the Dora-D framework. This is successful at least to the extent of providing an interactive search framework to find the preferred efficient solution.

Originally, it had seemed to me that man (more specifically, I) was not adept at handling objectives, except within a concept of Vagueness. which I have attempted to describe. It was not, as March suggests, only preferences that we would seem to construct, but Objectives as well. The Dora-D structure accommodates this concept well. It allows the use of a flexible range of latitude reducing and preference indicating mechanics, including delimiting methods such as Capping Weights (which I still consider a cognitively reliable concept), using various forms of decomposed preference, and even directly comparing real options of one form or another (which I would not now commend as a reliable elicitation method).

The quest for a less arbitrary basis for cognitive assumptions led, not only to consideration of empirical psychological findings and views, but the use of EP as a standardising criterion, or touchstone. My position was that, however powerful the human mind's facility for tackling difficult unbounded problems and identifying issues of importance, it is not capable (or, we should not assume that most are capable) of doing many of those things that we might think it should, in order to prime the models that the modern analyst builds to assist modern economic man. This went beyond my suspicion of Objectives. Man seems an efficient identifier of factors, but in analysis was a Comparator not a Calculator. Notwithstanding apparent holistic skills, he is not adept at precise detailed multidimensional comparison. His values are labile and, just as with sensory phenomena, he can only discriminate value cues with low "resolution", I suggested. I developed minimalist assumptions and have attempted to illustrate that powerful conclusions can be reached notwithstanding, and that that this can be done within the Dora-D framework. I believe some of these assumptions could be useful to other modellers. Cognitive limitations, perhaps, have been underplayed by other methodology designers, though I except from this Larichev and his collaborators. Even if I have not made my case, the minimalist perspective is safer, and conclusions that can nevertheless be drawn within its context are, by any standard, more powerful.

The notion of decomposition of decision problems is well understood by decision analysts, psychologists and thinking managers, even if it is sometimes left implicit;

though a more specific recognition of this process, aided, perhaps, by the classification I have suggested, might aid analysts in simplifying the resolution of some problems. The heuristic I have suggested for partitioning preferences of higher order to choices of lower order could be viewed as trivial, it is no more than the application of the principle of preferential independence, but its systematic application in what I have called Franklin Decomposition may be a practically useful modern development of Franklin's 250 year old suggestion. Franklin was not phased by [3,2] comparisons but Larichev pointed out the better psychological reliability of [1,1] comparisons. I have perceived such choices, what I have called Fundamentally Decomposed Preference, as the most basic preference building blocks and believe more general use can be made of the idea. I have illustrated how such choices can be generated in a manner which ensures that they are potent indicators, and then used within Dora-D to shrink valuation latitude; in some modes, without risking formulation infeasibility. However, I see no reason why the same concept cannot be employed within other solution methods.

I have speculated that designers of decision analysis techniques, sometimes, implicitly, may have approached value discovery as if the decision maker is a complicated inanimate black box, like many other systems that the OR discipline examines. We may ask him about certain things but, somehow, we do not expect him to know about the structure of his own values. Accordingly, the implication is that we must discover these in the same types of way as we might analyse an inanimate production system. I suggest that other analysts may find the notions of Conscious Process and Qualified Self Awareness, which I discuss, useful: specifically that, whilst some things may be elicited, it is not constructive, and may be misleading, to seek beyond a decision maker's informed and conscious statements. I have suggested that the fine structure and form of valuation falls into this class. Unless a decision maker asserts at least the possibility of, say, a particular non-linear or interactive structure, it is not merely unwise or unnecessary to delve for unconscious structure, but, perhaps, impertinent too. It smacks of accessing the opinions of a homunculus super-sovereign, within the mind of the decision maker, of whom he or she is unaware but to whom he or she is subordinate, and who talks only to the analyst.

The implication of this view is that we can be far simpler; matters of value should generally be transformable readily into linear relationships on the basis of straightforward expressions of intent by the decision maker. (However, the reader will recall that I cautioned that this is distinct from knowledge of the fine structure of pertinent *physical* relationships outwith the mind, about which even a well-informed decision maker may be ignorant). In any case, the evidence is that judges however introspectively sophisticated they seek to be, tend to be inferior to simple linear models of their behaviour. If it is true of a well trod judgement situation it should be even more true of a typical decision one. Thus whilst cognitive limitations may sometimes make life more difficult, this aspect of cognitive imprecision, which recognises a fruitless journey for what it is, makes it easier.

Similarly, "low resolution" of multidimensional value balance, can simplify the analysts task practically. I suggest that it is important to discriminate between the undiscerned and the indistinguishable. This is not, I believe, an issue of getting a better microscope. At a certain point on a delineated scale, which is coarse relative to the precision of the scales measuring the attributes contributing to it, our conscious sovereign decision maker is not able to tell in a sense which is independent of expression. A distinction does not exist. Though we cannot describe this as indifference in its classical sense, pragmatically it may be appropriate to treat it as such. The analyst may exploit this, but pursues the illusion of refinement at his or her peril. The attempt to use this property to generate archetypical options in the infinite option portfolio model, proved too demanding for a realistic condensation to a manageable number of "don't bother to look outside this list" options. However, the use of a coarser resolution, proved the principle of the method adopted and it would certainly be useful if one wished to provide a "material short-list" from a large but finite number of discrete efficient solutions, in a problem with the Basic Dora-D structure.

I gave the issue of Configurality some attention, as an interesting case in which a decision maker may make a conscious assertion concerning interaction between attributes. I commended the use of what I called the General Configural Model. This had two interesting properties. It could first be used to enabled transformation into a linear function which, whilst not being "proportional" to value, was strategically equivalent to such a scale. (It is actually quite difficult to explain what one means by a "scale of linear value" when non-specific configurality is involved). Additionally, it has the interesting property that all options which are efficient for a particular configurality, are efficient for all less disjunctive (or more conjunctive) configurality. This is a powerful conclusion, allowing a conservative view to be taken, whilst still working simply within the confines of the approach developed, and is perhaps exploitable elsewhere.

In my Portfolio Analysis case, I took a far more complex view of Return than is typical within established Portfolio Theory, whilst taking a simpler view of the stochastic aspects of risk. This was done with my Decision Maker's eyes open, because of data availability and manageability. It seems that residual variance tends to affects the proportions of an investment within a portfolio, but not generally whether the investment is above or below an inclusion threshold. The precise proportions are not very value sensitive, so I was not distressed by my crudity on this issue, given that I had incorporated Beta, by generating portfolios of imposed standard Beta. The ability to treat the potential components of return multi-dimensionally, seemed much more critical, feeling as I have suggested, that this fundamental issue is begged within MPT. (Once I have a reliable vector measure of Expected Returns, I will not need to worry too much about the issues of turning that information into portfolios, even with over-simplification of uncertainty). But we are in any event talking about an analytic framework, not my view of analytic treatment of a particular problem at one moment. The full panoply of MPT risk could, if an investment analyst wished it, be incorporated in a Dora-D portfolio model and analysed with the Frontier Probing algorithm.

10.2 Potentially publishable material

The basic method has already been published. The following lists concepts, ideas and issues covered in this thesis, which I suggest could be suitable for publication, or developed for publication.

1	Extension of Basic Dora-D to infinite option problems and Frontier Probing
2	Application of Dora-D to Financial Portfolio construction
3	Application of Dora-D to Research Portfolio construction: including use of the Reference Complements.
4	The General Configural Model, and the relationship of the membership of efficient option sets to conjunctive and disjunctive intensity. Working with linear decision models.
5	Formulating and evaluating MOLP structured problems using BPL principles
6	Shortlisting options by archetyping.
7	Choice as a Value Statement. Generating an optimum from a Holistic selection.
8	Revisiting Franklin. Classifying choice complexity and decomposing options into lower order choices. Specification of a modern algorithm based on Comparative Preference.
9	Generating potent Fundamentally Decomposed Choices for Dora-D and MOLP problem solving.
10	Normative Rationality for a Comparator. An operational criterion based on assumptions of minimalist and non-cardinal cognition assumptions.
11	Decision Making and Value concepts from an EP perspective. Generating minimalist mental facility assumptions.

10.3 Issues for further research

I have already indicated that I believe BPL-weight normalisation by contribution has more to offer and I have discussed some possibilities. Other areas that I have not touched upon is the feasibility of using the idea in the solution of Interval (Single) Objective Function problems and I believe a method akin to Frontier Probing could achieve this. I speculate whether it might also be used in a manner akin to some forms of Statistical procedure, such as Principle Components Analysis.

I believe that it might be possible to integrate the Dora-D methodology, with methods which express degree of preference in semantic expressions.

I feel confident that it would be relatively straightforward to modify Dora-D formulations to exploit "proximity" as a way to generating an option ranking, in similar style to, for example, Rivett (1977); albeit using a technique of very different character. It would be useful to pursue this. This could involve ordering of confidence of statements of closeness and would probably require an interval minimisation optimisation. Of additional interest, is the exploitable reliability of statements of the form "A is close in value to B" compared with "I prefer A to B". A combination approach should not be ruled-out. Indeed the tie-breaker suggestions made for the Basic Method might be construed as constituting this type of process.

There appears to be some relationship between the magnitude of the MCA(AP) of an efficient option and the likelihood of its optimality. It would be useful to research this further. This would enable a more sophisticated Information model than the present assumption based on prior equal likelihood.

Further empirical simulations to test the discriminatory efficacy of binary preference between [1,1] Larichev vectors, compared with the power of binary preference between [m,n] "efficient whole options", could be undertaken.

The statistical inter-relationships between assumptions of lack of discrimination of value expressed in information terms and the uncertainty of weights, and measures of composite value derived from multidimensional attributes, could also usefully be addressed. This might enable a firmer suggestion regarding appropriate thresholds of effective indifference, particularly if associated with the work suggested in the following paragraph. In the related area, further work could be done on methods for developing option archetypes.

I had felt constrained to make assumptions about some issues that, at least in principle, could be subjected to empirical testing. Could one design a method to properly measure the lability of value, and the ability or otherwise of a decision maker to discriminate multi-dimensional value, in a way that would enable numbers to be put on it, as Miller originally suggested for sensory inputs? A special feature that distinguishes recognition of composite value from the type phenomena considered by Miller, is the very fine discriminatory power that exists for single attributes because of their "no brain" character, which disappears as soon as one expands the process to two variables.

Even more untested is the "meaningfulness" of multiple objectives as useable intuitive cardinal concepts guiding action. With Objectives a central thread of much practical strategy formation (or is it promulgation?), this would seem an empirical research area of much potential importance.

Whilst I have not majored in this thesis on consideration of the impact of probability on decisions, I have suggested that Comparator-man should not be adept at cardinal probability. Work has been done on "encoding of frequency" but, though recognising that my knowledge of the specialist literature is limited, I am not aware of work which comments on the distinction between comparative likelihood and cardinal assessment and, given that the former can be confused with the latter, seeks to distinguish whether it is comparative frequency rather than a cardinal estimate that is actually encoded. This too would be topic on which empirical psychologists could assist decision analysts.

Finally, I would like experienced Evolutionary Psychologists to pursue some of the ideas relating to functional decision-related mental skills, which I have attempted to outline in this thesis, but that they seem not yet to have addressed.

Appendix A

Letter from Benjamin Franklin to Dr Joseph Priestly on Moral or Prudential Algebra

TO DR. PRIESTLEY.

LONDON, 19 September, 1772.

DEAR SIR :— In the affair of so much importance to you, wherein you ask my advice, I cannot, for want of sufficient premises, advise you *what* to determine; but, if you please, I will tell you *how.* When these difficult cases occur, they are difficult, chiefly because, while we have them under consideration, all the reasons *pro* and *con* are not present to the mind at the same time; but sometimes one set present themselves, and at other times another, the first being out of sight. Hence the various purposes or inclinations that alternately prevail, and the uncertainty that perplexes us.

To get over this, my way is, to divide half a sheet of paper by a line into two columns; writing over the one *pro,* and over the other *con;* then during three or four days' consideration, I put down under the different heads short hints of the different motives, that at different times occur to me, *for* or *against* the measure. When I have thus got them all together in one view, I endeavour to estimate their respective weights; and, where I find two (one on each side) that seem equal, I strike them both out. If I find a reason *pro* equal to some two reasons *con,* I strike out the three. If I judge some two reasons *con,* equal to some three reasons *pro,* I strike out the five; and thus proceeding I find at length where the balance lies; and if, after a day or two of farther consideration, nothing new that is of importance occurs on either side, I come to a determination accordingly. And though the weight of reasons cannot be taken with the precision of algebraic quantities, yet, when each is thus considered separately and comparatively, and the whole lies before me, I think I can judge better, and am less likely to make a rash step; and in fact I have found great advantage from this kind of equation, in what may be called *moral* or *prudential algebra.*

Wishing sincerely that you may determine for the best, I am ever, my dear friend, yours most affectionately,

B. FRANKLIN.

From *The complete works of Benjamin Franklin* (Bigelow, J) 1887 Putnam: New York 4 522-524.

References

Allen R, Athanassopoulos A, Dyson RG and Thanassoulis E (1997) Weights restrictions and value judgements in data envelopment analysis: Evolution, development and future directions. Ann Oper Res **73** 13-34

Alloy LB and Abramson LY (1979) Judgement of contingency in depressed and non-depressed students: Sadder but wiser? J Exp Psychol Gen **108** 441-485

P Andersen and NC Petersen (1993) A Procedure for Ranking Efficient Units in Data Envelopment Analysis. Man Sci **39** (10) 1261-1264

Arrow KJ (1951) Social choice and individual values. Wiley: New York, NY

Axlerod R (1976) Structure of Decision: The Cognitive Maps of Political Elites. Princeton University Press: Princeton NJ

Baron J (1988) Thinking and Deciding. CUP: Cambridge

Barron FH and Barrett BE (1996) Decision quality using ranked attribute weights. Man Sci **42** (11) 1515-1523

Barsalou L (1992) Cognitive Psychology: An overview for cognitive scientists. Eribaum: Hllsdale,NJ

Bazerman MH (1986) Judgement in Managerial Decision Making. Wiley

DE Bell (1977) A Decision Analysis of Objectives for a Forest Pest Control Problem. In *Conflicting Objectives in Decisions* (DE Bell, RL Keeney, H Raiffa Eds) pp 389-421 Wiley: Chichester 1977

Belton V (1992) An IDEA- Integrating data envelopment analysis with multiple criteria analysis. In *Proceedings of the Ninth International Conference on Multiple Criteria Making : Theory and Applications in Business, Industry and Commerce* (A Goicoeche, L Duckstein, and S Zionts, Eds) 71-79 Springer-Verlag: Berlin

Berry MA, Burmeister E and McElroy MB (1988) Sorting out risks using known APT factors. Finance Analysts J March-April 29-42

Blattberg RC and Hoch SJ (1990) Database models and managerial intuition. 50% model and 50% manager. Man Sci **36** (8) 887-899

Bouyssou D (1999) Using DEA as a tool for MCDM: some remarks. J Oper Res Soc **50** 974-978

Bowers KS (1981) Knowing more than we can say leads to saying more than we can know: On being implicitly informed. In *Towards a psychology of situations* (D Magnusson, Ed) 179-194 Lawrence Eribaum: Hillsdale, NJ

Bowman EH (1963) Consistency and optimality in managerial decision making. Man Sci 9 310-321

Buchanan JT, Henig EJ and Henig MI (1998) Objectivity and subjectivity in the decision making process. Annals of Operations Research **80** 333-345

Camerer C (1981) General conditions for success of bootstrapping models. Organ Behav Hum Perf **27** 411-422

Charnes A, Cooper WW, Lewin AY, and Seiford LM (1994) Data Envelopment Analysis: Theory, Methodology and Applications. Kluwer: Norwell MA

Charnes A, Cooper WW and Rhodes E (1978) Measuring the efficiency of decision making units. Eur. J. Opn. Res **2** 429-444

Charnes A, Cooper WW , Wei QL, Huang Z (1989) Cone Ratio Data Envelopment Analysis and Multi-Objective Programming. Int J of Syst Sci **20/7** 1099-1118

Charnov EL (1976) Optimal foraging: The marginal value theorem. Theor Popul Biol **9** 129-136

Cook WD and Green RH (2000) Project prioritization: a resource-constrained data envelopment analysis approach. Socio-Economic Planning Sciences **34** 85-99

Cook WD and Kress M (1985) Ordinal ranking with intensity of preference. Man Sci **41**/1 26-32

Cook WD and Kress M (1990) A data envelopment model for aggregating preference rankings. Man Sci **36**/11 1302-1310

Cosmides L, Tooby J and Barkow JH (1992) The adapted mind : Evolutionary Psychology and the generation of culture. Oxford University Press

Cosmides L and Tooby J (1987) From evolution to behavior: Evolutionary psychology as the missing link. In *The latest on the best: Essays on evolution and optimality* (J Dupre Ed) MIT Press: Cambridge, Mass 277-306

Cosmides L and Tooby J (1996) Are humans good intuitive statisticians after all? Rethinking some conclusions of the literature of judgement under uncertainty. Cognition **58**/1 1-73

Cottle S, Murray RF and Block FE (1986) Graham and Dodd's security analysis. (5th Edition). McGraw-Hill: New York

Damasio AR (1994) Descartes' error: emotion, reason and the human brain. Avon Books: New York

Darwin C (1859 and 1998) The Origin of Species by Means of Natural Selection, or The Preservation of Favoured Races in the Struggle for Life. Wordsworth Editions Ltd (reprint of 1st Edition)

Dawkins R (1986) The blind watchmaker: Why the evidence of evolution reveals a universe without design. Norton: New York

Dawes RM (1964) Social selection based on multidimensional criteria. J of Abnormal and Social Psychology **68** 104-109

Dawes RM (1979) The robust beauty of improper linear models in decision-making. Am Psychol **34** (7) 571-582

Dawes RM and Corrigan B (1974) Linear models in decision making. Psychol Bull **81** 571-582

Dawes RM, Faust D, Meehl P (1989) Clinical versus Actuarial Judgement. Science **243** 1668-1673

Debreu G, (1959) Theory of Value. Wiley, New York

Doyle J and Green RH (1993) Data Envelopment Analysis and Multiple Criteria Decision Making. Omega Int J Manage S **21** (6) 713-715

Dyson RG and Thanassoulis E (1988) Reducing Weight Flexibility in Data Envelopment Analysis. J Opl Res Soc **41** (9) 829-835

Ebert RJ and Kruse TE (1978) Bootstrapping the Security Analyst. J Appl Psychol **63** 110-119

Edwards W and Barron FH (1994) SMARTS and SMARTER: Improved Simple Methods for Multiattribute Utility Measurement. Organ Behav Hum Dec Proc **90** 306-325

Einhorn HJ (1970) The use of nonlinear, noncompensatory models in decision making. Psychol Bull **73** 221-230

Einhorn HJ and Hogarth RM (1975) Unit weighting schemes for decision making. Organ Behav Hum Perf **13** 171-192

Elster J (1979) Ulyssees and the Sirens. Cambridge University Press

Elster J (1986) Rational Choice. Blackwell: Oxford

Elton EJ and Gruber MJ (1987) Modern portfolio theory and investment analysis. John Wiley & Sons: New York, NY

Estes W (1976) The cognitive side of probability learning. Psychol Rev **83**(2) 37-64

Evans J St B T (1983) Thinking and Reasoning: Psychological Approaches. Routledge and Kegan Paul

Evans J St B T and Over DE (1996) Rationality and reasoning. Psychology Press: Hove

Fischoff B (1991) Value elicitation: Is there anything in there? Am Psychol **46**/8 621-641

Fischoff, B, Slovic, P and Lichtenstein, S (1980) Knowing what you want; Measuring labile values. In *Cognitive Processes in Choice and Decision Behavior* (Wallsten TS, Ed) Eribaum, Hillsdale 117-141

Forgas JP, George JM (2001) Affective Influences on Judgments and Behavior in Organizations: An Information Processing Perspective. Organ Behav Hum Dec **86** (1) 3-34

Fox CR and Tversky A (1998) A Belief-Based Account of Decision under Uncertainty. Man Sci **44**(7) 879-895

Franklin B (1772) Letter to Dr Joseph Priestly of 19 September on Moral or Prudential Algebra. In *The complete works of Benjamin Franklin* (Bigelow, J) 1887 Putnam: New York **4** 522-524. Quoted in full in Appendix A.

French S (1986) Decision theory; an introduction to the mathematics of rationality, Ellis Horwood Ltd, Chichester

Gigerenzer G and Todd PM (1999) Simple Heuristics that make Us Smart. Oxford University Press: New York

Goldberg LR (1968) Simple models or simple processes? Some research on clinical judgements. Am Psychol **23** (7) 483-496

Goldberg LR (1970) Man versus model of man: A rationale, plus some evidence, for a method of improving on clinical inference. Psychol Bull **73** 422-432

Goodall J (1986) The chimpanzees of Gombe: patterns of behavior. Belknap Press of Harvard University Press: Cambridge, MA

Goodwin P and Wright G (1991) Decision Analysis for management judgment. John Wiley and Sons: Chichester

Graham B (1973) The intelligent investor (4th Edition). Harper and Row: New York

Griggs RA (1983) The role of problem content in the selection task and in the THOG problem. In *Thinking and Reasoning: Psychological Approaches* (J St B T Evans ed) Routledge and Kegan Paul:London 16-43

Hale MI (1999) Mechanisms of the mind: An evolutionary perspective. Hale-van Ruth: Pittsburgh, PA

Hallerbach W (1994) Multi-attribute Portfolio Selection: A Conceptual Framework. Erasmus University, Rotterdam thesis.

Hallerbach W, H Ning H, Soppe A, Spronk J(2003) A framework for managing a portfolio of socially responsible investments. Eur J Op Res (in publication)

Hallpike CR (1979) Foundations of Primitive Thought. Oxford University Press

Harris M (1993) Culture, People, Nature- An Introduction to General Anthropology. (6th Edition) Harper Collins

Hasher L and Zacks RT (1984) Automatic processing of fundamental information: The case of frequency of occurrence. Am Psychol 39 1372-1388

Hayes, JR (1962) Human data processing limits in decision making. Electronic Systems Division Report ESD-TDR-62-48 July
Hill K (1982) Hunting and Human Evolution. J Hum Evol **11** 521-544

Hinde RA (1995) Individuals and culture. In *Origins of the Human brain* (Changeux J-P, Chavaillon, J, eds) Clarendon, Oxford 186-195

Hoffman PJ (1960) The paramorphic representation of clinical judgement. Psychol Bull **57** 116-131

Hoffman PJ (1968) Cue-consistency and configurality in human judgment. In *Formal Representation of Human Judgement.*(Kleinmuntz B ed) John Wiley and Sons, New York 53-90

Howell WC (1973) Representation of frequency in memory. Psychol Bull **80** 44-53

Huber BL and Huber O (1987) Development of the concept of comparative subjective probability. J Exp Child Psychol **44** 304-316

Hunter IML (1966) Mental calculation. In *Thinking and Reasoning: Selected Readings.* (PC Wason and PN Johnson-Laird, eds) Penguin Books

Jones CM and Jonides J (1992) Direct coding for frequency of occurrence. J Exp Psychol; Learning , Memory and Cognition **18** 368-378

Kahneman D and Tversky A (1971) Subjective probability: A judgment of representativeness. Cognitive Psychology **3** 430-454

Kahneman D and Tversky A (1979) Prospect theory: an analysis of decision under risk. Econometrica **47** 263-291

Kahneman, D and Tversky A (1984) Choices, Values, and Frames. Am Psychol **39**(4) 341-350

Kasanen E, Wallenius H and Wallenius J (2000) A study of high-level managerial decision processes, with implications of MCDM research. Eur J Op Res **120** 496-510

Keen PGW (1977) The evolving concept of optimality. In *Multiple Criteria Decision Making.* TIMS Studies in Man Scis 6 31-57

Keeney RL and Raiffa H (1976) Decisions with multiple objectives: Preferences and value trade-offs, John Wiley and Sons, New York

Khatri N and Ng HA (2000) The role of intuition in strategic decision making. Human Relations **53(1)** 57-86

Kirkwood CW (1997) Strategic Decision Making- Multiobjective Decision Analysis with Spreadsheets. Duxbury Press, Belmont CA

Korhonen P, Moshkovich HM and Wallenius J (1992) Multiple criteria decision support- a review. Eur J Oper Res **63** 361-375

Kleinmuntz B (1990) Why we still use our heads instead of formulas: towards an integrative approach. Psychol Bull **107** 296-310

Kleinmuntz DN (1990) Decomposition and the Control of Error in Decision-Analytic Models. In *Insights in decision Making* (Hogarth RM ed) University of Chicago Press: Chicago 107-126

Krebs JR (1973) Behavioral aspects in predation . In *Perspectives in Ethology.* (PPG Bateson, PH Klopfer eds) Plenuwm, New York 73-111

Larichev OI (1992) Cognitive Validity in the Design of Decision-Aiding Techniques. Journal of Multi-Criteria Decision Analysis **1** 127-138

Larichev OI and Moshkovich HM (1995) ZAPROS-LM- A method and system for ordering multiattribute alternatives. Eur J Op Res **82** 503-521

Laughlin WS (1968) Hunting: An integrating Biobehavior System and its Evolutionary Importance. In *Man the Hunter* (RB Lee and I Devore eds) Aldine Atherton: Chicago 304-320

Lee SM (1972) Goal Programming for Decision Analysis. Auerbach, Philadelphia

Lee W (1971) Decision Theory and Human Behavior. Wiley, New York

Lindblom CE (1959) The science of muddling through. Public Administration Review 79-88

March JG (1978) Bounded rationality, ambiguity, and the engineering of choice. Bell Journal of Economics **9** 587-608

Markowitz H (1952) Portfolio Selection. Journal of Finance **7**(1) 77-91

Markowitz H (1959) Portfolio selection: efficient diversification of investments. Blackwell: Cambridge, Mass

Maslow AH (1970) Motivation and Personality. (2nd Edition) Harper & Row

Miller GA (1956) The magical number seven, plus or minus two: Some limits on our capacity for processing information. Psychological Review **63** 81-97

Mithen S (1996) The prehistory of the mind. Thames and Hudson: London

Mintzberg H, Raisingham D and Théorêt A (1976) The structure of unstructured decision processes. Admin Sci Quart **21** 246-275

Nicholson N (1997) Evolutionary Psychology: Towards a New View of Human Nature and Organizational Society. Human Relations **9** 1053-1078

Nicholson N (1998) Seven deadly syndromes of management and organization: The view from evolutionary psychology. Managerial and Decision Economics **19** 411-426

Olson DL (1996) Decision Aids for Selection Problems. Springer-Verlag: New York

Oral M, Kettani O and Lang P (1991) A methodology for collective evaluation and selection of industrial R&D projects. Man Sci **37**(7) 871-883

Pinker S (1998) How the mind works. Allen Lane, Penguin Press: London

Pinker S (1995) Facts about human language relevant to its evolution. In *Origins of the Human brain* (J Chavaillon Ed) Clarendon: Oxford

Prietula MJ and Simon HA (1989) The experts in your midst. Harvard Bus Rev **67**(1) 120-124

Rivett BHP (1977) Multidimensional scaling for multiobjective policies. Omega **5** 367-379

Roy B (1968) Classement et Choix en Presence de Vues Multiples (La methode ELECTRE). Revue d'Informatique et de Recherche Operationelle **8** 57-75

Roy B (1977) Partial Preference Analysis and Decision-Aid: the Fuzzy Outranking Relation Concept. In *Conflicting Objectives in Decisions* (DE Bell, RL Keeney, H Raiffa Eds) pp 389-421 Wiley and Sons, Chichester 1977 40-75

Saaty T (1980) The Analytic Hierarchy Process. Mc Graw-Hill:New York

Samuelson PA (1938) A note on the pure theory of consumers' behaviour. Economica **5** 61-71

Saarilouma,P (1995) Chess players' thinking: A cognitive approach. Routledge:London

Sarrico CS, Hogan SM, Dyson RG and Athanassopoulos AD (1997) Data envelopment analysis and university selection. J Opl Res Soc **48** (12) 1163-1177

Sagan C (1977) Dragons of Eden. Random House: New York

Sen (1986) Behaviour and the Concept of Preference. In *Rational Choice* (ed J Elster) Blackwell: Oxford 60-82

Shepard RN (1987) Evolution of a Mesh between Principles of the Mind and Regularities of the World. In *The latest on the best: Essays on evolution and optimality* (J Dupre Ed) MIT Press: Cambridge, Mass 251-275

Simon HA (1955) A behavioral model of rational choice. Quarterly Journal of Economics **69** 99-118

Simon HA (1957) Rationality and administrative decision making. In *Models of Man.* Wiley: New York 196-206

Simon HA (1965) Administrative Behavior. (2nd Edition) Free Press: New York

Simon HA (!978) Rationality as Process and as Product of Thought. Am Econ Rev **68**(2) 1-16

Simon, HA (1979) Rational Decision Making in Business Organizations. American Economic Review **69** 493-512

Slovic P (1969) Analyzing the expert judge: A descriptive study of a stockbroker's decision processes. J of Applied Psychology **53** 255-263

Slovic P, Fischoff B and Lichtenstein S (1982) Facts versus fears: Understanding perceived risk. In *Judgment under uncertainty: heuristics and biases.* (D Kahneman, P Slovic and A. Tversky Eds) Cambridge University Press 463-489

Smith EA (1983) Anthropolological applications of Optimal Foraging Theory: A critical review. Curr Anthropol **24** 625-651

Smith P (1990) Data Envelopment Analysis applied to financial statements. Omega Int J Manage S **18**(2) 131-138

Spronk J and Hallerbach W (1997) Financial modelling: Where to go? With an illustration for portfolio management. Eur J Opn Res **99** pp 113-125

Standing L (1973) Learning 10000 pictures. Q J Exp Psychol **25** 207-222

Steuer RE (1977) An Interactive Multiple Objective Linear Programming Procedure. In *Multiple Criteria Decision Making.*TIMS Studies in Man Scis **6** 225-239

Stewart TJ (1992) A Critical Survey on the Status of Multiple Criteria Decision Making Theory and Practice. Omega Int J Manage S **20** 569-586

Stewart TJ (1996) Relationships between Data Envelopment Analysis and Multi-criteria Decision Analysis. J Opl Res Soc **47**(5) 654-665

Sutherland S (1992) Irrationality: The Enemy Within. Constable: London

Sutton PP (1999) Making Firm Decisions with Vague Objectives -Exploring a New Multiple Attribute Evaluation Technique. (Unpublished University of Bath transfer paper).

Sutton PP and Green RH (2002) A data envelopment approach to decision analysis. J Opl Res Soc **53** 1215-1224

Taylor RN (1975) Psychological Determinants of Bounded Rationality: Implications for Decision-Making Strategies. Decision Sciences **6** 409-429

The Times, The Good University Guide, 14 April 2000

Thompson RG, Langemeier LN, Lee CT, and Thrall RM (1990) The Role of Multiplier Bounds in Efficiency Analysis with Application to Kansas Farming. J of Econometrics **46**(1/2) 93-108

Tooby J and Cosmides L (1992) The Psychological Foundations of Culture. In *The adapted mind: Evolutionary Psychology and the generation of culture* (JH Barkow, L Cosmides and J Tooby Eds) Oxford University Press 19-136

Tversky A and Kahneman D (1971) Belief in the Law of Small Numbers. Psychol Bull **2** 105-10
Tversky A and Kahneman D (1974) Judgement under uncertainty: Heuristics and Biases. Science 185 1124-1131

Vaihinger H (1935) The Philosophy of "As if". Routledge and Kegan Paul: London

Wainer H (1976) Estimating coefficients in linear models: It don't make nevermind. Psychol Bull **83** 312-317

Washburn SL and Lancaster CS (1968) The Evolution of Hunting. In *Man the Hunter* (RB Lee and I Devore eds) Aldine Atherton: Chicago 293-303

von Winterfeldt D and Edwards W (1986) Decision Analysis and Behavioral Research. Cambridge University Press

Wong Y-H B and Beasley JE (1990) Restricting Weight Flexibility in Data Envelopment Analysis. J Opl Res Soc **41**(9) 829-835

Wright WF and Bower GH (1992) Mood effects on subjective probability assessment. Organ Behav Hum Dec Proc **52** 276-291

Wright G and Goodwin P (1999) Rethinking value elicitation for personal consequential decisions. J Mult-Crit Decis Anal **8** 3-10

Yntema DB, Torgerson WS (1961) Man-computer co-operation in decisions requiring common sense. IRE Trans Human Factors in Electronics **HFE-2** (1) 20-26

Zeleney M (1981) Multiple Criteria Decision Making. McGraw-Hill: New York

Zionts S (1981) A multiple criteria method for choosing among discrete alternatives. Eur J Oper Res **7**(1) 143-147